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Forward and Adjoint Acoustic Wave Equations with Vector Reflectivity

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Abstract

This work presents a method for forward and adjoint modeling full wavefields in acoustic media, based on a new formulation of the wave equation parameterized by vector reflectivity and velocity. This method is compared with the classical forward and adjoint modeling based on the second-order acoustic wave equation. We show that, if an estimate of reflectivity is known or obtained, the full acoustic seismic wavefield can be generated from velocity and reflectivity without requiring explicit knowledge of density. To validate the implementation numerically, we demonstrate that the forward and adjoint operators satisfy the adjoint test. From a geophysical standpoint, we compare seismograms and wavefields generated with both approaches over a known earth model, demonstrating the equivalence between the two methods. With these numerically and geophysically consistent operators based on vector reflectivity, we aim to incorporate them in full waveform inversion (FWI) schemes in future work.

Introduction

The acoustic wave equation is commonly expressed in terms of the medium's wave velocity or slowness and density. While velocity models can be estimated directly from seismic data, density typically must be inferred through empirical relationships, such as Gardner's equation, or obtained from independent geophysical methods. To address this limitation, Whitmore et al. (2020) proposed a wave equation formulated using velocity and vector reflectivity models.

Vector reflectivity is a physical quantity that can be estimated directly from seismic amplitude data and provides a direct representation of subsurface reflectivity. This property makes it a promising candidate for use in seismic inversion procedures such as FWI, where the goal is to reconstruct detailed images of the subsurface.

FWI relies on computing the gradient of an objective function, which requires both forward and adjoint modeling operators. The adjoint operator is responsible for backward wavefield propagation in time, and its numerical consistency with the forward operator is essential. When these operators satisfy the adjoint test (Claerbout and Abma, 1992), the gradient computation becomes reliable, which accelerates convergence and improves the overall stability of the inversion process.

Although the forward wave equation based on vector reflectivity has been introduced in the literature, the corresponding adjoint formulation remains largely unexplored. Furthermore, there is a lack of studies validating the numerical correctness of these operators and their adherence to the adjoint property.

In this work, we implement a forward and adjoint pair of acoustic wave equation operators formulated in terms of acoustic wave velocity and vector reflectivity. We demonstrate that these operators satisfy the adjoint test and are geophysically consistent with the classical formulation. The validation is performed using the adjoint test, and by comparing the resulting seismograms with those obtained from the classical acoustic wave equation. The results confirm the numerical and geophysical correctness of the proposed approach, establishing a robust foundation for future FWI strategies aimed at recovering the vector reflectivity.

Method and/or Theory

The forward modeling operator we implemented is a finite difference solver for the following second-order acoustic wave equation, based on the one proposed by Whitmore et al. (2020):

$$TU = \frac{1}{v_P^2}\frac{\partial^2 U}{\partial t^2} - \nabla^2 U + \left(2\mathbf{R} - \frac{\nabla v_P}{v_P}\right)\cdot\nabla U = S(\mathbf{x},t)\,\delta(\mathbf{x}-\mathbf{x}_s), \tag{1}$$

where T is the operator associated with this wave equation, $U(\mathbf{x},t)$ represents the wavefield at time t and position $\mathbf{x} = (x,z)$, $S(\mathbf{x},t)$ is the source term, and $\delta(\mathbf{x}-\mathbf{x}_s)$ is the Dirac delta function for a source located at position $\mathbf{x}_s$. This equation is expressed in terms of the propagation velocity of the acoustic wave $v_P(\mathbf{x})$ and the vector reflectivity $\mathbf{R} = [R_x(\mathbf{x}) \quad R_z(\mathbf{x})]^T$, a vector quantity that characterizes the medium's reflectivity in each direction, defined as:

$$\mathbf{R} = \frac{1}{2}\frac{\nabla z}{z}, \tag{2}$$

where $z(\mathbf{x}) = v_P(\mathbf{x})\rho(\mathbf{x})$ is the seismic impedance and $\rho(\mathbf{x})$ is the density model.

The adjoint operator solves the adjoint equation to (1) backwards in time:

$$T^*V = \frac{1}{v_P^2}\frac{\partial^2 V}{\partial t^2} - \nabla^2 V - \nabla\cdot\left[\left(2\mathbf{R} - \frac{\nabla v_P}{v_P}\right)V\right] = \sum_{i=1}^{n_{\mathrm{rec}}} R_i(\mathbf{x},t)\,\delta(\mathbf{x}-\mathbf{x}_r^i), \tag{3}$$

where T^* is the adjoint of T, $V(\mathbf{x},t)$ is the adjoint wavefield and $R_i(\mathbf{x},t)$ represents the receiver term for the ith receiver, which is located at the position $\mathbf{x}_r^i$ indicated by the delta function $\delta(\mathbf{x}-\mathbf{x}_r^i)$, and n_{rec} is the number of receivers.

