Submission code: 8BAL5NV7DJ

See this and other abstracts on our website: https://home.sbgf.org.br/Pages/resumos.php

UNO-Based Estimation of Upgoing Green's Functions in Marchenko Imaging

Victor Koehne (SENAI CIMATEC), Daniel Revelo Apraez (SENAI CIMATEC), Caio Leão (SENAI CIMATEC), Reynam C. Pestana (UFBA), Diego Barrera (SENAI CIMATEC and INCT-GP)

UNO-Based Estimation of Upgoing Green's Functions in Marchenko Imaging

Abstract

The Marchenko method, through multidimensional convolutions (MDCs) between a pre-processed shot gather and a first-arrival dataset, enables the retrieval of subsurface focusing functions. These can be used to reconstruct one-way Green's functions and perform zero-lag cross-correlation imaging that is free of internal multiples artifacts. A recent method in the literature employs a UNet to bypass Marchenko iterations by estimating the upgoing focusing function from its initial guess, followed by extra steps to compute the upgoing Green's function and generate an image. We propose an alternative strategy: using a U-shaped Neural Operator (UNO) network to directly estimate the upgoing Green's function from its first iterative approximation. This allows us to test whether UNO can capture more underlying physics and eliminate the need for further MDCs to build the Green's function from the focusing function. We demonstrate that a UNO trained on a subset of the Overthrust model can accurately predict the final upgoing Green's functions, delivering results comparable to the conventional approach. We further discuss the advantages and limitations of each network in the context of Marchenko-based imaging workflows.

Introduction

The Marchenko method enables the retrieval of Green's functions free of internal multiples by solving integral equations involving the reflection response $\mathbf{R}$ and focusing functions. Traditionally, this is done via an iterative Neumann series – conventional method – that performs multiple multidimensional convolutions (MDCs) between $\mathbf{R}$ and an estimated downgoing focusing function $\mathbf{f}^+$, followed by additional steps to obtain the upgoing Green's function $\mathbf{g}^-$ for imaging. Recently, Wang et al. (2025) proposed a data-driven alternative using a UNet to approximate the upgoing focusing function $\mathbf{f}^-$ directly from its initial approximation, and then apply the Marchenko equation, at the cost of two extra MDCs, to compute the $\mathbf{g}^-$ component required for imaging. Notably, van der Neut et al. (2015) showed a direct link between $\mathbf{g}^-$ and a first arrival, suggesting an alternative way to retrieve $\mathbf{g}^-$. However, leveraging this with a UNet is challenging, and a network with more embedded physics may yield more consistent and accurate results.

In this work, we propose a scheme that employs the U-shaped Neural Operator (UNO) network to directly estimate $\mathbf{g}^-$ from its first iterative approximation. The UNO architecture combines U-shaped encoding-decoding with nonlocal operators to enhance long-range dependencies and capture physical patterns (Rahman et al., 2023). The effectiveness of the proposed method is validated by comparing the predicted $\mathbf{g}^-$ with those obtained using the conventional scheme. Seismic images are then computed: the close similarity with those from the iterative scheme further confirms that the UNO model can deliver comparable imaging quality.

Theory

The iterative Neumann solution to the Marchenko equations involves computing multidimensional convolutions of $\mathbf{R}$ (deconvolved shot gathers after free-surface multiple removal) with the initial downgoing focusing function $\mathbf{f}_d^+$, which is estimated here using an eikonal solver from the subsurface point to the surface receivers ($\mathbf{x}_S$). Following van der Neut et al. (2015), the upgoing focusing function $\mathbf{f}^-$ and the upgoing Green's function $\mathbf{g}^-$ can then be computed by

$$\mathbf{f}^{-(K)} = \boldsymbol{\Theta}\mathbf{R}\sum_{k=0}^{K}\left(\boldsymbol{\Theta}\mathbf{R}^{\star}\boldsymbol{\Theta}\mathbf{R}\right)^{k}\mathbf{f}_d^{+} \qquad \text{and} \qquad \mathbf{g}^{-(K)} = \boldsymbol{\Psi}\mathbf{R}\sum_{k=0}^{K}\left(\boldsymbol{\Theta}\mathbf{R}^{\star}\boldsymbol{\Theta}\mathbf{R}\right)^{k}\mathbf{f}_d^{+}, \tag{1}$$

