A simple review of the Common-Reflection-Point (CRP) method

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Abstract

The Common-Reflection-Point (CRP) method combines a stacking traveltime operator with a source-receiver gather on which the stacking is performed. Such a procedure is seen to produce cleaner sections in which, most particularly are free from reflection-point-dispersal noise. In this paper, the CRP method is reviewed, with new, attractively simple expressions for the CRP traveltime operator and source-receiver gather being proposed. We hope that the present analysis may motivate a broader use of the CRP method for seismic imaging and processing purposes.

Introduction

Traveltime stacking is a well-recognized processing technique for seismic imaging purposes. As such, stacking operators that allow for more reliable and better-quality results are always in high demand. One of the main hurdles of stacking operators is *reflection point dispersal*, leading to lack of focusing on reflection points of interest. The CRP method combines a *stacking operator* with a corresponding *source-receiver gather* upon which the stacking is performed. Roughly speaking, CRP is not a new approach, but falling into the broader framework of Offset Continuation, designed to transform common-offset sections from one offset to another (see, e.g., Perroud et al., 1996; Santos et al., 1997; Coimbra et al., 20013; 2016).

As seen below, the traveltime stacking operator and source-receiver gather are expressed in the the form of analytic multi-parameter functions, with parameter extracted by coherency analysis (semblance) directly applied to the input data. For 2D seismic acquisition, a few of such CRP expressions are available in the literature, those being applied to several seismic processing purposes. Extensions to full 3D data are still a challenging task. Still considering 2D seismic sections, attractive, simplified expressions for the CRP traveltime and source-receiver gather expressions are here provided. Being derived by elementary results of plane-geometry, our expressions coincide and can easily replace those corresponding available counterparts.

Method

We consider 2D seismic data acquired on a horizontal plane. For simplicity, we assume continuous data, with data points $Q = (m, h, t)$ specified by scalar coordinates of midpoint m, half-offset h and time t coordinates. We consider a given data point $Q_0 = (m_0, h_0, t_0)$, referred as a *central point* supposed to belong to a primary-reflection event from a target depth reflector Σ, having P_Σ as the reflection point. Both Σ and P_Σ are throughout supposed fixed and nonidentified.

For a given half-offset $h \neq h_0$, our aim is to find data points $Q = (m, h, t)$ that are also primary reflections from Σ and moreover share the same reflection point P_Σ. As depicted in Figure 1, we consider the simple 2D earth model of a single reflector Σ, overlain by a homogenous medium of constant velocity V. Cartesian coordinates are such that the seismic line coincides with the x-axis.

CRP ZO-FO traveltime and midpoint: We suppose that our central point is a zero-offset (ZO) point $Q_{ZO} = (m_{ZO}, 0, t_{ZO})$, supposed to be a zero-offset (ZO). As above indicated, for a given half-offset $h \neq 0$, our aim is to find a corresponding finite-offset (FO) data point $Q = (m, h, t)$, that is also a primary-reflection point from Σ and moreover shares P_Σ as reflection point. Under those circumstances, the traveltime $t = t(m, h)$ along the ray $\boldsymbol{s}P_\Sigma\boldsymbol{r}$ is exactly given by the double-square-root (DSR) equation

$$t = \frac{1}{2}(t_s + t_r)\ , \tag{1a}$$

$$t_s = \sqrt{t_{ZO} + a_{ZO}(\Delta m_{ZO} - h)^2 + 4h^2/V^2}, \qquad t_s = \sqrt{t_{ZO} + a_{ZO}(\Delta m_{ZO} - h)^2 + 4h^2/V^2}, \tag{1b}$$

where $\Delta m_{ZO} = m - m_{ZO}$ is *midpoint dislocation* and, as indicated, V is the *constant velocity* of the overburden. Finally, a_{ZO} is the (horizontal) *midpoint slope* $t = t(m, h)$ evaluated at m_{ZO},

$$a_{ZO} = \left(\frac{\partial t}{\partial m}\right)(m_0, 0) = \frac{2\sin\alpha_{ZO}}{V}. \tag{2}$$

