



# SBGf Conference

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**Submission code: KPW7AJQ9PA**

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## High Resolution Relative Elastic Inversion

**Carlos Cunha (Petrobras), Paulo Carvalho (Petrobras), Alberto Carvalho (Petrobras),  
Edinalda Souza (Petrobras), Anna Lucia Oliveira (Petrobras), Quezia Cavalcante (Petrobras  
S.A.), Maria Clara Ciloni (Petrobras), Henrique Fraquelli (Petrobras)**

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This paper was prepared for presentation during the 19<sup>th</sup> International Congress of the Brazilian Geophysical Society held in Rio de Janeiro, Brazil, 18-20 November 2025. Contents of this paper were reviewed by the Technical Committee of the 19<sup>th</sup> International Congress of the Brazilian Geophysical Society and do not necessarily represent any position of the SBGf, its officers or members. Electronic reproduction or storage of any part of this paper for commercial purposes without the written consent of the Brazilian Geophysical Society is prohibited.

### Abstract Summary

We present a new approach for elastic inversion of seismic data with two major characteristics: reduced dependence on well data and higher resolution compared to conventional methods. The combination of these two aspects makes this method feasible for application in frontier exploration areas, where well data may be sparse (or even absent), as well as in production development prospects where complex reservoir connectivity can benefit from high-resolution inverted volumes.

The method is based on a set of linear equations that relate relative elastic parameters to relative integrated reflectivity. The term “relative” has different meanings for each of these attributes due to the particularities of their definitions. When well data is available, the relative elastic parameters can be directly compared to equivalent well-derived relative parameters for quality control (QC).

### Introduction

Processes designed to estimate elastic parameters from seismic data can be organized into two main groups: methods that are solely based on seismic data, which produce qualitative parameter volumes such as AVO (Amplitude Versus Offset) or AVA (Amplitude Versus Angle), and methods that are supported by well log information, which yield absolute values for the parameter volumes and are generally known as inversion methods. The absolute values resulting from the latter group allow for the estimation of petrophysical properties, making these methods a key process in the quantitative interpretation framework. However, relative elastic parameters can also be translated into relative petrophysical properties and provide quantitative information, provided there is sufficient knowledge about the specific geological context of the target area to constrain the statistical analysis (Msolo and Gidlow, 2015).

Some studies that are closer to the first group aim to define an inversion process similar to that of the second group, but with input data distinct from those used in conventional inversion processes (Connolly, 1999; Whitcombe et al., 2002; Rosa, 2010).

In the process described by Rosa (2010), the input data consists of integrated versions of conventional seismic data, and the result of the inversion process yields relative elastic properties.

This work generally follows the same principle, with some particularities:

- The input data consists of high-resolution relative-impedance volumes routinely generated for Petrobras' E&P projects (Cunha et al., 2019).
- The result of the inversion (relative elastic parameters) can be directly related to similar attributes derived from well logs, allowing for their use in quantitative processes.

### Relative Integrated Reflectivity

The reflection coefficient associated with a compressional wave incident at an angle relative to the normal of a locally flat interface can be expressed in terms of the elastic parameters of the underlying media using the following approximation (Aki and Richards, 1980):

$$R_\theta(it) = A + B \sin^2\theta + C \sin^2\theta \tan^2\theta; \quad (1)$$

where:

$$A = \frac{1}{2} \left[ \frac{\Delta v_P}{v_P} + \frac{\Delta \rho}{\rho} \right]; \quad B = A - 4k \frac{\Delta v_S}{v_S} + \left( \frac{1+4k}{2} \right) \frac{\Delta \rho}{\rho}; \quad C = \frac{1}{2} \frac{\Delta v_P}{v_P}; \quad k = \left( \frac{\tilde{v}_S}{\tilde{v}_P} \right)^2$$

In the above equations, the operator  $\Delta$  applied to a property of the medium corresponds to the difference between the property value below and above the interface, while the property without the operator  $\Delta$  refers to the average of these properties. The symbol  $\sim$  denotes a broader local average of the property (low-frequency trend).

Considering a time series of interfaces and rearranging the terms (according to Connolly, 1999), we obtain:

$$R_\theta(it) = \frac{1}{2} \left[ P_\theta \frac{\Delta v_P(it)}{v_P(it)} + Q_\theta \frac{\Delta v_S(it)}{v_S(it)} + S_\theta \frac{\Delta \rho(it)}{\rho(it)} \right] \quad (2)$$

where:

$$P_\theta = 1 + \tan^2 \theta; \quad Q_\theta = -8k \sin^2 \theta; \quad S_\theta = 1 - 4k \sin^2 \theta$$

