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Geometry-Based Ray Tracing Method for Complex Subsurface Models

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Abstract

Our objective in this work is to introduce a geometry-based ray-tracing method and demonstrate its application in characterizing of subsurface models. This algorithm is designed to minimize the number of operations required to determine the input and output points of ray paths in layered homogeneous geological models with intricate geometry. Implemented in C++, the algorithm utilizes an unstructured mesh of tetrahedral elements, which are widely used in structural finite element analysis. Preliminary results from modeling compressional wave propagation in a 2.5D nonhomogeneous and isotropic geological model have shown a significant capability for handling complex geological structures, particularly in modeling multiple reflected waves.

Introduction

The propagation of seismic waves in complex 3D heterogeneous structures is a complicated process. Analytical solutions of the elastodynamic equation for such media are not yet know. However, the most common approaches to investigating this process rely on methods based on direct numerical solutions of the elastodynamic equation, such as grid-point methods as well as approximate high-frequency asymptotic methods (Cerveny V., 2002). Based on an asymptotic solution of the elastodynamic equation, more commonly known as ray-series methods, asymptotic methods are only approximate. These methods can be applied to compute not only travel times but also the ray amplitudes, synthetic seismograms, and particle ground motions.

The seismic ray method can be divided into two fundamental parts: kinematic and dynamic. The kinematic part calculates seismic rays as characteristics of the eikonal equation, a nonlinear first-order differential equation for travel time, describing compressional or shear waves propagating in sufficiently smooth isotropic media. Meanwhile, the dynamic part consists of the evaluation of the vectorial values of amplitudes of the displacement vector. The propagation of seismic waves in inhomogeneous media would be incomplete without considering the interaction of these waves with geological layers interfaces. The exact process of reflection, transmission or refraction of plane waves at the interface requires a deep physical analysis and has only a local character. An extensive discussion on the calculation of rays associated to reflection, transmission and anomalous waves was presented in Cerveny V., 2002.

In the literature, many numerical methods exist to solve the ray-tracing system. However, the objective this work is to introduce an efficient computational inexpensive geometry-based ray-tracing method (GbRT) and demonstrate its application in the characterization of subsurface models. In the next section, we present this method, developed to compute ray paths in layered homogeneous geological models with significant heterogeneity.

A Geometry-Based Ray Tracing Method

In the kinematic ray-series approximation, the travel time of a high-frequency wave propagating through homogeneous media with velocity V is described by the eikonal equation, $(\nabla T)^2 = \boldsymbol{p} \cdot \boldsymbol{p}$, where $\boldsymbol{p}$ is the slowness vector (inverse of wave velocity V) and $T(\boldsymbol{x})$ represents the arrival time of a wavefront at a point $\boldsymbol{x}$ (Podvin and Lecomte, 1991; Schuster G. T., 2017). In more general cases, such as heterogeneous media, where the differential operator ∇ is not necessarily well-defined, a direct solution does not exist. For complex geological structures, layered models are commonly built with arbitrarily oriented interfaces that bound homogeneous layers.

In this context, the ray associated with the front of an elementary wave propagating through a homogeneous layer follows a straight line. This simplification enables the representation of the ray tracing method as a tree data structure. Each ray corresponds to a tree node containing information such as travel time, source and virtual receiver location at interfaces, unit direction vector, and elementary wave type. The travel time is determined by multiplying the path arc length by the inverse of the wave velocity. Each branch associated with the node represents a new ray originating from a Snell's law analysis at the interface.

The core aspect of this geometric perspective on the ray-tracing method is determining the final position of the ray at the lower interface. If a regular three-dimensional voxel grid of a layered seismic model is used, the Real-Line Voxelization algorithm, presented by Aleksandrov, Slatanov and Heslop (2021), describes the ray path as a sequence of input-output points at voxel faces This algorithm is characterized by minimizing the number of operations required to obtain the output points. If each interface is discretized in a regular triangular mesh, the Real-Line Voxelization algorithm is complemented by the Line-Interface Intersection algorithm (Möller and Trumbore,1997) to calculate the final intersection point. The drawback of this approach arises in rough interfaces. The regular voxel discretization is not necessarily the best option if the model contains an intrusion between layers, which is common in basalt and satl subsurface structures. To address this, we propose extending the Line-Interface Intersection algorithm to a non-uniform adaptive mesh that adjusts the mesh, focusing finer meshes in areas where it is necessary, and coarser meshes elsewhere, according to the users' choice.