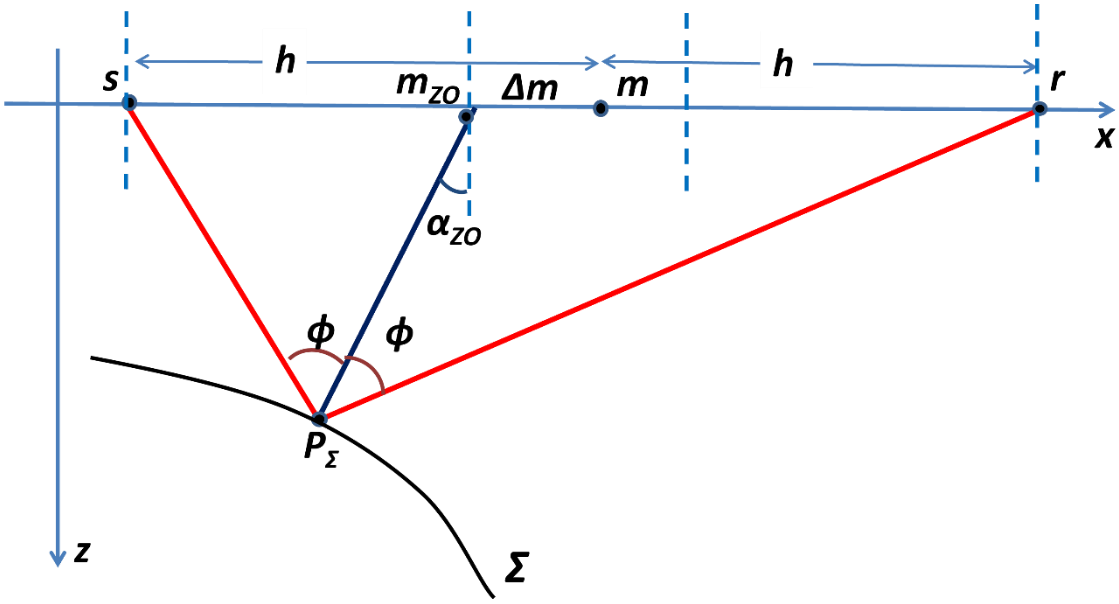

Figure 1: 2D model for the ZO-FO CRP situation

Derivation of CRP traveltime operator: From Figure 1, we verify that

$$\overline{\boldsymbol{s}P_\Sigma} = Vt_s, \qquad \overline{P_\Sigma\boldsymbol{r}} = Vt_r\,, \qquad \overline{P_\Sigma\boldsymbol{r}} = Vt_r\ , \tag{3a}$$

$$\overline{\boldsymbol{m}_0P_\Sigma} = \frac{V\,t_{ZO}}{2}, \qquad \overline{\boldsymbol{sm}_0} = h - \Delta m_{ZO}\ , \qquad \overline{\boldsymbol{m}_{ZO}\boldsymbol{r}} = h + \Delta m_{ZO}. \tag{3b}$$

Application of the *law of cosines* to the triangles $\boldsymbol{s}P_\Sigma\boldsymbol{m}_0$ and $\boldsymbol{m}_0P_\Sigma\boldsymbol{r}$, allow us to write

$$V^2{t_s}^2 = \left(\frac{V\,t_{ZO}}{2}\right)^2 + (h - \Delta m_{ZO})^2 - 2\left(\frac{V\,t_{ZO}}{2}\right)(h - \Delta m_{ZO})\sin\alpha_{ZO}, \tag{4a}$$

$$V^2{t_r}^2 = \left(\frac{V\,t_{ZO}}{2}\right)^2 + (h + \Delta m_{ZO})^2 + 2\left(\frac{V\,t_{ZO}}{2}\right)(h + \Delta m_{ZO})\sin\alpha_{ZO}, \tag{4b}$$

Application of the *bisection theorem* to the triangle $\boldsymbol{s}P_\Sigma\boldsymbol{r}$ produces, after some algebraic manipulations the expressions

$$t_r = \left(\frac{h+\Delta m_{ZO}}{h-\Delta m_{ZO}}\right)t_s, \text{ from which we obtain } t_r = \left(\frac{h-\Delta m_{ZO}}{2h}\right)t \text{ and } t_s = \left(\frac{h+\Delta m_{ZO}}{2h}\right)t. \tag{5}$$

Multiplying equations (4a) and (4b) by $h+\Delta m_{ZO}$ and $h-\Delta m_{ZO}$, respectively followed by summation of and further rearrangement, the sought-for CRP traveltime can be written as

$$t^2 = \frac{4h^2}{V^2} + \frac{2t_{ZO}^2 h^2}{h^2-(\Delta m_{ZO})^2}\,. \tag{7}$$

Derivation of CRP midpoint: To obtain the CRP midpoint expression, we use equations (4a) and (4b), however multiplying the first by $(h+\Delta m_{ZO})^2$ and the second by $(h-\Delta m_{ZO})^2$, followed by subtraction. We find

$$m = m_{ZO} + \frac{2a_{ZO}h^2}{t_{ZO}+\sqrt{t_{ZO}^2+4a_{ZO}^2h^2}}\,. \tag{8}$$

Substitution of the above into equation (5), leads to the CRP traveltime alternative expression

$$t_n^2 = \frac{t_{ZO}}{2}\left(t_{ZO} + \sqrt{t_{ZO}^2 + 4a_{ZO}^2h^2}\right). \tag{9}$$

Midpoint slope continuation: For our purposes, we will also need the *midpoint-slope continuation* equation

$$a = \left(\frac{\partial t}{\partial m}\right)(m,h) = \left(\frac{t_n^2}{t_{ZO}\,t}\right)a_{ZO}, \tag{10}$$

which relates the FO and ZO midpoint slopes a and a_{ZO}. Equation (10) can be obtained by differentiation of equation (5) with respect to midpoint.

