Estimation of viscoelastic parameters from zero-offset VSP data

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Introduction

There exist different expressions for seismic attenuation. We want to compare some of these models by doing parameter estimation on a zero-offset VSP dataset. The attenuation models are the Kolsky-Futterman model (Kolsky, 1956; Futterman, 1962), the standard linear solid model (Ben-Menahem and Singh, 1981), the power law (Strick, 1967), Azimi's second and third law (Azimi et al., 1968), the Cole-Cole model (Cole and Cole, 1941), Müller's (1981) model and Kjartansson's (1979) model.

In the modeling scheme we have used the geometric ray approximation (Ursin and Arntsen, 1985), for point-source, vertical wave propagation in a 1-D viscoelastic medium with plane wave reflection coefficients. Amundsen and Mittet (1994) used the same modeling algorithm to invert for complex velocities in a set of layers using zero-offset VSP-data. They did inversion with respect to each frequency component of the complex velocity. Our approach is to assume a formula for the complex velocity and then invert for the parameters in a few homogeneous layers. The inversion algorithm has been tested on a VSP-dataset from a well in the North Sea for different attenuation models. By minimizing the error energy we estimate the parameters in the different models.

Modeling and inversion algorithm

We consider wave propagation in a stack of viscoelastic layers in the vertical direction (normal to the layering), and we use the geometrical ray approximation described by Ursin and Arntsen (1985). Since we do not know the overburden effects, we shall use the modeled data

$$\hat{P}(z,\omega) = \frac{P(z_0,\omega)\mathfrak{L}(z_0,\omega)}{\mathfrak{L}(z_0,\omega) + \Delta\mathfrak{L}(z,\omega)} e^{i\omega\Delta\tau(z,\omega)}, \tag{1}$$

where $P(z_0,\omega)$ is the recorded data set at z_0, $\mathfrak{L}(z_0,\omega)$ is the normalized geometrical spreading, and

$$\Delta\mathfrak{L}(z,\omega) = \int_{z_0}^{z} A(\zeta,\omega)d\zeta, \qquad \Delta\tau(z,\omega) = \int_{z_0}^{z} \frac{d\zeta}{A(\zeta,\omega)} \tag{2}$$

are the incremental normalized geometrical spreading and traveltime, respectively. The complex propagation velocity, $A(z,\omega)$, is decomposed into phase velocity, $c(z,\omega)$, and attenuation, $\alpha(z,\omega)$, as follows

$$\frac{1}{A(z,\omega)} = \frac{1}{c(z,\omega)} + \frac{i\alpha(z,\omega)}{\omega}. \tag{3}$$

Note that $c(z,\omega)$ and $\alpha(z,\omega)$ are even and positive functions of ω.

In order to estimate parameters in a homogeneous layer, we take $P(z_0,\omega)$ at the top of the layer and compute the normalized error criterion

$$\frac{\Delta E}{E} = \frac{\sum_j\sum_\omega |P(z_j,\omega) - \hat{P}(z_j,\omega)|^2}{\sum_j\sum_\omega |P(z_j,\omega)|^2}. \tag{4}$$

The sum is taken over receivers in the layer and discrete frequencies in the data frequency band. In the following we estimate the unknown parameters in several seismic attenuation models by minimizing the error criterion above.

VSP data set

The VSP dataset consists of 114 seismic traces. The source in the experiment was located at 4 meters depth, 40 meters of horizontal distance from the well. Between 2900 meters and 4000 meters, 101 traces were collected, as shown in Figure 1. The ratio between the horizontal distance of the source and the depth of the geophones is about 1 %. For this reason the experiment is considered to be zero-offset. Three distinct layers can be detected between 3000 meters and 4000 meters by considering the amplitudes of the first arrivals, as shown in Figure 2. The slope of the curve indicates three different values of the attenuation coefficient, with interfaces at 3335 meters and 3650 meters depth. The sampling rate in the dataset is 1 kHz and the time window is ranging over 6 seconds. Since we restrict ourselves to the first arrivals, we use a smaller time window of 250 milliseconds in the inversion. The velocity is about 3000 m/s.

Numerical results for different attenuation models

For each attenuation model we estimated the parameters in the three different layers for which the data were recorded. The minimum values of the relative error energy $\Delta E/E$ are given in Table 1 for the different models which will be described below.

The complex velocity for the Kolsky-Futterman model is given by

$$\frac{1}{A(\omega)} = \frac{1}{c_0} + \frac{1}{\pi c_0 Q_0} \ln |\frac{\omega_0}{\omega}| + i\frac{\operatorname{sgn}(\omega)}{2c_0 Q_0}, \tag{5}$$

where $\omega_0 = 2\pi f_0$. For all the layers we fixed $f_0 = 50$ Hz, and minimized the error energy while varying the parameters Q_0 and c_0. The optimal parameter values were $Q_0 = 28$ and $c_0 = 3000.7$ m/s for the first layer, $Q_0 = 114$ and $c_0 = 3000.5$ m/s for the second layer, and $Q_0 = 35$ and $c_0 = 2999.7$ m/s for the third layer.

