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# Modeling Geological Structures Using the Monte Carlo Method

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## Abstract Summary

This study presents a numerical approach for simulating pressure distribution in subsurface geological structures, focusing on anticlines and synclines commonly associated with hydrocarbon accumulation. The method combines linear elasticity theory with the Beltrami-Michell equations, solved as a boundary value problem using the Monte Carlo method through random walks.

Geometries are defined via 2D Gaussian functions, extended with multiple layers of varying densities to simulate realistic heterogeneities. Results from regular-grid simulations show the method's effectiveness in capturing pressure gradients and identifying low- and high-pressure zones, supporting its applicability in geophysical exploration and reservoir evaluation.

The main purpose of this work is to demonstrate that the existence of a fluid accumulation zone requires a high-pressure zone acting mechanically as a sealing formation.

## Introduction

Accurate characterization of pressure distribution in the subsurface is a key factor in identifying potential hydrocarbon accumulation zones within sedimentary basins. Geological phenomena such as low- and high-pressure zones act as natural traps that, depending on their geological configuration and the elastic properties of the surrounding rocks, may favor the migration and retention of fluids such as oil, gas, and water.

The present modeling is based on post-stack/migration seismic data, from which information on P- and S-wave velocities, density, and reflector geometry is obtained. The studied model does not account for diagenesis or fracturing and is analyzed exclusively under static conditions, with no temperature changes. Moreover, petroleum geology principles are directly applied to the solution obtained as a result of the modeling based on the Beltrami-Mitchell model (Hantschel and Kauerauf, 2009; Mohriak, 2003).

This study proposes a direct physical modeling approach for such structures based on solid mechanics, particularly linear elasticity theory. The formulation relies on the Beltrami-Michell equations, which describe the distribution of stress tensor components in isotropic elastic media subjected to the gravitational forces. This system of partial differential equations is solved as a boundary value problem (BVP), with Dirichlet conditions.

The numerical solution is achieved using the Monte Carlo method. Rather than discretizing the differential equation directly, a large number of random walks are simulated from interior points of the domain ending at the boundary, where there are cells with a known pressure value. The statistical average of these samples provides an estimate of the pressure at the starting point, thereby connecting probabilistic simulation with physical modeling.

## Theoretical Foundations

The numerical modeling developed in this study is based on the theory of linear elasticity, which governs the mechanical behavior of continuous and isotropic media under stress. The relationship between the stress tensor $\sigma_{ij}$ and the strain tensor $\varepsilon_{ij}$ is described by the generalized Hooke's law:

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij}, \tag{1}$$

The mean rock pressure, $P_r$ is defined from the first invariant of the stress tensor:

$$P_r = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}). \tag{2}$$

The vertical stress due to overburden pressure, assuming variable density, $\rho(z)$ is expressed as:

$$\sigma_{zz}(z) = G\int_0^z \rho(z)\,dz, \tag{3}$$

where $G$ is the gravitational constant. Horizontal stress components are derived assuming lateral symmetry and use the $\gamma = V_S/V_P$ velocity ratio.

The Beltrami-Michell equations, derived from equilibrium and constitutive relations, govern the stress distribution and are given by :

$$\nabla^2\sigma_{ij} = \frac{3}{1+\nu}\frac{\partial^2 P_r}{\partial x_i \partial x_j} - \frac{\nu}{1-\nu}\delta_{ij}\nabla\cdot\mathbf{F} - \left(\frac{\partial F_i}{\partial x_j} + \frac{\partial F_j}{\partial x_i}\right), \tag{4}$$

where $\nu$ is the Poisson's ratio and $\mathbf{F}$ is the internal body force vector (Saad, 1975).

The approximate form of the (4) equation considers a smooth gradient for $P_r$ within the volume and when $\mathbf{F} = 0$. These considerations approximate equation (4) to the Laplace equation, $\nabla^2 P_r = 0$, which is the governing equation of the simplified experiment.

## Monte Carlo

The numerical solution of equation (4) uses the probabilistic approach named the Monte Carlo method. A large number $N$ of random walks are simulated starting from each internal point of a 2D grid. Each walk ends at a boundary where the pressure value is known. The pressure at the starting point is then estimated by the average of the boundary values reached:

$$P(x,z) \approx \frac{1}{N}\sum_{i=1}^{N} P_{\mathsf{b}}(x_i, z_i). \tag{5}$$

The random walk is implemented with equal probabilities for the four directions (north, south, east, west), within a regular mesh with spacing $\Delta x = \Delta z = 10$ m.

## Geological Structure Representation

We compare two inverse geological structures, the anticline and syncline, whose geometries are analytically defined using normalized Gaussian functions:

$$\text{Anticline:} \quad z(x) = z_0 - K\exp\left(-\frac{(x-x_0)^2}{s_x^2}\right) \tag{6}$$

$$\text{Syncline:} \quad z(x) = z_0 + K\exp\left(-\frac{(x-x_0)^2}{s_x^2}\right) \tag{7}$$

where $z_0$ is the base depth, $x_0$ the center of the structure, $K$ the amplitude, and $s_x$ the width parameter. Multiple geological layers are included by stacking additional Gaussian-shaped interfaces above and below the main structure, each with its own density $\rho_i$, which influences the resulting pressure field.