where $\boldsymbol{\Theta}$ is a mask separating events prior to the first arrival and $\boldsymbol{\Psi} = \mathbf{I} - \boldsymbol{\Theta}$. For $K = 0$, the initial estimates are given by $\mathbf{f}_0^- = \boldsymbol{\Theta}\mathbf{R}\mathbf{f}_d^+$ and $\mathbf{g}_0^- = \boldsymbol{\Psi}\mathbf{R}\mathbf{f}_d^+$. In this work, we perform $K = 5$ iterations, which corresponds to a total of 10 MDCs in the conventional scheme, as indicated in Figures 1(a) and 1(b) (red boxes of the schemes).

(a)

$$\mathbf{f}_d^+ \xrightarrow{\boldsymbol{\Theta}\mathbf{R}} \mathbf{f}_0^- \xrightarrow[\text{UNet}]{\text{Eq. 1, } K=5} \mathbf{f}^- \xrightarrow{\boldsymbol{\Theta}\mathbf{R}^{\star}} \mathbf{f}_m^+ \xrightarrow{+\mathbf{f}_d^+} \mathbf{f}^+ \xrightarrow{\boldsymbol{\Psi}\mathbf{R}} \mathbf{g}^-$$

(b)

$$\mathbf{f}_d^+ \xrightarrow{\boldsymbol{\Psi}\mathbf{R}} \mathbf{g}_0^- \xrightarrow[\text{UNet or UNO}]{\text{Eq. 1, } K=5} \mathbf{g}^-$$

Figure 1: Comparison of strategies to estimate $\mathbf{g}^-$. (a) Conventional approach, where $\mathbf{f}^-$ is first obtained using either the iterative Marchenko method (red box) or a UNet (orange box), as in Wang et al. (2025). (b) Direct estimation of $\mathbf{g}^-$ from $\mathbf{g}_0^-$, using either the iterative method (red box) or the proposed approach (violet box).

To perform Marchenko imaging, we apply a zero-lag cross-correlation condition at each focal point $\mathbf{x}_I$. In the frequency domain, the imaging condition is expressed as (Matias et al., 2018)

$$I_{cc}(\mathbf{x}_I) = \sum_{\mathbf{x}_S}\sum_{\omega} \hat{G}^-(\mathbf{x}_I, \mathbf{x}_S)\, \hat{f}_d^{\dagger}(\mathbf{x}_I, \mathbf{x}_S). \tag{2}$$

As is evident from Eq. 2, evaluating the imaging condition I_{cc} requires knowledge of the upgoing wavefield $\mathbf{g}^-$. In the conventional method (Figure 1(a) - red box), and for our tests, retrieving $\mathbf{f}^-$ involves performing 10 MDCs using Eq. 1. Wang et al. (2025) demonstrated that $\mathbf{f}^-$ can be estimated using a UNet, yielding a stable loss curve throughout training. Subsequent steps are then applied to compute $\mathbf{g}^-$, as also summarized in Figure 1(a) (orange box). In that setup, the network is trained using $10\,\%$ of the image points, with $5\,\%$ allocated for validation and the remaining $85\,\%$ for testing, as shown in Figure 2(a).

In this work, based on Eq. 1 for $\mathbf{g}^-$, we propose an alternative approach to directly estimate $\mathbf{g}^-$ from its initial approximation $\mathbf{g}_0^-$, as illustrated in Figure 1(b) (violet box). Considering the complexity of $\mathbf{g}^-$ and the potential limitations of using a UNet for this task, we introduce the UNO network as a more suitable alternative. The training, validation, and testing parameters used in this setup follow the same configuration adopted in Wang et al. (2025). The UNO topology consists of a 4-level encoder-decoder structure where each level contains operator blocks that integrate spectral convolutions (via FFT), pointwise convolutions, and MLPs with residual connections, coupled with skip connections between corresponding encoder-decoder levels. Then, as depicted in Figure 1(b), the proposed scheme eliminates the need for additional steps to compute $\mathbf{g}^-$, thereby saving the cost of two MDCs (see Figure 1(a)). This experiment also aims to evaluate UNO's capacity to incorporate more physical steps. In the next section, we compare the outputs of $\mathbf{g}^-$ and the resulting migrated images obtained using both UNet and UNO within the proposed framework.

