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Robust Blind \(\ell_p \)-Norm Deconvolution for Sparse Reflectivity Estimation in Non-Gaussian Noise

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Abstract Summary

Seismic data can exhibit non-Gaussian noise characteristics in certain geological environments, undermining the Gaussian assumption typically employed by classical signal processing techniques. This paper proposes a robust blind deconvolution approach for seismic signals contaminated with impulsive noise, modeled using α-stable distributions. The method is formulated through the minimization of the ℓ_p-norm of the residual error, providing enhanced robustness to outliers compared to traditional ℓ_2-norm-based methods. To promote sparse reflectivity, an ℓ_1-norm regularization term is added to the cost function, which is defined based on a classical convolutional model of the seismic trace.

Introduction

Traditionally, seismic deconvolution is formulated as an estimation problem based on the minimization of the mean squared error (ℓ_2-norm), under the assumption that the noise affecting the data is Gaussian (Robinson and Treitel, 1981). However, in complex geological environments, seismic data may be contaminated with impulsive non-Gaussian noise (*spikes*) during acquisition, significantly undermining the Gaussian assumption (Lima, 2022; Yue and Peng, 2015). A more appropriate model for this type of noise is the family of α-stable distributions, which generalize the Gaussian by allowing for heavy tails and better capturing the statistical behavior of impulsive disturbances (Shao and Nikias, 1993; Yue and Peng, 2015). In such scenarios, the ℓ_2-norm becomes unsuitable due to its high sensitivity to outliers, often resulting in poor seismic resolution and the masking of true reflectivity events.

Moreover, even when the additive noise is Gaussian, the distribution of the residual error in the inverse problem may exhibit heavy-tailed behavior due to model mismatch, wavelet estimation uncertainty, or regularization effects. Thus, adopting a robust cost function based on the ℓ_p-norm can enhance performance by better accommodating these deviations.

In this work, we propose a novel blind deconvolution method based on the minimization of the ℓ_p-norm of the residual error, combined with ℓ_1-norm regularization to enforce sparsity in the reflectivity profile. Unlike traditional approaches that rely on Gaussian noise assumptions and the ℓ_2-norm, our formulation is designed to be robust against impulsive noise modeled by α-stable distributions. The method simultaneously estimates both the seismic wavelet and the reflectivity series from the observed trace. We validate its effectiveness through numerical experiments involving synthetic data, highlighting the improved performance in scenarios with heavy-tailed noise. Additionally, we provide a statistical analysis based on Monte Carlo simulations to quantify the robustness of the proposed approach across different values of p.

Method and/or Theory

In reflection seismology, the recorded signal can be described by the classical convolutional model. In discrete time, a seismic trace is given by:

$$x_n = w_n * r_n + v_n = \sum_{\ell=0}^{L} r_\ell w_{n-\ell} + v_n, \tag{1}$$

where n denotes the discrete time index, w_n is the emitted source signal (seismic wavelet), r_n is the reflectivity function of the subsurface, and v_n is additive noise that accounts for environmental disturbances and acquisition uncertainties. This model can also be expressed in matrix form as:

$$\mathbf{x} = \mathbf{W}\mathbf{r} + \mathbf{v} = \mathbf{R}\mathbf{w} + \mathbf{v}, \tag{2}$$

where $\mathbf{R}$ and $\mathbf{W}$ are convolution matrices constructed from $\mathbf{r}$ and $\mathbf{w}$, respectively.

In blind deconvolution, both the reflectivity $\mathbf{r}$ and the wavelet $\mathbf{w}$ are unknown, and only the seismic trace $\mathbf{x}$ is available. In this work, an initial estimate of the wavelet is obtained directly from the seismic trace under a zero-phase assumption. Because this estimation step is susceptible to contamination by impulsive noise, a time-domain window of length L is subsequently applied to truncate and smooth the estimated wavelet, serving as a simple yet effective form of preprocessing. The parameter L is user-defined and should ideally match the time support of the actual source signal. Based on this initial wavelet estimate, the objective is to jointly refine both unknown signals by minimizing a cost function that reflects the discrepancy between the observed data and the modeled trace. To this end, we define a cost function that integrates robustness to outliers and sparsity promotion. Specifically, it minimizes the ℓ_p-norm of the residual error to reduce sensitivity to impulsive noise, while an ℓ_1-norm regularization term encourages sparsity in the reflectivity:

$$J(\mathbf{r}, \mathbf{w}) = \frac{1}{p}||\mathbf{e}||_p^p + \lambda||\mathbf{r}||_1 = \frac{1}{p}\sum_{n=0}^{N-1} |x_n - \hat{x}_n|^p + \lambda \sum_{n=0}^{N-1} |r_n|, \tag{3}$$

where $\hat{x}_n$ is the reconstructed signal, $\lambda > 0$ is a regularization parameter that balances the data fidelity and sparsity terms, and p controls the robustness of the loss to heavy-tailed errors.

