
**Submission code: BP9NGQM706**

See this and other abstracts on our website: https://home.sbgf.org.br/Pages/resumos.php

# New pure P-wave equation for reverse time migration in tilted transversely isotropic media

**Lucas Silva Bitencourt (CPGG - UFBA), Reynam Da Cruz Pestana (CPGG - UFBA)**

# New pure P-wave equation for reverse time migration in tilted transversely isotropic media

Please, do not insert author names in your submission PDF file.



---

## Abstract Summary

To decrease the information loss when imaging a complex geological medium, it is crucial to consider anisotropy, particularly the transversely isotropic medium, as it is the most common anisotropy in exploration geophysics. However, the elastic wave equations that govern such medium describe $P$ and $S$ waves that are coupled. This results in shear wave energy in seismic modeling and, consequently, in artifacts in the seismic image. Thus, a new acoustic wave equations for pure $P$-wave in the tilted transversely isotropic (TTI) media based on the decomposition of the elastic wave equation in the vertical transversely isotropic (VTI) media is proposed. This new equation correctly decouples the $P$ and $S$ wavefields. Additionally, the pure $P$-wave equation is efficacious and stable for seismic modeling and migration of complex geological structures, regardless of whether $\delta > \epsilon$. The efficacy of our method is demonstrated by modeling synthetic TTI data found in the literature.

## Introduction

Alkhalifah (2000) deduced a pseudoacoustic wave equation for vertical transversely isotropic (VTI) media, using the exact dispersion relation of the elastic wave equation in such media (Tsvankin, 1996), but it generates $SV$-wave artifact. Therefore, efforts were made to develop pure $qP$-wave equations (Pestana et al., 2012), and such equations eliminated the artifact generated by the SV wave. The generalization from VTI to tilted transversely isotropic (TTI) media is done by rotating the symmetry axis so that it can vary freely in space (Zhan et al., 2012).

However, the pseudoacoustic wave equations, although kinematically accurate and useful for imaging, are unphysical and provide dynamically inaccurate wavefields. To circumvent this issue, acoustic wave equations in TTI media were presented throughout the years; the work of Zhang et al. (2011) is of note. Nevertheless, these equations describe $P$ and $S$ waves that are coupled. This results in shear wave energy in seismic modeling and in artifacts in the seismic image (Pestana et al., 2011).

Consequently, developing a pure $P$-wave equation that is physically accurate is crucial to improve the quality of seismic imaging without generating artifacts. Recently, Zhang et al. (2022) proposed a notable anisotropic-Helmholtz decomposition method that numerically decouples the $P$ and $S$ wavefields and generates correct units, phases, and amplitudes compared with input elastic wavefields. Thus, in this work, we extend their work and propose a new acoustic wave equation for $P$-wave in TTI media.

## Method

Let the 3D elastic VTI wave equation in the $\mathbf{k}-t$ domain in the matrix form,

$$\rho\frac{\partial^2\mathbf{U}}{\partial t^2}=-A\mathbf{U}+\mathbf{F}, \tag{1}$$

in which

$$\begin{gathered}
c_{11}=(1+2\varepsilon)\,\rho v_P^2,\;\; c_{33}=\rho v_P^2,\;\; c_{44}=\rho v_S^2,\;\; c_{66}=(1+2\gamma)\,\rho v_S^2,\\
c_{13}=\rho\sqrt{\left[(1+2\delta)\,v_P^2-v_S^2\right]\left(v_P^2-v_S^2\right)}-\rho v_S^2,\\
A=K_1^\dagger L_1K_1+K_2^\dagger L_2K_2+K_3^\dagger L_3K_3+K_4^\dagger L_3K_4,\\
K_1=\begin{bmatrix} ik_x & 0 & 0\\ 0 & ik_y & 0\\ 0 & 0 & ik_z\end{bmatrix},K_2=\begin{bmatrix} ik_y & 0 & 0\\ 0 & ik_x & 0\\ 0 & 0 & 0\end{bmatrix},K_3=\begin{bmatrix} ik_z & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & ik_x\end{bmatrix},K_4=\begin{bmatrix} 0 & 0 & 0\\ 0 & ik_z & 0\\ 0 & 0 & ik_y\end{bmatrix},\\
L_1=\begin{bmatrix} c_{11} & (c_{11}-2c_{66}) & c_{13}\\ (c_{11}-2c_{66}) & c_{11} & c_{13}\\ c_{13} & c_{13} & c_{33}\end{bmatrix},L_2=\begin{bmatrix} c_{66} & c_{66} & c_{66}\\ c_{66} & c_{66} & c_{66}\\ c_{66} & c_{66} & c_{66}\end{bmatrix},L_3=\begin{bmatrix} c_{44} & c_{44} & c_{44}\\ c_{44} & c_{44} & c_{44}\\ c_{44} & c_{44} & c_{44}\end{bmatrix},
\end{gathered} \tag{2}$$

