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## **New digital filters optimized for cosine and sine transforms**

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## New digital filters optimized for cosine and sine transforms

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### Summary

The cosine and sine transforms play a fundamental role in the data analysis of geophysics for electrical and electromagnetic methods. There are many efficient numerical algorithms for evaluating these transforms. However, for geophysics of electrical and electromagnetic methods, the digital linear filter algorithm appears to be the most suitable due to its simplicity and efficiency. In this work, we present a new set of optimized filters for the cosine and sine transforms, both with 80 points. Through the direct data problem of 1D mCSEM, we attest to the high performance of these new filters.

### Introduction

In many problems in geophysics related to electrical and electromagnetic methods, the Fourier transform can be broken down into cosine and sine transforms, and in general, such transforms do not have analytical solutions; therefore, the application of numerical methods is necessary to obtain approximate solutions for these transforms. In this context, several geophysical problems related to electrical and electromagnetic methods (RIJO, 2002) are solved numerically through digital linear filters associated with the cosine and sine transforms. The existence of hydrocarbons in the Amazon Basin located at the mouth of the Amazon River (SBGf Bulletin, 2024) may require the application of seismic and non-seismic geophysical methods in the exploration of hydrocarbons in deep and ultra-deep waters. Among the non-seismic methods, the Controlled Source Electromagnetic Marine method – mCSEM stands out. This method, when used in conjunction with seismic methods, can confirm the existence of hydrocarbons, map the outline of a reservoir, and help reduce ambiguity in geological interpretation. The electromagnetic components of this method can be obtained through cosine and sine transforms, however, these transforms do not have analytical solutions. In light of this scenario, this work aims to present digital linear filters of 80 points for both cosine and sine transforms, obtained according to the methodology proposed by Almeida (2002), and apply these filters to the 1D mCSEM method in the simulation of a geo-electric model for hydrocarbon exploration in deep and ultra-deep waters.

### Methodology

Using the methodology proposed by Almeida (2002), we assume for the calculation of the optimal digital linear filters of the cosine and sine transforms, obtained through the Wiener-Hopf least squares method described by Koefoed & Dirks (1979), the cosine and sine transforms with analytical solutions, respectively (SPIEGEL, 1976):

$$\int_0^{+\infty} k_x^2 \text{Exp}(-ak_x) \cos(k_x x) dk_x = 2a(a^2 - 3x^2)/(a^2 + x^2)^3 \quad (1)$$

$$\int_0^{+\infty} k_x^2 \text{Exp}(-ak_x) \text{sen}(k_x x) dk_x = 2x(3a^2 - x^2)/(a^2 + x^2)^3 \quad (2)$$

For the calculation of the maximum amplitude of the relative error (ALMEIDA, 2002), we used cosine and sine transforms with analytical solutions, respectively (SPIEGEL, 1976):

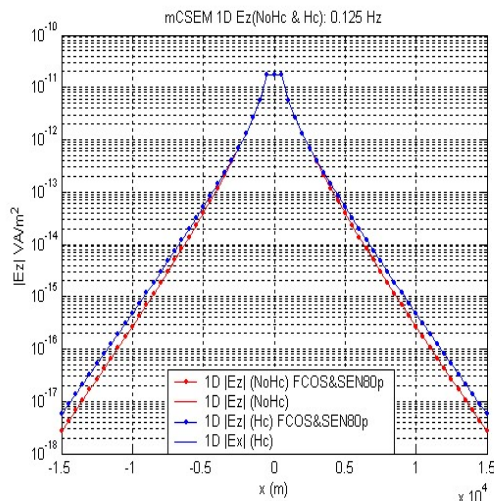
$$\int_0^{+\infty} \text{Exp}(-ak_x^2) \cos(k_x x) dk_x = \text{Exp}(-x^2/4a) \sqrt{\pi/a} (1/2) \quad (3)$$

$$\int_0^{+\infty} 4k_x/(1 + 4k_x^2) \text{sen}(k_x x) dk_x = \text{Exp}(-x/2) (\pi/2) \quad (4)$$

