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A Physics-Informed Kolmogorov-Arnold Network Approach to the Seismic Ray Tracing

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Abstract Summary

Seismic ray tracing is a key technique in imaging and reservoir studies, typically solved by numerical methods that may become unstable in complex velocity models. Physics-Informed Neural Networks (PINNs) offer improved physical consistency but suffer from limited generalization and convergence. We propose a Physics-Informed Kolmogorov–Arnold Network (PIKAN) that uses trainable activation functions and parametric inputs to model continuous families of raypaths. The model is trained with a composite loss combining data-driven Mean Squared Error, physics loss via automatic differentiation, and a data-aware weighting scheme to improve learning in regions under-sampled by raypaths. The velocity model is constructed using B-spline interpolation from sparsely sampled parametric data, with adaptability to complex models. Experiments demonstrate high predictive accuracy $(R^2 > 0.99)$ and strong agreement with Runge-Kutta solutions. PIKAN also generalizes to unseen conditions, confirming its robustness for seismic modeling in realistic scenarios.

Introduction

Seismic ray tracing is a significant technique of wavefield modeling of wave propagation in multi-media for seismic imaging and reservoir characterization. It refers to solving a system of nonlinear differential equations by approximating Snell's law for ray estimation of the trajectories and the travel times in heterogeneous and anisotropic velocity models. Although traditional numerical solvers, such as modified Newton-based methods, provide accurate solutions, they lose stability or consume much computational effort when a high velocity contrast or spatial variability exists (Koketsu and Sekine, 1998; Wang, 2014).

In recent years, efforts have been made to develop other methods based on Physics-Informed Neural Networks (PINNs) for ray tracing problems. These frameworks incorporate the governing physical equations in the loss function directly, such that the network can learn solutions without access to labeled data (Duarte et al., 2023). However, PINNs continue to suffer from key limitations: they are challenging to converge and generalize, which often discourages the use of initial conditions – like source position or shooting angle – as input variables. Therefore, the network learns a single raypath with fixed initial conditions, restricting its generalization and scalability.

To mitigate these challenges, we employ Physics-Informed Kolmogorov–Arnold Networks (PIKANs) for seismic ray tracing. Unlike common networks, KANs utilize trainable activation functions, improving interpolation and interpretability (Liu et al., 2025). This allows the model to generalize over a number of initial conditions and learn continuous families of raypaths. Besides, we introduce a data-aware regularization approach that adaptively reweights the physics loss by the local data density (Xiang et al., 2022), promoting the physical consistency in regions under-sampled by raypaths and data-guided refinement elsewhere. Our contributions provide an improved and more effective solution for seismic ray tracing in complex velocity models.

Theory and Method

We can represent a ray tracing in a $v(s)$ medium by a system of differential equations (Margrave, 2003)

$$\begin{aligned}\frac{d\mathbf{s}}{dt} &= v(\mathbf{s})^2\mathbf{p} \\ \frac{d\mathbf{p}}{dt} &= -v(\mathbf{s})\nabla\frac{1}{v(\mathbf{s})},\end{aligned} \tag{1}$$

where t is time, $\mathbf{s}$ is position coordinates (x, z), and $\mathbf{p}$ is the slowness vector (p_x, p_z). Solving this system in time using a second-order Runge-Kutta method, we were not only able to derive the raypath variables utilized in model output (x, z, p_x, p_z) but also their respective derivatives needed in the calculation of the physics loss, which helps the model acquire knowledge of the fundamental physical equations of the ray tracing problem.

In this work, we employ an inhomogeneous P-wave parametric velocity model given by the equation $v(x, z) = 1.5 + 0.3x + z$. To ensure consistency with the grid-based numerical solvers used in our ray tracing, we first get a coarse subset of the complete velocity array via a constant stride of 20, and then reconstruct the complete velocity model by two-dimensional B-spline filtering. This approach enables building a continuous and differentiable velocity field even from sparsely sampled or discontinuous models. Notably, our proposed method, which incorporates B-spline filtering for velocity model reconstruction, is not limited to analytical or continuous models; it can also be successfully applied to complex and non-smooth velocity models, thus making it a versatile approach for application to both synthetic and real geological settings.