Considering an unstructured mesh composed of tetrahedron elements (Figure 1). If each geometric element in the mesh has triangle faces, let a ray Ω, with origin in $\boldsymbol{O}$, arc-length s and normalized direction $\boldsymbol{d}$ defined as,

$$\boldsymbol{R}(s) = \boldsymbol{O} + s\,\boldsymbol{d}, \tag{1}$$

and a triangle defined with its vertices $\boldsymbol{V}_1, \boldsymbol{V}_2$ and $\boldsymbol{V}_3$. The ray/triangle intersection problem is completely solved if the point (u, v) on the surface is uniquely determined such that

$$\boldsymbol{O} + s\,\boldsymbol{d} = (1 - u - v)\boldsymbol{V_1} + \boldsymbol{V_2} + \boldsymbol{V_3} \tag{2}$$

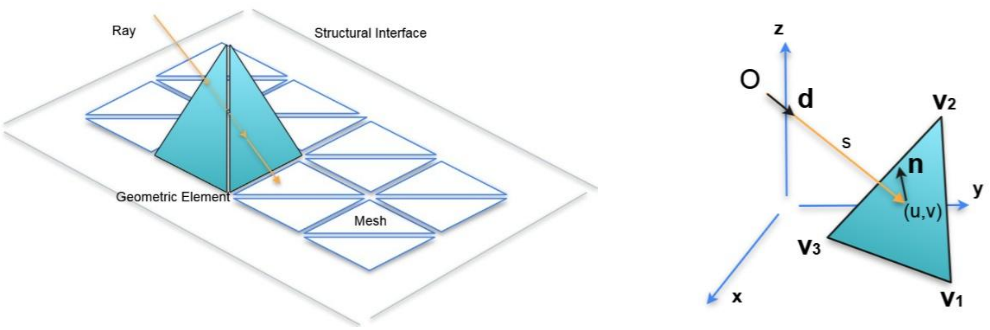

Figure 1: A simple regular three-dimensional tetrahedral grid of a seismic model with a single structural interface (left). Notation used in the line-triangle intersection scheme (right).

Denoting $\boldsymbol{E}_1 = \boldsymbol{V}_2 - \boldsymbol{V}_1$, $\boldsymbol{E}_2 = \boldsymbol{V}_3 - \boldsymbol{V}_1$ and $\boldsymbol{T} = \boldsymbol{O} - \boldsymbol{V}_1$, the solution to Equation (2) is obtained by

$$\begin{pmatrix} s \\ u \\ v \end{pmatrix} = \frac{1}{(\boldsymbol{d} \times \mathbf{E}_2) \cdot \boldsymbol{E}_1} \begin{pmatrix} (\boldsymbol{T} \times \boldsymbol{E}_1) \cdot \boldsymbol{E}_2 \\ (\boldsymbol{d} \times \boldsymbol{E}_2) \cdot \boldsymbol{T} \\ (\boldsymbol{T} \times \boldsymbol{E}_1) \cdot \boldsymbol{d} \end{pmatrix} = \frac{1}{\boldsymbol{P} \cdot \boldsymbol{E}_1} \begin{pmatrix} \boldsymbol{Q} \cdot \boldsymbol{E}_2 \\ \boldsymbol{P} \cdot \boldsymbol{T} \\ \boldsymbol{Q} \cdot \boldsymbol{d} \end{pmatrix}, \tag{3}$$

where $\boldsymbol{P} = \boldsymbol{d} \times \boldsymbol{E_2}$ and $\boldsymbol{Q} = \boldsymbol{T} \times \boldsymbol{E}_1$. One advantage of this method is that the plane equation need not be computed nor be stored, which can amount to significant memory savings. To ensure numerical stability, the test that eliminates parallel rays must compare $\boldsymbol{n} \cdot \boldsymbol{d}$ to a small interval around of machine precision, with a properly adjust of ε-value. This algorithm is complemented by the approach presented by Aleksandrov, Slatanov and Heslop (2021), which considers the new geometrical element and describes the ray path as a sequence of points at tetrahedral faces.

A part of the algorithm requires an unstructured mesh composed of tetrahedral elements. This kind of mesh has a considerable benefit in modeling intricate geological structures. To illustrate this benefit, Figure 2 shows a structural model representing the interfaces of a geological formation composed by low velocity layers (LVs) within thick (1 km) basalt layer, as well as sills and dykes. A zoom-in of an unstructured tetrahedral mesh of model composed to approximately 350K elements is presented in the right side of the figure. The irregular geometry of the unstructured mesh and the variable cell sizes-particularly fine near principal interfaces and coarser in the interior layers-accelerate the geometry ray-tracing algorithm compared to the Real-Line Voxelization algorithm.