If we perform a discrete time integration (cumulative sum) on both sides of equation (2), we get:

$$\sum_1^{it} R_\theta(it') = \frac{1}{2} \left[ P_\theta \sum_1^{it} \frac{\Delta v_P(it')}{v_P(it')} + Q_\theta \sum_1^{it} \frac{\Delta v_S(it')}{v_S(it')} + S_\theta \sum_1^{it} \frac{\Delta \rho(it')}{\rho(it')} \right] \quad (3)$$

To clarify the next steps, we make the transition from discrete to continuous. Thus, each term on the right side of equation (3) can be expressed as:

$$\sum_1^{it} \frac{\Delta v_P(it')}{v_P(it')} \cong \int_0^t \frac{1}{v_P(t')} \frac{\partial v_P(t')}{\partial t} dt' = \ln v_P(t) - \ln v_P(0)$$

We can then define the Integrated Reflectivity  $R_\theta^I(t)$  as:

$$R_\theta^I(t) = 2 \int_0^t R_\theta(t') dt'$$

Which takes the following form:

$$R_\theta^I(t) = P_\theta [\ln v_P(t) - \ln v_P(0)] + Q_\theta [\ln v_S(t) - \ln v_S(0)] + S_\theta [\ln \rho(t) - \ln \rho(0)] \quad (4)$$

We also define the Relative Integrated Reflectivity  $R_\theta^{IR}(t)$  as the Integrated Reflectivity subtracted from its moving average over an interval  $2\Delta t$ :

$$R_\theta^{IR}(t) = R_\theta^I(t) - \frac{1}{2\Delta t} \int_{t-\Delta t}^{t+\Delta t} R_\theta^I(t') dt'. \quad (5)$$

Each term in the integral in Equation (5) takes the following form:

$$\frac{1}{2\Delta t} \int_{t-\Delta t}^{t+\Delta t} \ln v_P(t') dt' = \ln \bar{v}_P(it); \quad \frac{1}{2\Delta t} \int_{t-\Delta t}^{t+\Delta t} \ln v_P(0) dt' = \ln v_P(0)$$

where  $\ln \bar{v}_P(it)$  corresponds to the moving average of the logarithm of  $v_P$  over the interval  $2\Delta t$ , with similar definitions for the other properties ( $v_S$  and  $\rho$ ).

With the appropriate substitutions, Equation (5) becomes:

$$R_\theta^{IR}(t) = P_\theta [\ln v_P(t) - \ln \bar{v}_P(t)] + Q_\theta [\ln v_S(t) - \ln \bar{v}_S(t)] + S_\theta [\ln \rho(t) - \ln \bar{\rho}(t)],$$

where the absence of terms related to the values of the properties at the origin ( $t = 0$ ) is noted.

After a simple algebraic manipulation, this equation takes the form:

$$R_\theta^{IR}(t) = \beta [P_\theta \ln(v_P^R(t) + 1) + Q_\theta \ln(v_S^R(t) + 1) + S_\theta \ln(\rho^R(t) + 1)] \quad (6)$$

where:

$$v_P^R = \frac{v_P(t) - \bar{v}_P(t)}{\bar{v}_P(t)}, \quad v_S^R = \frac{v_S(t) - \bar{v}_S(t)}{\bar{v}_S(t)}, \quad \rho^R = \frac{\rho(t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

Here,  $v_P^R$ ,  $v_S^R$  and  $\rho^R$ , are relative properties defined according to the equations above, and  $\beta$  is a scaling factor introduced to account for the fact that the reflection coefficients obtained from seismic data have arbitrary amplitude values.

### Elastic Inversion for Relative Properties

The factors  $P_\theta$ ,  $Q_\theta$  and  $S_\theta$  in equation (6) can be rearranged to obtain a more general equation:

$$R_\theta^{IR}(t) = P_\theta A_P(t) + Q_\theta A_S(t) + S_\theta A_\rho(t), \quad (7)$$

with the following options for the desired inversion attributes:

Attributes →	$v_P^R, v_S^R$ e $\rho^R$	$I_P^R, I_S^R$ e $\rho^R$	$I_P^R, (v_P/v_S)^R$ e $\rho^R$
$A_P(t)$	$\ln(v_P^R(t) + 1)^\beta$	$\ln(I_P^R(t) + 1)^\beta$	$\ln((v_P/v_S)^R(t) + 1)^\beta$
$A_S(t)$	$\ln(v_S^R(t) + 1)^\beta$	$\ln(I_S^R(t) + 1)^\beta$	$\ln(I_P^R(t) + 1)^\beta$
$A_\rho(t)$	$\ln(\rho^R(t) + 1)^\beta$	$\ln(\rho^R(t) + 1)^\beta$	$\ln(\rho^R(t) + 1)^\beta$
$P_\theta$	$1 + \tan^2\theta$	$1 + \tan^2\theta$	$1 + \tan^2\theta - 8k \sin^2\theta$
$Q_\theta$	$-8k \sin^2\theta$	$-8k \sin^2\theta$	$8k \sin^2\theta$
$S_\theta$	$1 - 4k \sin^2\theta$	$4k \sin^2\theta - \tan^2\theta$	$4k \sin^2\theta - \tan^2\theta$