The complex velocity for the standard linear solid is given by

$$\frac{1}{A(\omega)} = \frac{1}{c_0}\sqrt{\frac{1 - i\omega\tau_0}{1 - i\omega\tau_\epsilon}}. \tag{6}$$

We found the minimum error energy for the layers by fixing the parameter τ_0 for each layer and varying the parameters τ_ϵ and c_0. The optimal parameter values were $\tau_0 = 3.8 \cdot 10^{-3}$ s, $c_0 = 3000.8$ m/s and $\tau_\epsilon = 4.16 \cdot 10^{-3}$ s for the first layer, $\tau_0 = 3 \cdot 10^{-5}$ s, $c_0 = 3000.5$ m/s $\tau_\epsilon = 9.2 \cdot 10^{-5}$ s for the second layer, and $\tau_0 = 3 \cdot 10^{-4}$ s, $c_0 = 2999.9$ m/s and $\tau_\epsilon = 4.3 \cdot 10^{-4}$ s for the third layer.

The complex velocity for the power law is given by

$$\frac{1}{A(\omega)} = \frac{1}{c_0} + a|\omega|^{\gamma-1}\tan\left(\frac{\gamma\pi}{2}\right) + i\frac{a|\omega|^{\gamma}}{\omega}. \tag{7}$$

The minimum error energy was found by fixing the exponent γ and varying a and c_0 for each of the layers. The optimal parameter values were $\gamma = 1.2$, $c_0 = 3000.7$ m/s and $a = 2.05 \cdot 10^{-6}\ \mathrm{m}^{-1}$ for the first layer, $\gamma = 1.4$, $c_0 = 3000.5$ m/s and $a = 2.25 \cdot 10^{-7}\ \mathrm{m}^{-1}$ for the second layer, and $\gamma = 0.9$, $c_0 = 3000.0$ m/s and $a = 7.95 \cdot 10^{-6}\ \mathrm{m}^{-1}$ for the third layer.

The complex velocity for Azimi's second law is given by

$$\frac{1}{A(\omega)} = \frac{1}{c_0} - \frac{2}{\pi}\frac{a\ln|\beta_2\omega|}{1-\beta_2^2\omega^2} + i\frac{a\operatorname{sgn}(\omega)}{1+\beta_2|\omega|}. \tag{8}$$

The minimum error energy was found by fixing β_2 and varying a and c_0 for each of the layers. The optimal parameter values were $\beta_2 = -1.1 \cdot 10^{-4}$ s, $c_0 = 3000.8$ m/s and $a = 4.40 \cdot 10^{-6}\ \mathrm{sm}^{-1}$ for the first layer, $\beta_2 = -1.5 \cdot 10^{-4}$ s, $c_0 = 3000.5$ m/s and $a = 1.25 \cdot 10^{-6}\ \mathrm{sm}^{-1}$ for the second layer, and $\beta_2 = 0.5 \cdot 10^{-4}$ s, $c_0 = 2999.9$ m/s and $a = 8.30 \cdot 10^{-6}\ \mathrm{sm}^{-1}$ for the third layer.

The complex velocity for Azimi's third law is given by

$$\frac{1}{A(\omega)} = \frac{1}{c_0} + \frac{a\beta_3\sqrt{|\omega|}}{1+\beta_3^2|\omega|} - \frac{2}{\pi}\frac{a\ln|\beta_3^2\omega|}{1-\beta_3^4\omega^2} + i\frac{a\operatorname{sgn}(\omega)}{1+\beta_3\sqrt{|\omega|}}. \tag{9}$$

The minimum error energy was found by fixing β_3 and varying a and c_0 for each of the layers. The optimal parameter values were $\beta_3 = 5 \cdot 10^{-6}\ \mathrm{s}^{1/2}$, $c_0 = 3000.7$ m/s and $a = 5.85 \cdot 10^{-6}\ \mathrm{sm}^{-1}$ for the first layer, $\beta_3 = 3 \cdot 10^{-6}\ \mathrm{s}^{1/2}$, $c_0 = 3000.5$ m/s and $a = 1.45 \cdot 10^{-6}\ \mathrm{sm}^{-1}$ for the second layer, and $\beta_3 = 1.0 \cdot 10^{-2}\ \mathrm{s}^{1/2}$, $c_0 = 3000.0$ m/s and $a = 5.12 \cdot 10^{-6}\ \mathrm{sm}^{-1}$ for the third layer.