Results

To evaluate the effectiveness of the proposed methodology, we use a slice of the Overthrust velocity model shown in Figure 2(a) and perform a test consistent with that presented in Wang et al. (2025). Both UNet and UNO are trained for 600 epochs, with the corresponding loss curves displayed in

Figures 2(b) and (c). We observe that UNO converges more smoothly over the epochs, which is expected given its enhanced capacity to incorporate physical knowledge. While UNet also converges, its loss exhibits higher variance. Figure 3 shows the initial estimate of $\mathbf{g}^-$ followed by its final estimate using the conventional method (Figure 1(a) with red box). Red arrows highlight internal multiple events that are successfully attenuated between Figures 3(a) and 3(b). Figure 3(c) presents the final UNet result using the direct estimation, where attenuation is observed and artifacts appear. The UNO result is shown in 3(d), which is similar to 3(b) and has few or no extra artifacts.

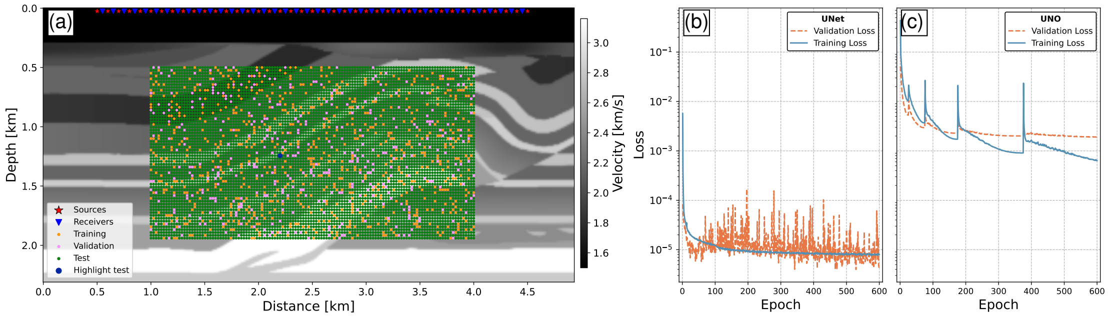

Figure 2: (a) Slice of the Overthrust velocity model showing the training (orange), validation (pink), and test (green) sets. The dark blue circle highlights the focal point for $\mathbf{g}^-$ in Figure 3. Loss curves for UNet with Adam optimizer (b) and UNO with AdamW + CosineAnnealingWarmRestarts (c).

In Figure 4, we compare the migrated images. The red arrows and the region inside the red rectangle highlight internal multiple artifacts present in Figure 4(a), which are attenuated by conventional Marchenko imaging in Figure 4(b). Figure 4(c) shows the UNet result, which achieves good reconstruction overall, but introduces some “granulation” (cyan arrows) due to the extra artifacts in $\mathbf{g}^-$ mentioned earlier. Figure 4(d) presents the UNO image, which is clear and closely resembles the conventional result in Figure 4(b). This highlights UNO’s capability to incorporate more physics into its training process.

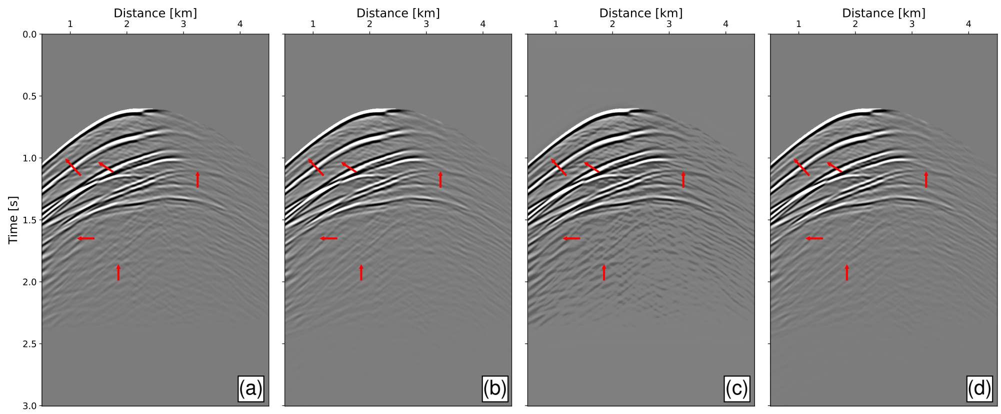

Figure 3: $\mathbf{g}^-$ for: first estimation (a), after 10 MDCs (b), UNet inference (c), and UNO inference (d).