The system of equations defined by the set of available  $\theta$  values can be initially inverted to obtain:  $A_P(t)$ ,  $A_S(t)$ ,  $A_\rho(t)$  and subsequently, the relative properties can be derived, such as:

$$(v_P^R(t) + 1)^\beta = \exp[A_P(t)] \quad \rightarrow \quad v_P^R(t) = \sqrt[\beta]{\exp[A_P(t)]} - 1$$

The factor  $\beta$  can be adjusted directly if there are wells within the area covered by the data, or statistically through nearby wells.

It is also possible to generate volumes with absolute values if trend volumes exist (obtained from velocity models and empirical relationships), for example:

$$v_P(t) = \bar{v}_P(t) [v_P^R(t) + 1] \quad (8)$$

### Validation with synthetic data

To validate the inversion process described above, reflectivity profiles were generated for four reflection angles based on the density, compressional sonic, and shear sonic profiles for the well W-2, using the Zoeppritz equations.

Figure 1 shows the result of integrating these reflectivity profiles at the top, and at the bottom, the relative integrated reflectivity, which results from subtracting a trend curve defined by a Lagrange polynomial fit. In this example, the average of the seventh and eighth-order polynomials was used.

From the linear system represented in Equation 7, these profiles were inverted to obtain  $I_P^R(t)$ ,  $I_S^R(t)$  and  $\rho^R(t)$ , and subsequently, using Equation 8, to obtain para  $I_P(t)$ ,  $I_S(t)$  and  $\rho(t)$ . These inverted profiles are compared with the original ones in Figure 2.

## Relative Elastic Inversion of Data from a Deep Water Turbidite system

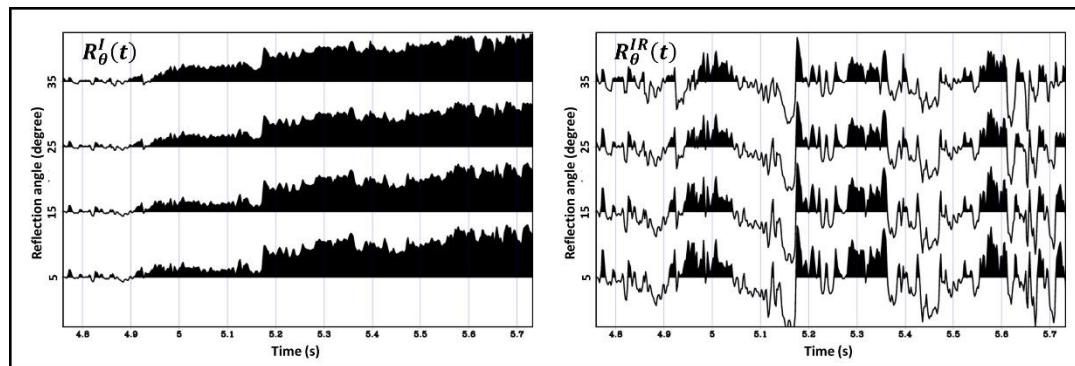
We applied the relative elastic inversion to data from a deep water turbidite prospect in Brazil. Figures 3 and 4 show some results in the vicinity of the same well of Figure 2.

### Conclusions

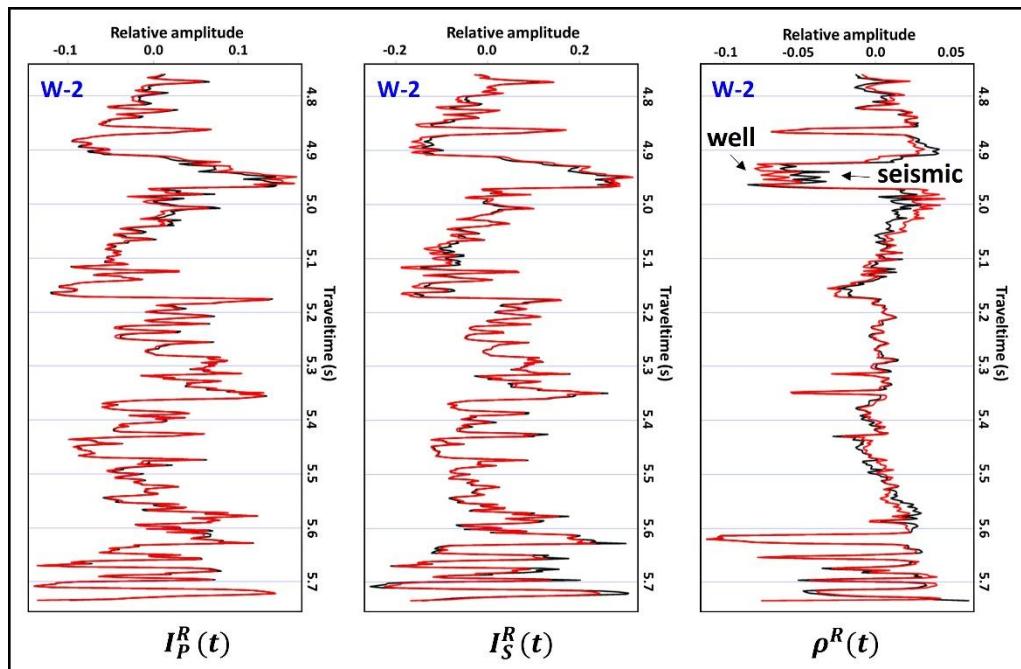
The results obtained from the application of this new inversion method, both to synthetic and field data demonstrate its potential to appraisal of prospects in frontier exploration areas.