The code for the operators was implemented with Python and the Devito package (Louboutin et al., 2019; Luporini et al., 2020).

Results

We validated the correctness of the backward propagation operator as an adjoint operador with the adjoint test on multiple models, whose results are on Table 1. The relative errors are of the order of 10^{-16} to 10^{-15}, which are close to the machine accuracy of a double precision floating point of 2.22×10^{-16}. This indicates that our implementation of the adjoint operator is numerically consistent.

Model	$\lvert\langle TU,V\rangle - \langle U,T^*V\rangle\rvert$	$\frac{\langle TU,V\rangle - \langle U,T^*V\rangle}{\langle TU,V\rangle}$
Homogeneous	9.09494702×10^{-13}	6.15281153×10^{-16}
2 layers	1.54614099×10^{-11}	2.71205816×10^{-15}
5 layers	7.27595761×10^{-12}	3.98379234×10^{-16}

Table 1: Results of the adjoint test for different models.

In order to verify the geophysical consistency of the operators based on T and T^*, we compared their output with the output of operators that uses the classical second-order acoustic wave equation, $\tilde{T}U = S(\mathbf{x}, t)\,\delta(\mathbf{x} - \mathbf{x}_s)$, where the operator $\tilde{T}$ is defined by

$$\tilde{T}U = \frac{1}{v_P^2}\frac{\partial^2 U}{\partial t^2} - \rho\nabla\cdot\left(\frac{1}{\rho}\nabla U\right) = S(\mathbf{x}, t)\,\delta(\mathbf{x} - \mathbf{x}_s), \tag{4}$$

and its adjoint equation $\tilde{T}^*V = \sum_{i=1}^{n_{\mathrm{rec}}} R_i(\mathbf{x}, t)\,\delta(\mathbf{x} - \mathbf{x}_r^i)$, where $\tilde{T}^*$ is the adjoint of $\tilde{T}$,

$$\tilde{T}^*V = \frac{1}{v_P^2}\frac{\partial^2 V}{\partial t^2} - \nabla\cdot\left[\frac{1}{\rho}\nabla(\rho V)\right] = \sum_{i=1}^{n_{\mathrm{rec}}} R_i(\mathbf{x}, t)\,\delta(\mathbf{x} - \mathbf{x}_r^i). \tag{5}$$

We ran both pairs of operators over a homogeneous earth model with a single source and a receiver and compared the generated wavefields, from which some snapshots are displayed on Figure 1. We also modeled the seismograms in Figure 2 for a 2 layers earth model. It can be observed from these images that the output of the operators based on T and T^* closely matches the output of the operators based on $\tilde{T}$ and $\tilde{T}^*$.