and $\varepsilon$, $\delta$ and $\gamma$ are the Thomsen's parameters (Thomsen, 1986). Zhang et al. (2022) show that

$$\mathbf{U}=\mathbf{U}^P+\mathbf{U}^S,\;\; \mathbf{U}^P=DD^T\frac{1}{D^2}\mathbf{U},\;\; \mathbf{U}^S=\left(I-DD^T\frac{1}{D^2}\right)\mathbf{U}, \tag{3}$$

in which $\mathbf{U}^P$ and $\mathbf{U}^S$ are the $P$- and $S$-wave fields, and $D=\begin{bmatrix}k_x & k_y & rk_z\end{bmatrix}^T$, in which $r=\frac{\sqrt{\left[(1+2\delta)v_P^2-v_S^2\right]\left(v_P^2-v_S^2\right)}}{(1+2\varepsilon)v_P^2-v_S^2}$. Hence, we apply equation 3 to equation 1 and obtain

$$\rho\frac{\partial^2\mathbf{U}^P}{\partial t^2}=-DD^T\frac{1}{D^2}\left[A\mathbf{U}-\mathbf{F}\right],\;\; \rho\frac{\partial^2\mathbf{U}^S}{\partial t^2}=-\left(I-DD^T\frac{1}{D^2}\right)\left[A\mathbf{U}-\mathbf{F}\right]. \tag{4}$$

Afterwards, we define $P=\frac{1}{\rho}D^T\frac{1}{D^2}A\mathbf{U}^P$ and $S_P=\frac{1}{\rho}D^T\frac{1}{D^2}A\mathbf{U}^S$. Thus, substituting them into equation 4, we obtain the new acoustic pure $P$-wave equation

$$\frac{\partial^2P}{\partial t^2}=-\frac{1}{\rho}D^T\frac{1}{D^2}ADP+F, \tag{5}$$

By substituting equation 2 into equation 5, we obtain

$$\frac{\partial^2P}{\partial t^2}=-v_P^2\left[(1+2\varepsilon)\,k_r^2+k_z^2-\frac{2(\varepsilon-\delta)\left(1-\frac{v_S^2}{v_P^2}\right)}{1+2\varepsilon-\frac{v_S^2}{v_P^2}}\frac{k_r^2k_z^2}{k_r^2+r^2k_z^2}\right]P+F. \tag{6}$$

To compute it, we propose an approach based on Huang et al. (2023), such that

$$\frac{\partial^2P}{\partial t^2}=-v_P^2\left[(1+2\varepsilon)\,k_r^2+k_z^2-\frac{4(\varepsilon-\delta)\left(1-\frac{v_S^2}{v_P^2}\right)}{\left(1+2\varepsilon-\frac{v_S^2}{v_P^2}\right)(1+r^2)}\frac{k_r^2k_z^2}{k_r^2+k_z^2}\right]P+F. \tag{7}$$