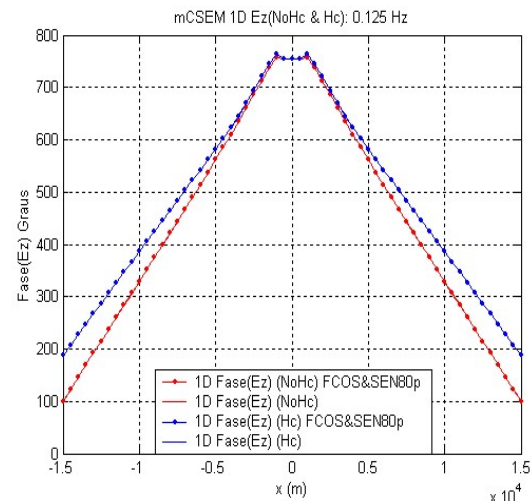
Thus, in a given iteration, we obtain an approximate solution of the cosine transform (Eq.03) (or sine (Eq.04)), based on the optimal digital linear filter potential obtained through the cosine transform (Eq.01) (or sine (Eq.02)), according to the method presented above. From this approximate solution of the cosine transform (Eq.03) (or sine (Eq.04)), and the analytical solution of the cosine transform (Eq.03) (or sine Eq.04), we calculate the relative error between these solutions, both regarding the cosine transform and the sine. Now, from this relative error, we calculate its maximum amplitude, thus obtaining what Almeida (2002) calls the maximum amplitude of the relative error, which is associated with the optimal digital linear filter potential. For the following iteration, the procedure is similar to that described above, thus obtaining an error, maximum amplitude of the relative error, and a digital linear filter potential associated with this error. In this way, between the two errors, we choose the smaller of the two, consequently the digital linear filter associated with this smaller error. Therefore, from the last pre-established iteration, we obtain the optimal digital linear filter of the cosine (or sine) transform. The parameters of these filters ( $a_0$ : initial abscissa,  $t$ : increment of the abscissas, and the weights) are in Tables I and II.

## Results

We used the 1D mCSEM geo-electric model proposed by Rijo (2005), which is expressed by the upper half-space, air; by the sea layer with a thickness  $h_1$  equal to 1500 m and electrical resistivity  $\rho_1$  equal to 0.3  $\Omega\text{m}$ ; by the sedimentary layer with a thickness  $h_2$  equal to 950 m and resistivity  $\rho_2$  equal to 0.8  $\Omega\text{m}$ ; by the heterogeneity layer with hydrocarbons (Hc), thickness  $h_3$  equal to 50 m and resistivity  $\rho_3$  equal to 10  $\Omega\text{m}$ , and finally by the lower half-space with resistivity  $\rho_4$  equal to 0.8  $\Omega\text{m}$ . The horizontal electric dipole is located at coordinates  $x = 0$  m and  $z = 1470$  m, and operating at a frequency of 0.125 Hz. The receivers are fixed on the seabed with a spacing of 500 meters. We applied cosine filters (Table I) and sine filters (Table II) on the  $z$  component of the electric field,  $E_z$  VA/m<sup>2</sup> (RIJO, 2002; SOUZA, 2007; SILVA 2012), obtaining its amplitudes  $|E_z|$  (Hc) (1D model with heterogeneity) and  $|E_z|$  (NoHc) (1D model without heterogeneity), Fig.1, and their phases, in degrees,  $\text{phase}(E_z)(Hc)$  and  $\text{phase}(E_z)(NoHc)$ , Fig.2, and we compared these results with those obtained through the methodology proposed in Almeida (2024).



**Figure 1:** Amplitudes  $|E_z|$  (Hc) e  $|E_z|$  (NoHc), 1D mCSEM.



**Figure 2:** Phases  $(E_z)(Hc)$  e  $(E_z)(NoHc)$ , 1D mCSEM.

Through the proposed digital linear cosine and sine filters, the amplitudes  $|E_z|$  (Hc)  $FCOS\&SEN80p$  and  $|E_z|$  (NoHc)  $FCOS\&SEN80p$  were obtained, Fig.1, and the phases  $(E_z)(Hc)$   $FCOS\&SEN80p$  and  $(E_z)(NoHc)$   $FCOS\&SEN80p$ , Fig.2. Furthermore, through the methodology proposed in Almeida (2024), the amplitudes  $|E_z|$  (Hc) and  $|E_z|$  (NoHc) were obtained, Fig.1, and the phases  $(E_z)(Hc)$  and  $(E_z)(NoHc)$ , Fig.2.

## Conclusions

Both in Fig.1 and in Fig.2, a perfect fit is shown between the values ( $|E_z|$  and phase  $(E_z)$ ) calculated through the proposed cosine and sine filters and the values calculated through the methodology proposed in Almeida (2024). We chose  $E_z$  because in the  $k_x$  and  $k_y$  domains it only uses the sine and cosine transforms, respectively (SILVA, 2012).

Through cosine and sine filters, it is possible to enable the inverse problem of 1D mCSEM, as well as the direct problems of 2.5D and 3D mCSEM. Furthermore, these filters are perfectly suited for the inverse problems of 2.5D and 3D mCSEM, given that the cosine and sine transforms are calculated a large number of times in these inverse problems; thus, by applying the filters, it is possible to minimize the computation time of these problems.