### Acknowledgements

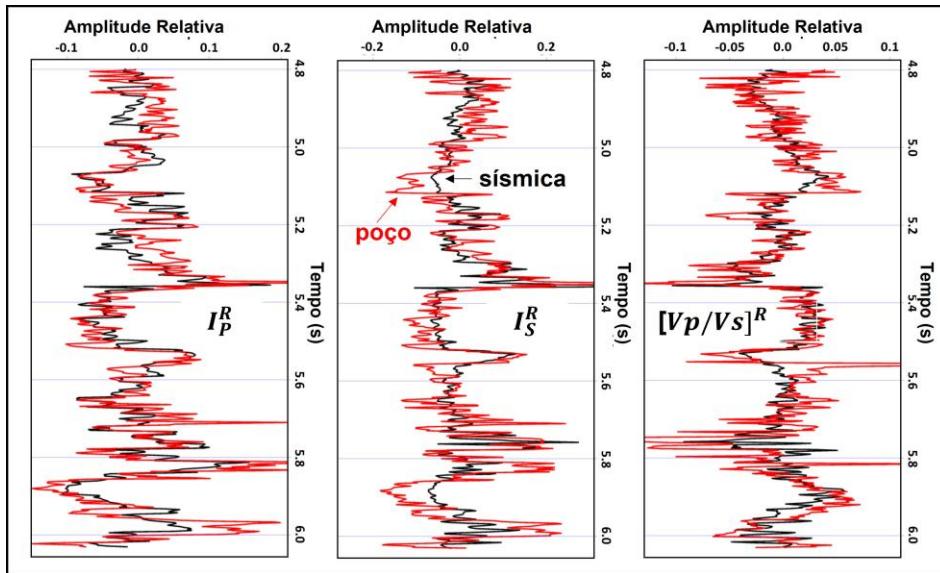
We thank Petrobras for providing the resources to develop this work and the permission to publish it.



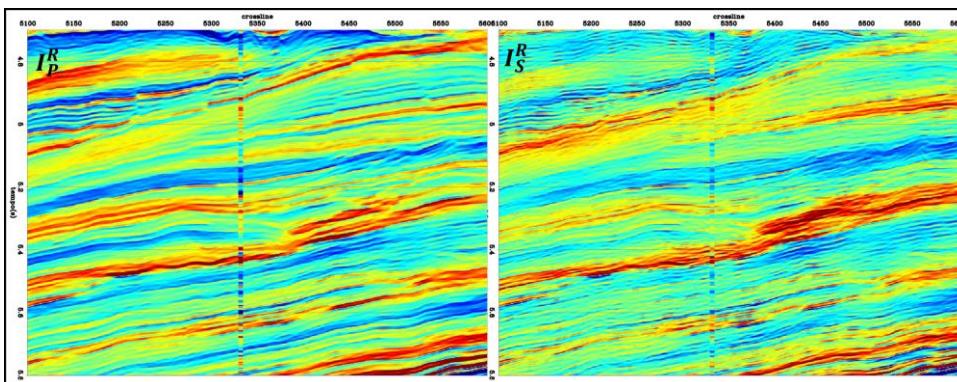
**Figure 1:** Left: integrated reflectivity logs for four reflection angles for well W-2. Right: relative integrated reflectivity logs derived from the logs at the left by subtraction and division by the respective running average curves.



**Figure 2:** inversion results for the relative integrated reflectivity logs show in Figure 1.



**Figure 3:** Relative elastic inversion results at well-2 location (also shown in Figure 4) for the deep-water prospect. No well information was used for the inversion.



**Figure 4:** Relative elastic inversion results for the deep-water prospect. No well information was used for the inversion. Left: P impedance. Right: S impedance.

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