CRP FO-FO traveltime and midpoint: We are now ready to generalize expressions (7-9) to the case where of an FO central point $Q_0=(m_0,h_0,t_0)$, $(h_0\neq 0)$. As before, Q_0 is assumed to be a primary-reflection data point from Σ having P_Σ as reflection point. For a given half offset $h\neq h_0$, the CRP problem is to find the midpoint and traveltime pairs $(m_h, t_{h)}$ and such that $Q_h =$ $--(m_h,h,t_h)$, is a primary reflection data point from Σ with reflection point P_Σ. For convenience, we set V_0 to denote the velocity of the homogeneous overburden of Σ. Under conceptual consideration of the ZO central data point $Q_{ZO}=(\boldsymbol{m}_{ZO},0,t_{ZO})$, the midpoint slope continuation (10) can be seen to relate a_0 and a_h (midpoint slopes associated with Q_0 and Q_h) with a_{ZO} (corresponding midpoint slope associated with Q_{ZO}). With obvious notations, we can write

$$a_{ZO} = \left(\frac{t_{ZO}\,t_0}{t_{n_0}^2}\right)a_0 = \left(\frac{t_{ZO}\,t_h}{t_{n_h}^2}\right)a_h, \quad \left(\text{with} \quad t_{n_0}^2 = t_0^2 - \frac{4h^2}{V_0^2} \quad \text{and} \quad t_{n_h}^2 = t_h^2 - \frac{4h^2}{V_0^2}\right). \tag{11}$$

Substituting into equation (8) and (9), we obtain

$$t_{n_0}^2 = \left(\frac{t_{ZO}^2}{2t_{n_0}^2}\right)\left(t_{n_0}^2 + \sqrt{t_{n_0}^4 + 4t_0^2a_0^2h_0^2}\right), \quad \text{and} \quad t_{n_h}^2 = \left(\frac{t_{ZO}^2}{2t_{n_0}^2}\right)\left(t_{n_0}^2 + \sqrt{t_{n_0}^4 + 4t_0^2a_0^2h^2}\right). \tag{12}$$

$$m_0 = m_{ZO} + \frac{2t_0a_0h}{t_{n_0}^2+\sqrt{t_{n_0}^4+4t_0^2a_0^2h^2}} \quad \text{and} \quad m_h = m_{ZO} + \frac{2t_0a_0h}{t_{n_0}^2+\sqrt{t_{n_0}^4+4t_0^2a_0^2h^2}}, \tag{13}$$

Taking into account the relations

$$t_{n_h}^2 = t_{n_0}^2\left(\frac{t_{n_h}^2}{t_{n_0}^2}\right) \quad \text{and} \quad m_h - m_0 = (m_h - m_{ZO}) - (m_0 - m_{ZO}), \tag{14}$$

we obtain our final expressions for the CRP traveltime and midpoint, namely

$$t_h^2 = \frac{4h^2}{V_0^2} + t_{n_0}^2 \left[\frac{t_{n_0}^2 + \sqrt{t_{n_0}^4 + 4t_h^2 a_0^2 h^2}}{t_{n_0}^2 + \sqrt{t_{n_0}^4 + 4t_0^2 a_0^2 h_0^2}} \right]. \tag{15}$$

$$m_h = m_0 + \left(\frac{2t_0 a_0 h}{t_{n_0}^2 + \sqrt{t_{n_0}^4 + 4t_0^2 a_0^2 h^2}} - \frac{2t_0 a_0 a_0 h_0}{t_{n_0}^2 + \sqrt{t_{n_0}^4 + 4t_0^2 a_0^2 h_0^2}} \right). \tag{16}$$

The CRP traveltime and midpoint above coincide with the ones available in the literature (see, e.g., Coimbra et al. (2016), Perroud et al. (1996) and Santos et al. (1997).

Conclusions

The CRP method aims to produce stacked sections which primary reflections are significantly enhanced. Those sections and free from reflection-point dispersion noise. CRP stacking relies, not only on traveltime operator, but also on dedicated source-receiver pairs designed for single reflection point illumination. In this way, reflection-point dispersal is very much attenuated. In this paper a new version of the expressions of CRP travelime and midpoint is presented besides being attractively simple, have a more straightforward intuitive derivation. We hope that such good properties may contribute to a better understanding of the CRP method and motivate its use for a variety of seismic processing and imaging purposes.

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