Submission code: BV8GKPJY7K

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Preliminary Results of Integrating Airborne Geophysical Datasets Using Equivalent Sources on the Sphere

Arthur Siqueira-Macedo (USP), Leonardo Uieda (IAG | USP)

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Abstract

Geoscientists are increasingly adopting the equivalent source method to process and interpolate magnetic data, particularly in airborne geophysical surveys. However, conventional Cartesian-based approaches face significant limitations when applied to regional and global scales due to the Earth's curvature, which complicates data interpolation and integration. This work adapts the magnetic equivalent source method to spherical coordinates. To account for the geometric differences between Cartesian and spherical systems, we introduce modifications to the mathematical formulations. We incorporate the gradient-boosting technique to tackle computational efficiency in datasets with millions of observations. We evaluate the performance of the method through synthetic data tests, identifying conditions in which it can struggle, such as variations in data density and geological complexity. The adapted method will be integrated into open-source software, broadening its accessibility and impact.

Introduction

Magnetic field data are essential for imaging subsurface structures in geophysics (Blakely, 1995; Langel and Hinze, 1998). A widely used processing method is the equivalent source technique (Dampney, 1969), which models the field using virtual sources to enable operations like continuation and reduction to the pole. While typically implemented in Cartesian coordinates, this approach loses accuracy over large areas due to Earth's curvature. To address this, the equivalent source method can be applied on the sphere (Langel and Hinze, 1998; Von Frese et al., 1981).

To reduce the computational cost associated with the large volume of data typically found in regional and continental-scale airborne magnetic surveys, we integrate the gradient-boosting strategy proposed by Soler and Uieda (2021) and Uppal et al. (2025), which improves efficiency through windowing and iterative refinement, into the spherical equivalent sources formulation.

We test the method on synthetic airborne magnetic datasets to assess its accuracy, computational performance, and suitability for regional-scale applications. These experiments aim to highlight the advantages and limitations of the spherical formulation and guide future improvements for handling large, irregular datasets

Methods

Equivalent sources in spherical coordinates

The modeling of magnetic fields on the sphere relies on a local reference system defined by radial, colatitudinal, and longitudinal unit vectors. The equations for the radial (r), colatitudinal (θ), and longitudinal (ϕ) components of the magnetic field of a dipole in spherical coordinates is given by Langel and Hinze (1998).

The equivalent sources technique reconstructs the observed magnetic field by inferring a distribution of hypothetical sources, commonly referred to as an equivalent layer, that replicates the measured data with fidelity (Cordell, 1992; Dampney, 1969; Oliveira Jr. et al., 2023). When working in spherical coordinates, the observations $\mathbf{d}^o$ acquired at positions (ϕ_i, θ_i, r_i) can be described by a harmonic expansion of the form:

$$d_i^o \approx f_i(\mathbf{c}) = \sum_{j=1}^{P} a_{ij} c_j, \tag{1}$$

in which a_{ij} quantifies the effect of the j-th source on the i-th observation, and c_j denotes the physical property (e.g., magnetic moment amplitude) of the source positioned at (ϕ_j, θ_j, r_j).

This relation can be expressed compactly in matrix form as:

$$\mathbf{d} = \mathbf{A}\mathbf{c}, \tag{2}$$

in which $\mathbf{d}$ is the predicted data vector, $\mathbf{c}$ is a column vector containing the source parameters, and $\mathbf{A}$ is the sensitivity (Jacobian) matrix whose elements a_{ij} are the total field anomaly of a dipole with unit magnetic moment.

To estimate the source parameters $\mathbf{c}$, we solve a regularized linear inverse problem. This is commonly achieved by minimizing a cost function that balances data misfit and model norm. The solution is obtained through the normal equations:

$$(\mathbf{A}^\mathsf{T}\mathbf{A} + \mu\mathbf{I})\mathbf{c} = \mathbf{A}^\mathsf{T}\mathbf{d}^o, \tag{3}$$

in which μ is a regularization parameter that controls the trade-off between fitting the data and stabilizing the solution, and $\mathbf{I}$ is the identity matrix.

Gradient-boosting in spherical coordinates

We adopt the gradient boosting strategy proposed by Soler and Uieda (2021) and Uppal et al. (2025) to fit equivalent source models in a scalable and memory-efficient manner. This approach solves large linear inverse problems by iteratively adjusting local models based on the residuals from previous steps. In this formulation, the predicted data are modeled as the sum of contributions from K overlapping windows:

$$\mathbf{d} = \sum_{k=1}^{K} \mathbf{A}_k \mathbf{c}_k, \tag{4}$$

in which $\mathbf{A}_k$ is the Jacobian matrix associated with the k-th window, and $\mathbf{c}_k$ contains the source magnetic moment amplitude for that window. The K windows are designed to overlap, ensuring full coverage of the survey area and allowing local refinements in regions of redundancy.

The algorithm operates iteratively, fitting each window to the current residuals and updating the residuals ($\mathbf{r}_k$) accordingly:

$$\mathbf{r}_k = \mathbf{r}_{k-1} - \mathbf{d}_k, \tag{5}$$

in which $\mathbf{r}_0 = \mathbf{d}_o$ is the initial residual vector, and $\mathbf{d}_k = \mathbf{A}_k\mathbf{c}_k$ is the contribution of the k-th window to the predicted data.