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Table I - Cosine filter

$a_0 = -19.9899144600843 \quad t = 0.303336292120218$			
Pesos do filtro de 1 a 80			
1	2.485113303934135E-008	41	1.572107369764826E-004
2	-9.244308354882383E-008	42	1.082057849190820E-004
3	2.043779708735897E-007	43	2.806105808039508E-004
4	-3.229734890936459E-007	44	2.084218683584285E-004
5	4.273745757377869E-007	45	5.020072914822015E-004
6	-4.842962667994802E-007	46	3.985953047787535E-004
7	5.120083445606555E-007	47	8.999955410789142E-004
8	-4.906297277767509E-007	48	7.578269225205428E-004
9	4.690317463759542E-007	49	1.616719051169505E-003
10	-4.069393362990943E-007	50	1.433782352209870E-003
11	3.814228663053104E-007	51	2.909577802308264E-003
12	-3.047414626964230E-007	52	2.701472822284509E-003
13	3.067292379670296E-007	53	5.245118004907079E-003
14	-2.182102643707155E-007	54	5.071769074128196E-003
15	2.721100292988756E-007	55	9.469052579329239E-003
16	-1.551451408187239E-007	56	9.489982284054645E-003
17	2.931498119149351E-007	57	1.711058211807379E-002
18	-1.109822017444683E-007	58	1.768952285688356E-002
19	3.897944695696852E-007	59	3.090390689535458E-002
20	-7.485844036161920E-008	60	3.276280510055300E-002
21	5.993579838970209E-007	61	5.553510375551612E-002
22	-2.880375105539423E-008	62	5.97115533383853E-002
23	9.921589484198024E-007	63	9.776043624746653E-002
24	5.922626080679326E-008	64	0.103433687930817
25	1.696534972757659E-006	65	0.159287282142184
26	2.507181598816889E-007	66	0.147889469813456
27	2.942665383644049E-006	67	0.185438370740711
28	6.667383547825766E-007	68	4.211751576698009E-002
29	5.140801934446867E-006	69	-0.140286652900759
30	1.545844229243875E-006	70	-0.699713270360336
31	9.021410862421492E-006	71	-0.935220243072245
32	3.354666382183098E-006	72	-8.778056313552508E-002
33	1.588657766989038E-005	73	1.95499126636198
34	6.998091973009412E-006	74	-1.57803946790642
35	2.806167454235985E-005	75	0.645962350129143
36	1.421703854389556E-005	76	-0.190560945449149
37	4.970813187557050E-005	77	4.897564970267435E-002
38	2.833829461791140E-005	78	-1.149541718585231E-002
39	8.828841546345580E-005	79	2.217111385630405E-003
40	5.568316099278159E-005	80	-2.547966128774666E-004

Table II – Sine filter

$a_0 = -9.60965350662927 \quad t = 0.214074798638745$			
Pesos do filtro de 1 a 80			
1	1.362737606956797E-007	41	1.170368114930927E-002
2	-8.178486116138456E-007	42	5.469252085224543E-002
3	2.637754708936981E-006	43	4.242178091105896E-002
4	-5.885156467918189E-006	44	0.106402525333697
5	1.018330410960819E-005	45	0.111053059918389
6	-1.432838045065062E-005	46	0.207436637425196
7	1.676275239907786E-005	47	0.238108117517733
8	-1.578627882380251E-005	48	0.358260635194268
9	1.025044660375015E-005	49	0.365106807148972
10	8.859285534409179E-007	50	0.367368189339183
11	-1.776666544818996E-005	51	7.574198576008981E-002
12	4.111256404950335E-005	52	-0.396538664586427
13	-7.038065965031008E-005	53	-0.993789863340858
14	1.070090215006399E-004	54	-0.676366217408265
15	-1.497779115119835E-004	55	1.02015053861528
16	2.021715700056121E-004	56	1.18787913834167
17	-2.612473936295858E-004	57	-2.10014509127376
18	3.348294167073051E-004	58	1.45108431230409
19	-4.152144807369142E-004	59	-0.686482442694308
20	5.195973061646214E-004	60	0.280967740563066
21	-6.281215900566697E-004	61	-0.116153127256239
22	7.790178421518488E-004	62	5.267996100831550E-002
23	-9.227047316382044E-004	63	-2.663624113139941E-002
24	1.145732730951485E-003	64	1.470399533468361E-002
25	-1.327221811053279E-003	65	-8.592240321566141E-003
26	1.666058296285163E-003	66	5.161179352882024E-003
27	-1.871954304238830E-003	67	-3.105608434478935E-003
28	2.406612107257071E-003	68	1.827179528484088E-003
29	-2.579133983437050E-003	69	-1.023599196773532E-003
30	3.468051095472880E-003	70	5.264671044582002E-004
31	-3.437396552625559E-003	71	-2.324987951844238E-004
32	5.015584125890062E-003	72	7.272156906589094E-005
33	-4.340327462931217E-003	73	1.760696486519881E-006
34	7.350176387308552E-003	74	-2.637429206292491E-005
35	-4.944574893471921E-003	75	2.613416957817724E-005
36	1.108129280454076E-002	76	-1.725352078161171E-005
37	-4.353239282661370E-003	77	8.520944111141081E-006
38	1.754761798973071E-002	78	-3.091724732388354E-006
39	-4.237072615500930E-004	79	7.453818138232652E-007
40	2.979993539595359E-002	80	-9.007018714231406E-008