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## **2D inversion of synthetic data from Multi-Frequency Electromagnetic Method**

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## 2D inversion of synthetic data from Multi-Frequency Electromagnetic Method

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### Abstract

This work presents a study on two-dimensional inversion models of Multi-Frequency Electromagnetic Method (EMMF) data, aiming at the characterization of geological structures with electrical conductivity contrasts. The inversion scheme is based on the Gauss-Newton method, incorporating the Marquardt strategy and a smoothness regularization to enhance stability and regularization throughout the iterative process. The Jacobian matrix, essential for linearizing the inverse problem, is computed using the Self Adjoint method, which allows for accurate and efficient derivative calculations without compromising computational performance. Numerical experiments were conducted on models containing both conductive and resistive targets to evaluate the reconstruction capability of the proposed approach. The results showed improved resolution and accuracy in recovering conductive targets, which is attributed to the higher sensitivity of the EMMF data.

### Introduction

Inductive electromagnetic methods are widely used in Geophysics to investigate the electrical properties of the subsurface, with applications in areas such as mineral prospecting and groundwater exploration. One such method is the Multi-Frequency Electromagnetic Method (EMMF), which employs a large square loop as a source and operates over a broad frequency range. Its principle relies on a reference table (Ábaco) containing the real and imaginary parts of the radial component of the magnetic field, measured at long distances — a feature that is uncommon among inductive methods.

Since its introduction by Dr. Carlos Alberto Dias (Dias, 1968; Sato, 1979), several studies have contributed to the advancement of EMMF. Works by da Silva e Silva (2012), Nunes (2014), and da Piedade (014a) have respectively addressed two-dimensional modeling, the influence of topography on field measurements, and the first 1D and 2D data inversions. However, the approach used by Da Piedade to compute the Jacobian matrix — based on perturbations and parallel programming — proved inefficient for 2D inversion, due to the high computational cost associated with repeatedly solving the forward problem.

This study focuses on the two-dimensional inversion of EMMF data for the characterization of subsurface geological structures with electrical conductivity contrasts. The inversion algorithm is based on the Gauss-Newton method, combined with the Marquardt strategy and a global smoothness regularization to ensure stability and convergence throughout the iterative process. To improve computational efficiency, the Jacobian matrix is calculated using the Self-Adjoint method, enabling accurate and efficient sensitivity analysis without the need for repeated forward problem solutions. Synthetic experiments involving both conductive and resistive targets were performed, demonstrating better resolution and accuracy for conductive anomalies due to the higher sensitivity of the EMMF to such features. The results confirm the potential of the proposed methodology for improving inversion performance, particularly in applications involving conductive geological targets.

### Methodology

In the forward modeling of EMMF data for 2D media, Maxwell's equations are solved for homogeneous, isotropic, plane-parallel layers in terms of electrical resistivity, as well as embedded bodies with anomalous resistivities relative to the surrounding layers. Both the layers and these bodies are

assumed to be infinite in the Cartesian  $y$ -direction. The presence of these anomalous structures introduces both vertical and lateral variations in resistivity, which characterizes the medium as two-dimensional (2D). The electromagnetic fields are computed using numerical techniques, typically the Finite Element Method (FEM), which provides field components at discrete points in the model domain (Jin, 2014).

The forward modeling code used in this study was originally developed in the master thesis by Nunes (2014) and later adapted for inversion applications by da Piedade (2014a). The code calculates the magnetic components  $H_r$  and  $H_z$ , generated by a circular loop source, for various frequencies and receiver positions (da Silva e Silva, 2012).

The inversion program for the EMMF method, used in this work, employs a modified Gauss-Newton algorithm combined with the Marquardt strategy (Marquardt, 1963) to minimize the data misfit functional. This functional is defined as

$$\Phi(p) = \sum_{i=1}^N [y_i - f_i(p)]^2 + \alpha R(p), \quad (1)$$

where  $y_i$  is the  $i$ -th observed geophysical data point (EMMF data),  $f_i(p)$  is the predicted data from the mathematical model for a given parameter set  $p$ , and  $R(p)$  is the regularization functional. The parameter vector  $p$  corresponds to the subsurface electrical resistivity distribution to be estimated. The regularization functional introduces additional constraints to the inverse problem, aiming to stabilize the solution by minimizing variations in the parameter values. In this work, a global smoothness regularization is employed. The scalar  $\alpha$  is the regularization parameter, which controls the relative importance of the regularization term in the inversion process. In summary, the inversion seeks to minimize the total functional to find a model that best fits the data while maintaining reasonable smoothness in the resistivity distribution.

## Results

In both models (Figures 1 and 3), synthetic EMMF data were generated using nine frequencies ranging from 3 Hz to 316 Hz. The transmitting loop, responsible for inducing the electromagnetic fields, was positioned at the origin of the coordinate system and had a radius of 340 meters. A total of 51 receivers were uniformly distributed along the profile, covering distances from 3.2 km to 5.6 km away from the center of the source.

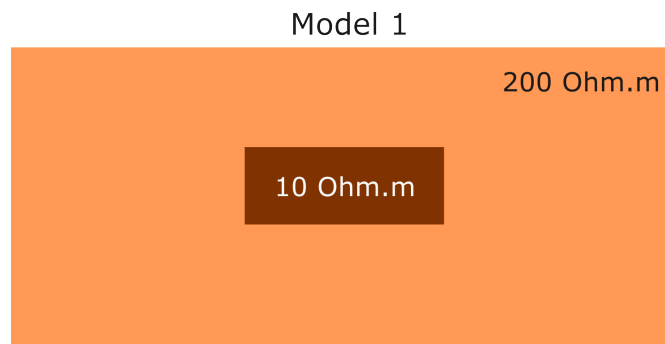


Figure 1: Model 1 to generate EMMF synthetic data to inversion with a conductive target (10 Ohm-m) in a half-space with 200 Ohm-m.