# Results

The mesh definition is the same for any structure like the anticline and syncline. The simulations were constructed with necessarily uniform spacing $\Delta x = \Delta z = 10$ m. The mesh spans a 1000 m $\times$ 1000 m domain and serves as the computational grid for the Monte Carlo simulation.

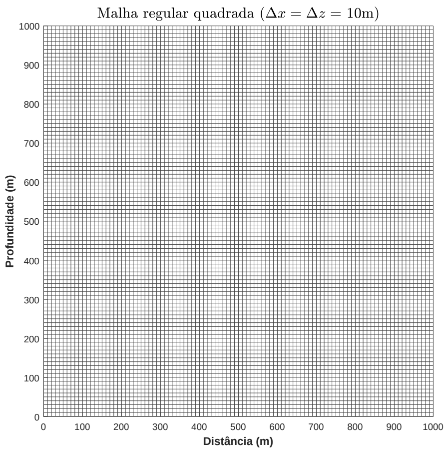


Figure 1: Regular square mesh used in both models.

## Anticline and Syncline Models

Figure 2 shows the result of the modeling aiming to represent an anticline, a fluid accumulation zone, which requires a high-pressure layer at its top, on the outer part. If a low-pressure layer is present at the top, the anticline cannot function as a fluid accumulator.

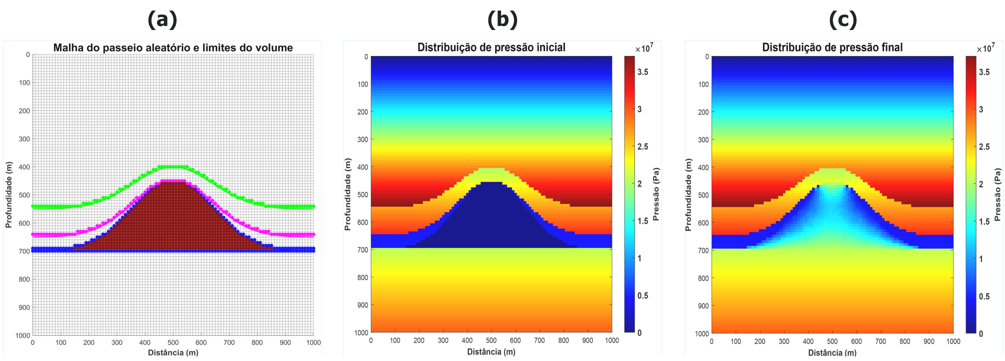


Figure 2: Anticline results: (a) Boundaries and mesh. (b) Initial pressure. (c) Final pressure.

Figure 3 shows the result of the modeling aiming to represent a syncline, a zone that does not appear to act as a fluid accumulator, even in the simulation—unlike the anticline case. As we can observe, a high-pressure layer is not sufficient to create the low-pressure zone required for fluid accumulation. The only possible locations for fluid presence are in the horizontally flattened lateral flanks.

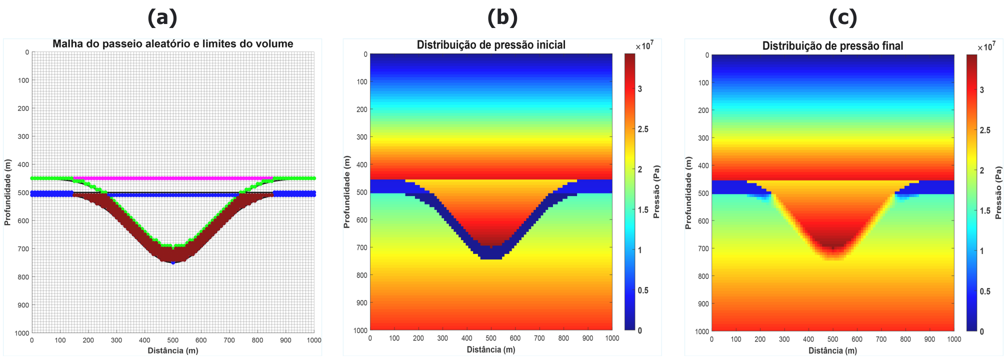


Figure 3: Syncline results: (a) Boundaries and mesh. (b) Initial pressure. (c) Final pressure.

## Conclusions

This study demonstrates the use of the Monte Carlo method for pressure modeling in geological structures, offering direct application in reservoir geophysics. By integrating physical theory (linear elasticity and Beltrami-Michell equations) with probabilistic computation (random walks), the method provides a powerful, flexible, and computationally efficient tool for analyzing complex geological environments.

Introducing multiple geological layers with distinct densities allows the simulation of realistic stratigraphic heterogeneity, producing more accurate pressure fields.

The anticline model reproduced natural accumulation zones, while the syncline model revealed pressure retreats, highlighting their potential as hydraulic traps or barriers.

Compared to conventional deterministic methods, the proposed approach offers numerical efficiency, stability, and adaptability, as outlined by Dimov (2008); Kroese et al. (2011); Rubinstein and Kroese (2017). It also opens avenues for future use of MCMC algorithms (e.g., Metropolis-Hastings) in inversion and calibration with real data.

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