The loss function is a combination of the data loss and the weighted physics loss. Data loss $\mathcal{L}_{\text{MSE}}$ is the Mean Squared Error (MSE) between the predicted output of the PIKAN model and ground truth raypath variables. The physics loss is computed via automatic differentiation (autograd) to obtain the time derivatives $\frac{dx}{dt}, \frac{dz}{dt}, \frac{dp_x}{dt}, \frac{dp_z}{dt}$ from the model outputs and compare them with their analytically determined values. An initial condition loss is also included to ensure that at $t = 0$ the model predictions for position match the known source location. The physics loss can be expressed as the weighted sum of the squared discrepancies between the predicted and ground truth derivatives, along with the penalty associated with the initial conditions ($\mathcal{L}_{\text{IC}}$):

$$\mathcal{L}_{\text{physics}} = \frac{1}{n}\sum_{i=1}^{n} w_i \left\| \frac{d\hat{\mathbf{r}}_i}{dt} - \frac{d\mathbf{r}_i}{dt} \right\|^2 + \mathcal{L}_{\text{IC}}, \tag{2}$$

where $\mathbf{r}_i = [x_i, z_i, p_{x_i}, p_{z_i}]$, $\hat{\mathbf{r}}_i$ is the model prediction and w_i is a sample-specific weighting factor to emphasize critical regions with low raypath data density. Therefore, the total loss function used during training is defined as:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{MSE}} + \mathcal{L}_{\text{physics}} \tag{3}$$

Its performance was evaluated by Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), Root Mean Squared Error (RMSE), and the R-squared statistic (R^2).

Results

Initially, we investigated the influence of the grid size hyperparameter on the accuracy of the PIKAN model in seismic ray tracing problems. From Figure 1, we can see that decreasing the grid size (i.e.,

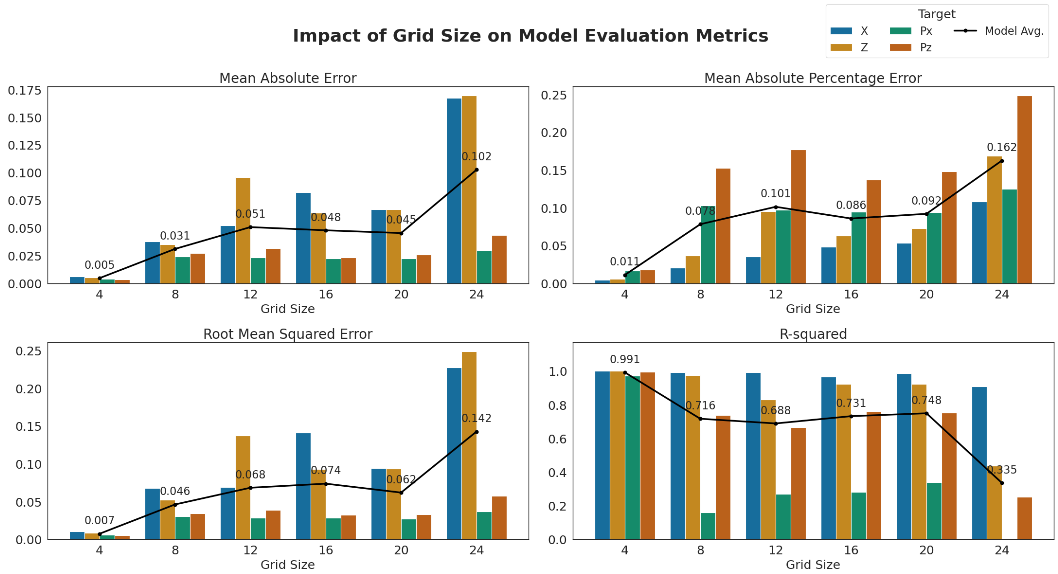

Figure 1: Influence of the grid size hyperparameter on PIKAN performance for seismic ray tracing.