To minimize the cost function $J(\mathbf{r}, \mathbf{w})$ with respect to both $\mathbf{r}$ and $\mathbf{w}$, we compute the partial derivatives. The i-th component of the gradient with respect to $\mathbf{r}$ is:

$$\frac{\partial}{\partial r_i} J(\mathbf{r}) = -\sum_{n=0}^{N-1} w_{n-i} e_{p_n} + \lambda sgn(r_i), \tag{4}$$

where $e_{p_n} = |x_n - \hat{x}_n|^{p-1} sgn(x_n - \hat{x}_n)$. In matrix form, the gradient becomes:

$$\frac{\partial J(\mathbf{r})}{\partial \mathbf{r}} = -\mathbf{W}^T \mathbf{e_p} + \lambda sgn(\mathbf{r}), \tag{5}$$

and the gradient with respect to the wavelet is:

$$\frac{\partial J(\mathbf{w})}{\partial \mathbf{w}} = -\mathbf{R}^T \mathbf{e_p}. \tag{6}$$

The gradient descent update rules for both the signals are then given by:

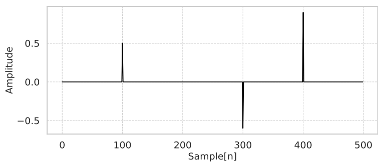
$$\begin{cases} \mathbf{r}^{k+1} &= \mathbf{r}^k - \mu_r(-\mathbf{W}^T\mathbf{e_p} + \lambda sgn(\mathbf{r^k})), \\ \mathbf{w}^{k+1} &= \mathbf{w}^k - \mu_w(-\mathbf{R}^T\mathbf{e_p}), \end{cases}$$

where μ_r and μ_w are the step sizes for the reflectivity and wavelet updates, respectively.

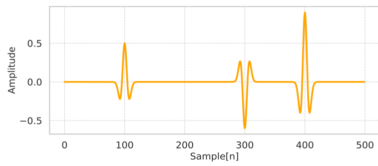
Results

0.1 Experiment

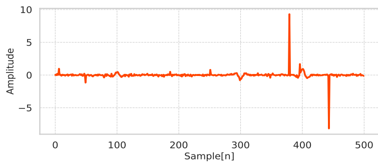
To evaluate the performance of the proposed deconvolution method, we conducted numerical simulations using synthetic seismic data. The reflectivity signal is defined as a sparse sequence with nonzero coefficients at selected time indices, as shown in Fig. 1a. The synthetic seismic trace is obtained by convolving this reflectivity with a Ricker wavelet of central frequency $f_0 = 25$ Hz and length $L = 51$ samples, as illustrated in Fig. 1b. Impulsive noise is then added to the synthetic trace. The noise is generated from an α-stable distribution with parameters $\alpha = 1.5$, $\beta = 0$, $\delta = 0$, and $\gamma = 0.05$, as shown in Fig. 1c.



(a)

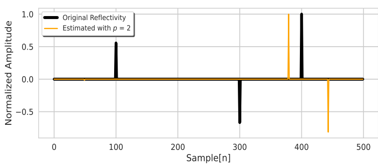


(b)

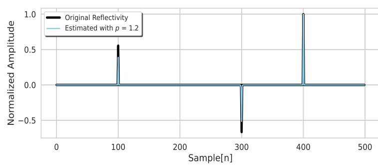


(c)

Figure 1: (a) Original sparse reflectivity sequence; (b) seismic trace obtained by convolution with a Ricker wavelet ($f_0 = 25$ Hz); (c) seismic trace with added α-stable noise ($\alpha = 1.5$, $\gamma = 0.05$).



(a)



(b)

Figure 2: Estimated reflectivity signals: (a) result for $p = 2$ ($\lambda = 2$, $\mu_r = 0.0001$, $\mu_w = 0.0001$); (b) result for $p = 1.2$ ($\lambda = 1.8$, $\mu_r = 0.0001$, $\mu_w = 0.0001$).

Figure 2 presents the estimated reflectivity signals obtained by minimizing the cost function for two values of p. Fig. 2a shows the result for $p = 2$, corresponding to the classical least-squares approach, while Fig. 2b shows the result for $p = 1.2$. In both cases, we performed $N = 2000$ iterations, and empirically selected the regularization and step-size parameters λ, μ_r e μ_w.

0.2 Statistical Analysis

To evaluate the robustness of the proposed method, we conducted a Monte Carlo simulation using the Pearson correlation coefficient (ρ) as the performance metric. The experiment used the same seismic trace described in Fig. 1b, with 20 different realizations of additive α-stable noise. We evaluated the performance of the blind deconvolution method for values of p in the interval $1 \leq p \leq 2$, with increments of 0.1.

Figure 3 shows the box plots of ρ obtained for different values of p in the interval $[1.0, 2.0]$. As p decreases from 2.0 to approximately 1.2, the median correlation steadily increases, indicating improved robustness to impulsive noise. This reflects the method's enhanced ability to suppress outliers when using lower-order norms. However, for values of p closer to 1.0, the dispersion of results becomes more pronounced, suggesting a potential trade-off between robustness and estimation stability. These results highlight the importance of selecting an appropriate p value depending on the expected noise characteristics.

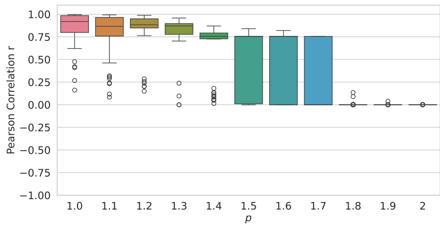


Figure 3: Box plot of the Pearson correlation coefficient (r) for each tested value of p, based on 20 Monte Carlo runs with α-stable noise realizations.

Conclusions

This work presented a robust blind deconvolution method for seismic signals under non-Gaussian noise, modeled via α-stable distributions. The approach is based on ℓ_p-norm minimization with ℓ_1 regularization to promote sparsity. Numerical results show that values of $p < 2$ yield better reconstruction than the classical ℓ_2 approach, highlighting greater robustness to impulsive noise. The results also emphasize the importance of selecting p according to the noise characteristics to balance robustness and stability.

Acknowledgments

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