To obtain the aforementioned equation in the TTI media, we can combine the methods proposed by Zhan et al. (2012) and Bitencourt and Pestana (2024). In the 2D case, we obtain

$$\frac{\partial^2P}{\partial t^2}=-v_P^2\left[\tilde{A}_{11}k_x^2+\tilde{A}_{33}k_z^2+\tilde{A}_{13}k_xk_z+\tilde{A}_{1333}\frac{k_xk_z^3}{k_x^2+k_z^2}+\tilde{A}_{3333}\frac{k_z^4}{k_x^2+k_z^2}\right]P, \tag{8}$$

in which

$$\tilde{A}_{11} = A_{11} + A_{1111}, \quad \tilde{A}_{33} = A_{33} - A_{1111} + A_{1133}, \quad \tilde{A}_{13} = A_{13} + A_{1113},$$
$$\tilde{A}_{1333} = A_{1333} - A_{1113}, \quad \tilde{A}_{3333} = A_{1111} + A_{3333} - A_{1133},$$
$$A_{11} = 1 + 2\varepsilon\cos^2\theta, \quad A_{33} = 1 + 2\varepsilon\sin^2\theta, \quad A_{13} = -2\varepsilon\sin 2\theta, \quad A_{1111} = \frac{1}{4}\gamma\sin^2 2\theta,$$
$$A_{3333} = \frac{1}{4}\gamma\sin^2 2\theta, \quad A_{1113} = \frac{1}{2}\gamma\sin 4\theta, \quad A_{1333} = -\frac{1}{2}\gamma\sin 4\theta, \tag{9}$$
$$A_{1133} = \gamma\left(1 - \frac{3}{2}\sin^2 2\theta\right), \quad \gamma = -\frac{4(\varepsilon-\delta)\left(1 - \frac{v_S^2}{v_P^2}\right)}{\left(1 + 2\varepsilon - \frac{v_S^2}{v_P^2}\right)(1 + r^2)}.$$

## Results

Initially, we investigate the dispersion relation of the proposed equation. For this purpose, we use two homogeneous models with $v_0 = 3\,\mathrm{km/s}$ and $v_S = 0\,\mathrm{km/s}$. Figure 1 shows, in both models, a comparison of the proposed equation with the equations from Tsvankin (1996), Zhan et al. (2012), and Huang et al. (2023). It is evident that the proposed equation is more precise than the one from Zhan et al. (2012) and is as precise as the pseudoacoustic equation from Huang et al. (2023).

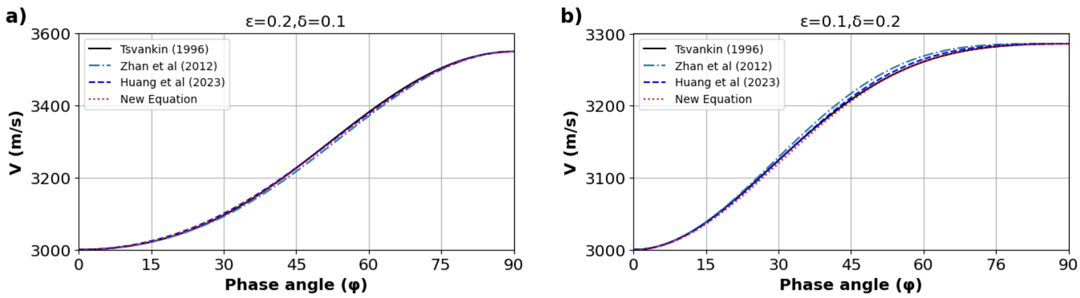


Figure 1: Comparison of the dispersion relations for different values of $\varepsilon$ and $\delta$.

Finally, we model the Marmousi TTI model. The wavefield is propagated for $t = 1.2\,\mathrm{s}$, with $\Delta t = 2\,\mathrm{ms}$ and a peak frequency of $15\,\mathrm{Hz}$. Figure 2 shows a comparison of snapshots computed with the equation from Zhan et al. (2012) and with the proposed equation; and Figure 3 shows a profile extracted at $x = 6250\,\mathrm{m}$, in which the result obtained using the equation from Huang et al. (2023) is added. It can be seen that the proposed equation provides a stable and consistent result.

## Conclusions

In this work, a new pure P-wave equation in TTI media was proposed. We have shown that it is efficacious and stable for seismic modeling of complex inhomogeneous and anisotropic media. These results are significant, as they can improve the quality of seismic imaging in complex geological structures without increasing the computational cost in relation to the currently available methods.