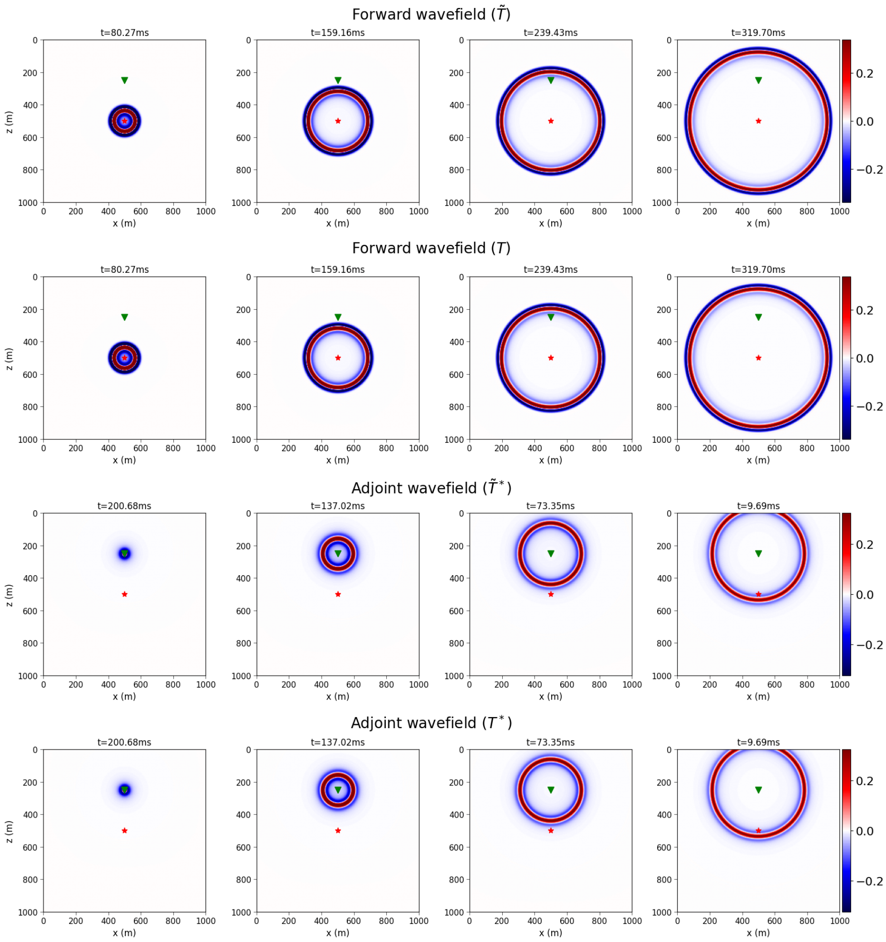

Figure 1: Wavefields generated by the forward and adjoint operators in a homogeneous model. The source position is indicated by a star and the receiver position is indicated by a triangle.

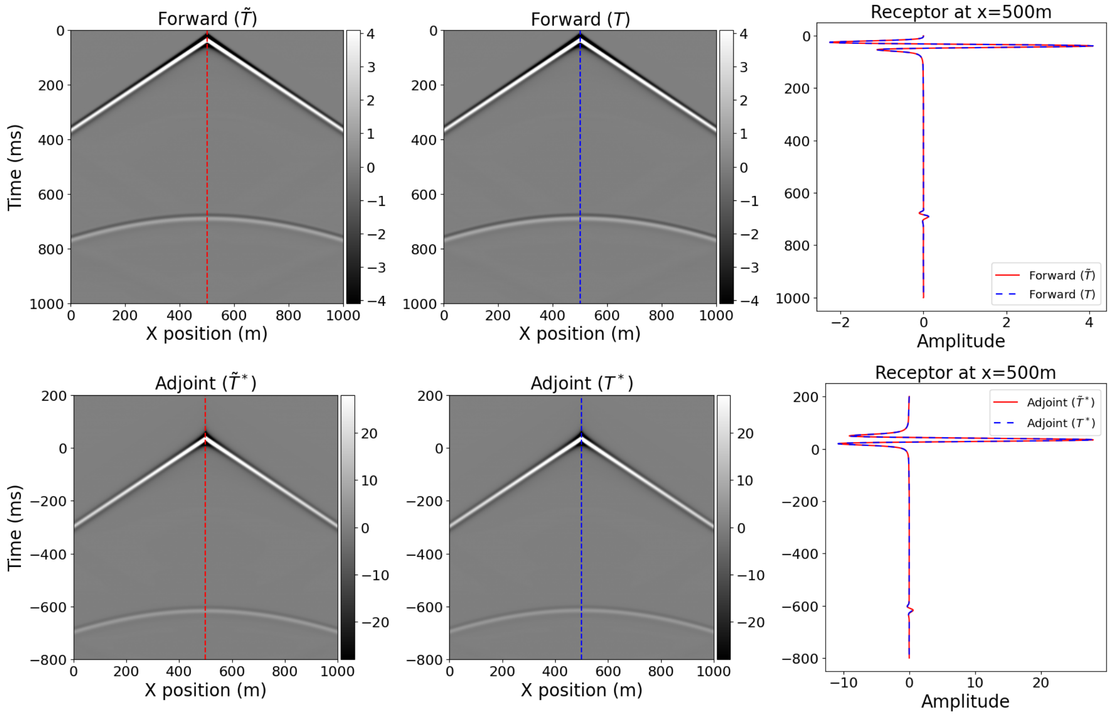

Figure 2: Seismograms generated by the forward and adjoint operators using the classical wave equation (left), the equations (1) and (3) (middle) and the amplitudes recorded by the receptor at $x = 500$m (right).

Conclusions

We implemented forward and backward propagation operators based on the equation proposed by Whitmore et al. (2020). The numerical and geophysical consistency of our implementation was demonstrated through the adjoint test and by comparing its output with operators that uses the classical acoustic wave equation and its adjoint equation. In future work, we will incorporate these operators in a FWI scheme to recover the vector reflectivity field.

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