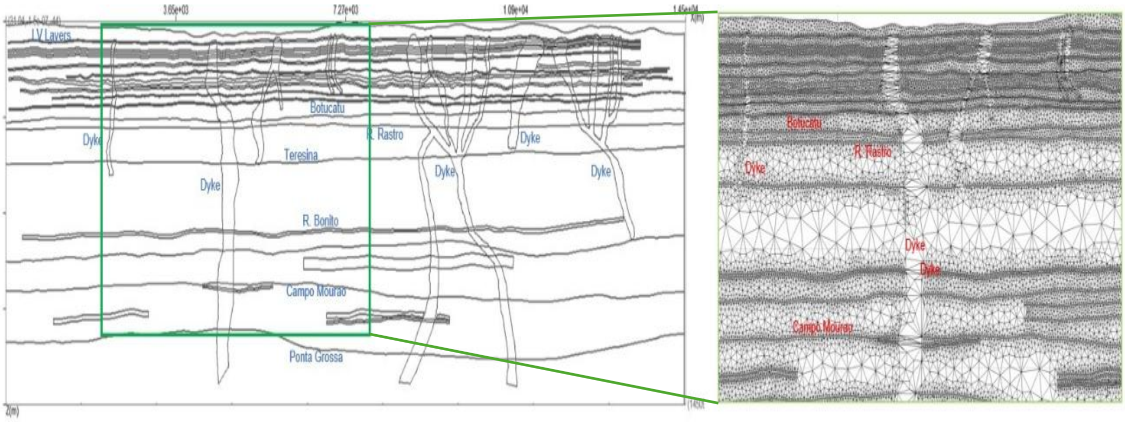

Figure 2: Vertical view of a 2.5D structural model (left) and detailed view of the unstructured tetrahedral mesh (right).

To demonstrate the potential of the method, the algorithm was implemented in C++, and the next section presents the results obtained using the described approach to simulate multiple reflected wave propagation in a simplified basalt model.

Results

One of the main goals in implementing of the GbRT method is the ability to model and characterize multiple reflected waves from thin layers. These seismic waves are events that have been reflected more than once and can make it more difficult to identify deeper seismic events in real seismic data. Intrabasalt low-velocity (vesicular) layers are characterized by a lower seismic velocity compared to the surrounding material (massive basalt) and are known for generating a series of internal multiples.

Consider a single-layer homogeneous 2.5D model of basalt with depth $1000m$ and extension $5000m$, characterized by an average compressional velocity of 5600 m/s, which includes two flat low-velocity zones (LV_1 and LV_2) with a velocity $3800\ m/s$. This model is used to simulate multiple reflected waves using the GbRT method described in the previous section. With a single compressional source located at the top and center of the model, the ray paths associated with direct wave and the multiples generated in LV_1 and LV_2, are showed in the Figure 3. The LV_1 and LV_2 layers, measuring $50m$ and $25m$ in thickness and situated at depths of $250m$ and $350m$, respectively, have been shown to have the potential to generate multiples waves, as illustrated by the ray-path set.

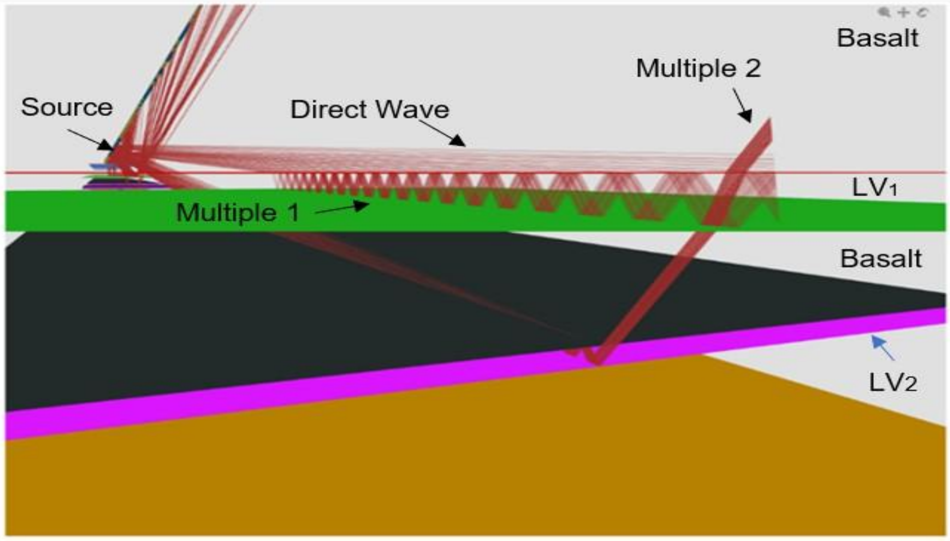

Figure 3: Ray path for a simplified model with two intrabasalt low-velocity (vesicular) layers. The red line represents the ray paths associated with multiple reflected waves generated within LV_1 and LV_2 layers.

Conclusions

We present an efficient computationally inexpensive GbRT method to model elementary wave propagation in intricate geological structures. Using an unstructured adaptive mesh composed of tetrahedral elements, which allows for modeling complex geometries, the algorithm demonstrates optimal results in simulating multiple reflective waves within thin intrabasalt low-velocity layers in a simplified geological model.

Acknowledgments

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