4 and 8) resulted in substantial accuracy improvements in predictions based on considerably lower errors (MAE, MAPE, and RMSE) and higher R^2 scores. However, increasing the grid size to 20 or 24 led to a decline in the model's performance. Following the identification of grid size 4 as optimal, we trained and tested the convergence behavior of the model further. Training showed quick initial drops in the total loss and physics loss, with subsequent stabilization. In addition, training and validation data losses were always low and closely following one another during training, indicating that the model learned the underlying physics constraints well without overfitting.

Metric	**X**	**Z**	**Px**	**Pz**
R^2	0.9998	0.9997	0.9952	0.9991
MAPE	0.0036	0.0042	0.0069	0.0086
MAE	0.0051	0.0038	0.0016	0.0015
RMSE	0.004690	0.002431	0.001778	0.003084

Table 1: Evaluation metrics for PIKAN predictions on seismic ray tracing.

Table 1 illustrates a snapshot of the final evaluation metrics of every output of the model (X, Z, Px, and Pz). The very high R^2 values invariably and the extremely low error metrics illustrate the accuracy of the model in forecasting seismic raypaths. Besides, the qualitative assessment based on the visual comparison shown in Figure 2 also substantiates the quantitative results. The PIKAN model-calculated raypaths follow closely the reference raypaths calculated with the Runge-Kutta scheme, offering further evidence of its performance and robustness in seismic ray tracing.

Conclusions

We have proposed a novel approach for seismic ray tracing based on Physics-Informed Kolmogorov-Arnold Networks (PIKANs), which incorporate physical constraints directly into the training process

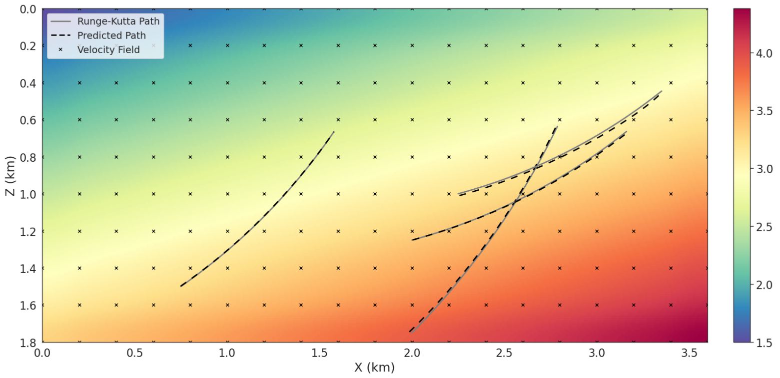

Figure 2: Comparison between PIKAN predicted raypaths (dashed lines) and reference raypaths generated by the Runge-Kutta method (solid lines) across different initial conditions.

to predict the raypaths in a parametric velocity field. We tested our method by evaluating its predictions against classical Runge-Kutta numerical integration, demonstrating strong agreement in both trajectories and their derivatives. Including initial conditions such as source position and shooting angle as model inputs was crucial, enabling the model to generalize well to raypaths not seen during training, demonstrating effective interpolation capabilities. The results confirm that the use of physical constraints during training and the data-informed regularization contributed to highly accurate predictions and maintained strong training stability, even in the presence of sparse data. For this application, the use of small grid sizes allowed more accurate learning of physically consistent and smooth raypaths. These findings highlight the potential of PIKANs as an accurate and cost-effective alternative for seismic ray tracing. Future work includes extending the method to non-parametric velocity models, integrating advanced deep learning techniques to enhance generalization, and exploring model interpretability to support geophysical analysis.

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