We emphasize that the original UNet-based approach, which estimates $\mathbf{f}^-$ followed by two additional MDCs prior to imaging, is equivalent to the proposed UNO-based estimation of $\mathbf{g}^-$, and can be trained in fewer epochs, as shown in Wang et al. (2025). This conventional scheme is also better suited when the final goal is to retrieve the full Green’s function, which requires both $\mathbf{f}^-$ and $\mathbf{f}^+$. In contrast, our proposed scheme focuses exclusively on imaging through $\mathbf{g}^-$, highlighting UNO’s ability to incorporate more physics into the training process. Although this comes at a higher training cost compared to the former approach, it may be offset in 3D applications, where the time savings from avoiding two additional MDCs per image point can build up to a meaningful level.

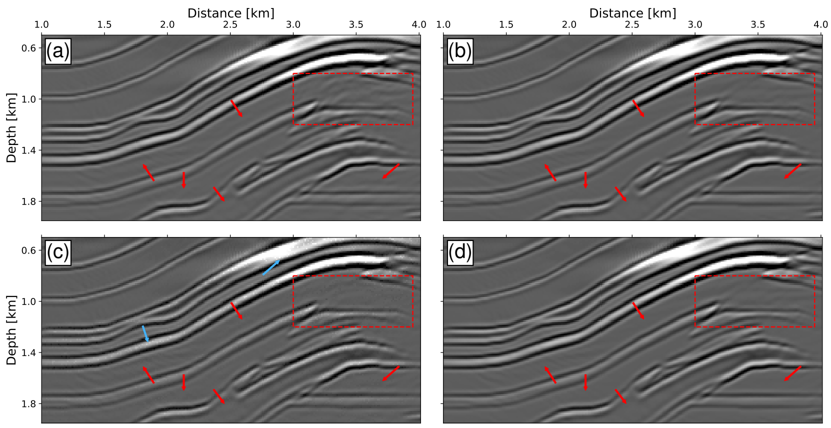

Figure 4: Migrated images for: single scattering (a), iterative method (b), UNet (c), and UNO (d).

Conclusions

In this work, we explored neural network-based alternatives to the iterative Marchenko scheme for internal multiple attenuation and imaging. By reproducing the UNet-based estimation of $\mathbf{f}^-$ from its first iteration, we confirmed its effectiveness in generating accurate Marchenko images after two additional MDCs, as previously proposed. We also validated the UNO architecture for the same task, achieving comparable imaging results. Building on this, we proposed a novel scheme that directly estimates the final upgoing Green's function from its initial approximation, bypassing the final MDCs. Our results show that UNO outperforms UNet in this direct $\mathbf{g}^-$ estimation, producing cleaner images with fewer artifacts and better agreement with conventional Marchenko results. This highlights UNO's enhanced ability to incorporate physical constraints during training. Although this scheme incurs higher training costs, it may offer computational advantages in large-scale 3D scenarios where repeated MDCs become expensive. The choice between $\mathbf{f}^-$- and $\mathbf{g}^-$-based strategies should depend on the final objective–whether full Green's function retrieval or imaging–and the trade-off between training time and runtime efficiency.

Acknowledgments

This research was conducted within the framework of the Marchenko project, with financial support from CENPES/Petrobras and CNPq through the INCT-GP. We would like to thank CENPES and SENAI CIMATEC for granting permission to publish this research.

References

Matias, M. M. A., R. da C. Pestana, and J. van der Neut, 2018, Marchenko imaging by unidimensional deconvolution: Geophysical Prospecting, **66**, 1653–1666.

Rahman, M. A., Z. E. Ross, and K. Azizzadenesheli, 2023, U-NO: U-shaped Neural Operators: ArXiv preprint.

van der Neut, J., I. Vasconcelos, and K. Wapenaar, 2015, On Green's function retrieval by iterative substitution of the coupled Marchenko equations: Geophys. J. Int., **203**, 792–813.

Wang, N., M. Ravasi, and T. Alkhalifah, 2025, Accelerating Marchenko Imaging via Self-Supervised Focusing Function Prediction: 1–5.