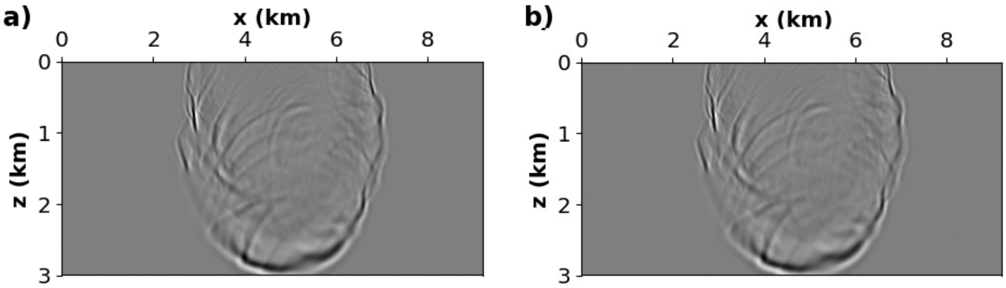


Figure 2: Comparison of wave field snapshots at time $t = 1.2\,\mathrm{s}$: a) Zhan et al. (2012), and b) proposed equation.

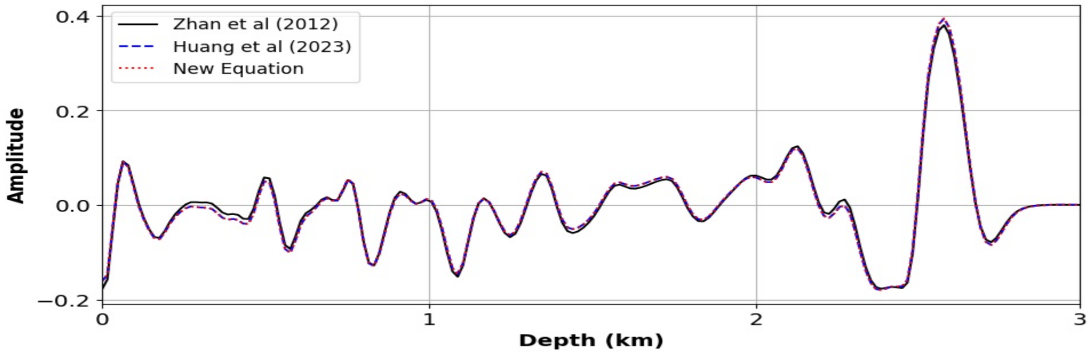


Figure 3: Profile extracted from wavefield at $x = 6250\,\mathrm{m}$ and $t = 1.2\,\mathrm{s}$.

## References


Alkhalifah, T., 2000, An acoustic wave equation for anisotropic media: Geophysics, **65**, 1239–1250.

Bitencourt, L. S., and R. C. Pestana, 2024, Simplified TTI pure qP-wave equation implemented in the space domain and applied for reverse time migration in TTI media: Geophysics, **89**, no. 1, C17–C28.

Huang, J., Q. Mao, X. Mu, J. Yang, M. S. Ivan, Z. Liu, and S. Zhang, 2023, Least-squares reverse time migration based on an efficient pure qp-wave equation: Geophysical Prospecting, **72**, 1290–1311.

Pestana, R. C., B. Ursin, and P. L. Stoffa, 2011, Separate P- and SV-wave equations for vti media: 81st Annual International Meeting, SEG, Expanded Abstracts, 163–167.

———, 2012, Rapid expansion and pseudo spectral implementation for reverse time migration in VTI media: Journal of Geophysics and Engineering, **9**, no. 3, 291–301.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, 1954–1966.

Tsvankin, I., 1996, P-wave signatures and notation for transversely isotropic media: An overview: Geophysics, **61**, 467–483.

Zhan, G., R. C. Pestana, and P. L. Stoffa, 2012, Decoupled equations for reverse time migration in tilted transversely isotropic media: Geophysics, **77**, no. 2, T37–T45.

Zhang, L., L. Liu, F. Niu, J. Zuo, D. Shuai, W. Jia, and Y. Zhao, 2022, A novel and efficient engine for P-/S-wave-mode vector decomposition for vertical transverse isotropic elastic reverse time migration: Geophysics, **87**, no. 4, S185–S207.

Zhang, Y., H. Zhang, and G. Zhang, 2011, A stable TTI reverse time migration and its implementation: Geophysics, **76**, no. 3, WA3–WA11.