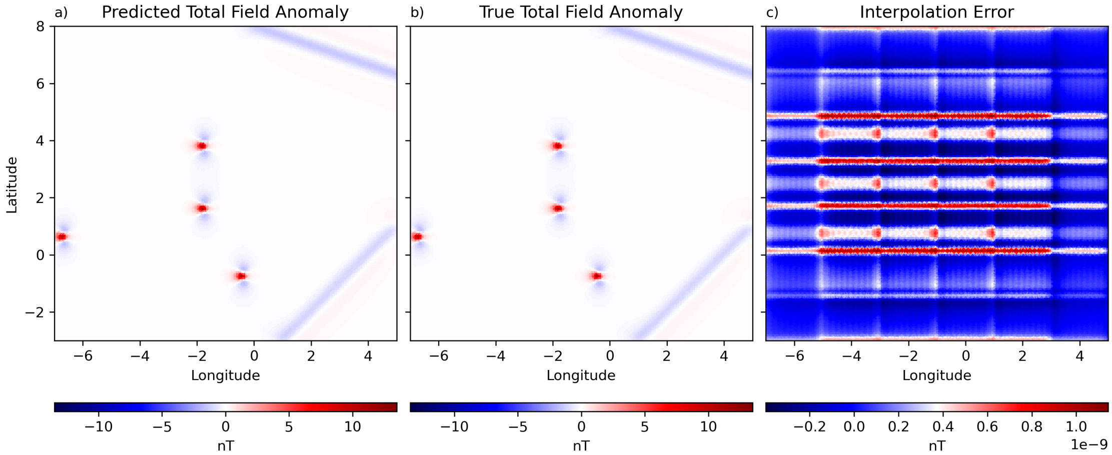

Figure 1: Inversion of synthetic total field anomaly (TFA) data using spherical equivalent sources and the gradient boosting method. **a)** Predicted TFA by the spherical equivalent source model. **b)** Reference (true) TFA from forward modeling of the model. **c)** Interpolation error between predicted and reference data, with amplitudes near 10^{-9} nT, indicating an accurate interpolation.

To reduce computational cost and memory usage, each window uses only the sources and data located within its bounds to compute a reduced Jacobian matrix $\tilde{\mathbf{A}}_k$. Nevertheless, predictions are still evaluated at all observation points through forward modeling:

$$d_k^i = \sum_{j=1}^{M_k} c_{kj}\, \mathbf{b}_{kj} \cdot \hat{F}, \tag{6}$$

where $\mathbf{b}_{kj}$ is the magnetic field contribution of the j-th source in the k-th window, and $\hat{F}$ is the direction of the Earth's main field, typically given by the IGRF.

To improve convergence and reduce potential artifacts, the order in which the windows are processed is randomized. The global solution vector $\mathbf{c}$ is updated incrementally by adding each local estimate $\mathbf{c}_k$ to the corresponding positions. Because the forward modeling operation is linear, the final predicted data can be obtained using the full set of coefficients directly in the forward modeling.

This strategy allows efficient modeling of large datasets by breaking the problem into smaller overlapping parts, avoiding the need to construct or store a full sensitivity matrix.

Results

The synthetic dataset consists of 51182 data points distributed in a pattern similar to that of an airborne magnetic survey. To simulate realistic measurement conditions, Gaussian noise with a 0 nT mean and standard deviation of 0.1 nT was added to the data.

Figure 1 shows that the proposed method accurately reconstructs the total field anomaly. The predicted and true anomalies (panels a and b) are visually similar, indicating precise reproduction of the magnetic signal. The interpolation errors (panel c) are negligible, around 10^{-9} nT. The checkerboard pattern in the residuals reflects the block-based nature of the gradient boosting algorithm. However, the existence of this pattern has no effect on the predicted grid.

The use of gradient boosting made the processing feasible and efficient for this amount of data on a modest computer. In contrast, applying the traditional equivalent sources method would have exceeded the available memory limits, as it requires storing and manipulating a full 51182×51182 sensitivity matrix, which would require approximately 21 Gb of RAM using 64-bit floating point numbers. The inversion and forward modeling tasks were completed in approximately 9 minutes on a personal computer with an Intel® Core™ i7-2600 CPU @ 3.40GHz (8 cores) and 12 GB of DDR3 RAM.

Conclusions

In this work we tackled the problem of using equivalent sources in spherical coordinates for large datasets. This work demonstrates that adapting the gradient-boosting method to spherical coordinates is a viable and effective strategy for processing magnetic data over regional to continental scales. Integrating the gradient-boosting strategy enables the inversion of large airborne datasets using limited computational resources, without the need to construct a full sensitivity matrix.

Synthetic tests confirm that the spherical formulation preserves signal fidelity and achieves low residuals even in the presence of noise, validating its suitability for real-world applications. However, challenges remain in areas with highly irregular data coverage or complex geological signatures, where further refinement, such as adaptive windowing or data weighting, may be needed.

In future work, we aim to incorporate the dual-layer approach proposed by Uppal et al. (2025), apply the method to more realistic synthetic and real datasets, and address the challenges posed by the convergence of longitude lines near the poles. The implementation developed in this project will be integrated into open-source geophysical software, facilitating reproducibility and fostering broader adoption in the geoscience community.

Acknowledgments

This research was supported by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) 88887.947899/2024-00 and by start-up grant PRPI 22.1.09345.01.2 from Universidade de São Paulo. Special thanks to the members of the CompGeoLab (IAG/USP) for their valuable insights and continued support throughout this work.

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