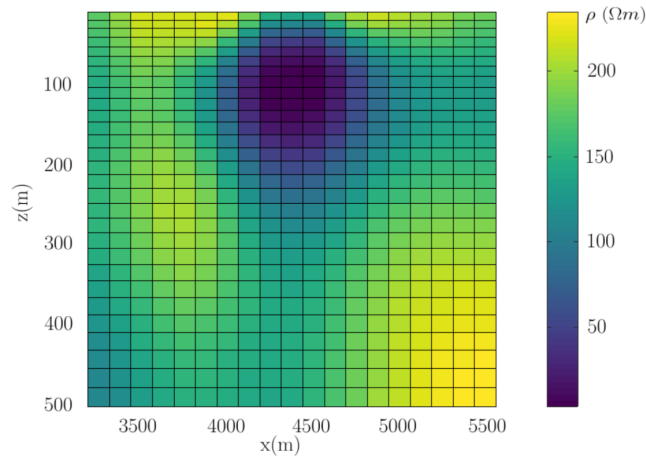


Figure 2: Inversion result of synthetic model 1, with Global Smoothness regularization. The regularization parameter was  $\alpha = 0.001$  and the start model was 200 ohm-m.

The inversion results showed a clear distinction in the recovery of the subsurface targets. Conductive bodies were accurately identified (Figures 2 and 4), demonstrating the method's high sensitivity to variations in electrical conductivity, especially for more conductive structures. In contrast, the resistive target was not well resolved, likely due to the reduced sensitivity of the EMMF response to high-resistivity anomalies under the given frequency range and acquisition geometry.

These results highlight the effectiveness of the method in detecting conductive features, while also pointing to limitations in resolving resistive structures, suggesting the need for further adjustments in the inversion strategy or acquisition parameters when resistive targets are of interest.

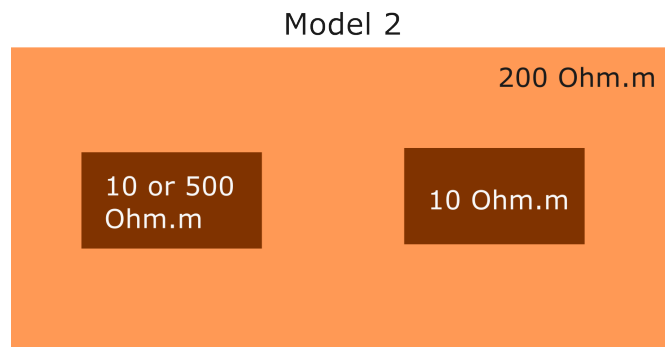


Figure 3: Model 2 to generate synthetic data to inversion. The model has two blocks, the one on the left side could be more conductive or more resistive than the 200 Ohm-m half space.

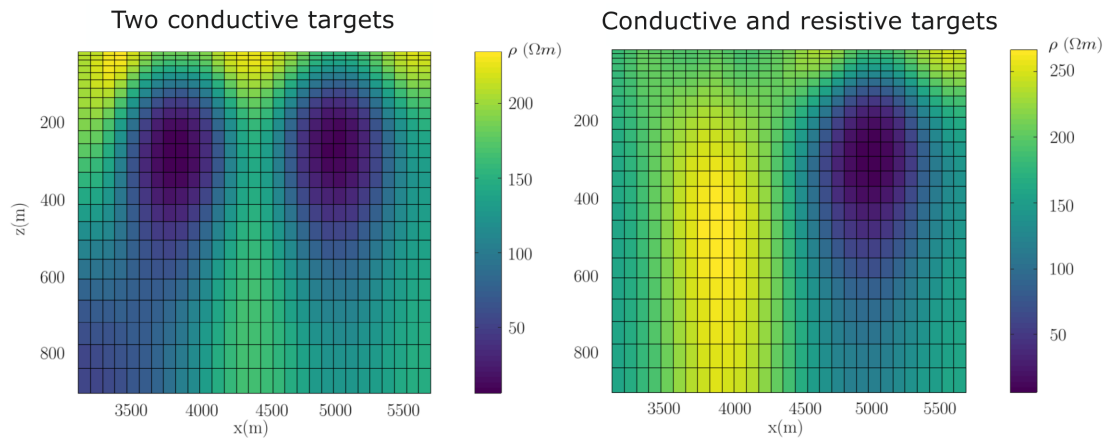


Figure 4: Inversion results of synthetic model 2. The regularization parameter was  $\alpha = 0.001$  and the start model was 200 ohm-m.

## Conclusion

This work presented a two-dimensional inversion study of Multi-Frequency Electromagnetic Method (EMMF) data, employing a modified Gauss-Newton algorithm with Marquardt regularization and a global smoothness constraint. The Jacobian matrix was efficiently calculated using the self-adjoint method, significantly reducing computational costs. Synthetic tests with multiple conductive and resistive targets demonstrated the effectiveness of the proposed methodology. Conductive anomalies were accurately recovered, highlighting the high sensitivity of the EMMF data to conductive structures. Conversely, resistive targets proved more challenging to resolve, indicating potential limitations of the method under the current acquisition and regularization settings. These results underscore the potential of the developed inversion scheme for characterizing subsurface conductivity variations while pointing to areas for future improvements, such as optimizing regularization parameters and frequency selection to better detect resistive features.

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