The complex velocity for the Cole-Cole model is given by

$$\frac{1}{A(\omega)} = \frac{1}{c_0}\sqrt{\frac{1-(i\omega\tau_0)^b}{1-(i\omega\tau_\epsilon)^b}}. \tag{10}$$

The minimum error energy was found by fixing τ_0 and b, and varying τ_ϵ and c_0 for each of the layers. The optimal parameter values were $\tau_0 = 3.8 \cdot 10^{-3}$ s, $c_0 = 3000.7$ m/s, $\tau_\epsilon = 4.05 \cdot 10^{-3}$ s and $b = 0.55$ for the first layer, $\tau_0 = 3.0 \cdot 10^{-5}$ s, $c_0 = 3000.5$ m/s, $\tau_\epsilon = 5.75 \cdot 10^{-5}$ s and $b = 0.75$ for the second layer, and $\tau_0 = 3.0 \cdot 10^{-4}$ s, $c_0 = 3000.0$ m/s, $\tau_\epsilon = 3.6 \cdot 10^{-4}$ s and $b = 0.20$ for the third layer.

The complex velocity for Müller's model is in the high frequency approximation is given by

$$\frac{1}{A(\omega)} = \frac{1}{c_0}e^{\frac{1}{2}|\frac{\omega_0}{\omega}|^\gamma(\cot(\gamma\frac{\pi}{2})+isgn(\omega))}. \tag{11}$$

The minimum error energy was found by fixing γ and varying ω_0 and c_0 for each of the layers. The optimal parameter values were $\gamma = 0.321$ s, $c_0 = 3000.7$ m/s and $\omega_0 = 4.4 \cdot 10^{-3}$ s^{-1} for the first layer, $\gamma = 0.361$ s, $c_0 = 3000.5$ m/s and $\omega_0 = 1.1 \cdot 10^{-4}$ s^{-1} for the second layer, and $\gamma = 0.263$ s, $c_0 = 3000.0$ m/s and $\omega_0 = 1.7 \cdot 10^{-4}$ s^{-1} for the third layer.

The complex velocity for Kjartansson's model is given by

$$\frac{1}{A(\omega)} = a|\omega|^{\gamma-1}\tan\left(\frac{\gamma\pi}{2}\right) + i\frac{a|\omega|^\gamma}{\omega}. \tag{12}$$

The minimum error energy was found by varying γ and a for each of the layers. The optimal parameter values were $\gamma = 1.26$ and $a = 1.5 \cdot 10^{-6}$ m^{-1} for the first layer, $\gamma = 1.88$ and $a = 4.4 \cdot 10^{-8}$ m^{-1} for the second layer, and $\gamma = 0.72$ and $a = 2.0 \cdot 10^{-5}$ m^{-1} for the third layer.

Conclusions

From Table 1 it is seen that all models give good fit to the data. The error energy is almost the same for all models, but there is some variation between the layers. The best fit is obtained for layer 3 while layer 2 gives the worst fit. Azimi's second model gives the best fit in all layers.

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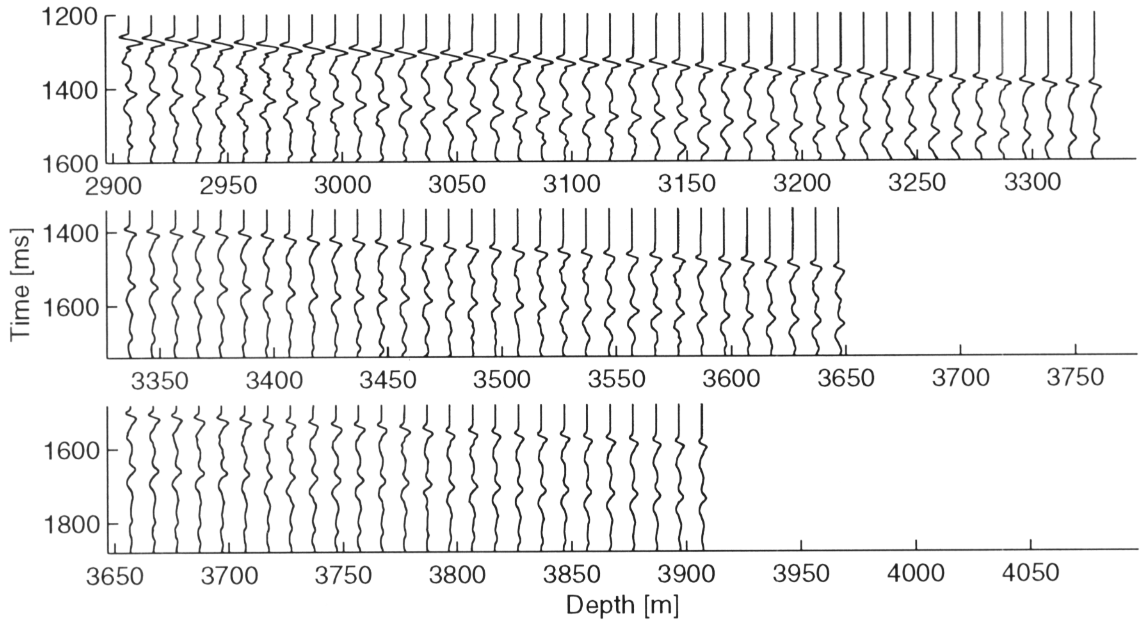

Figure 1: Corrected seismic traces.

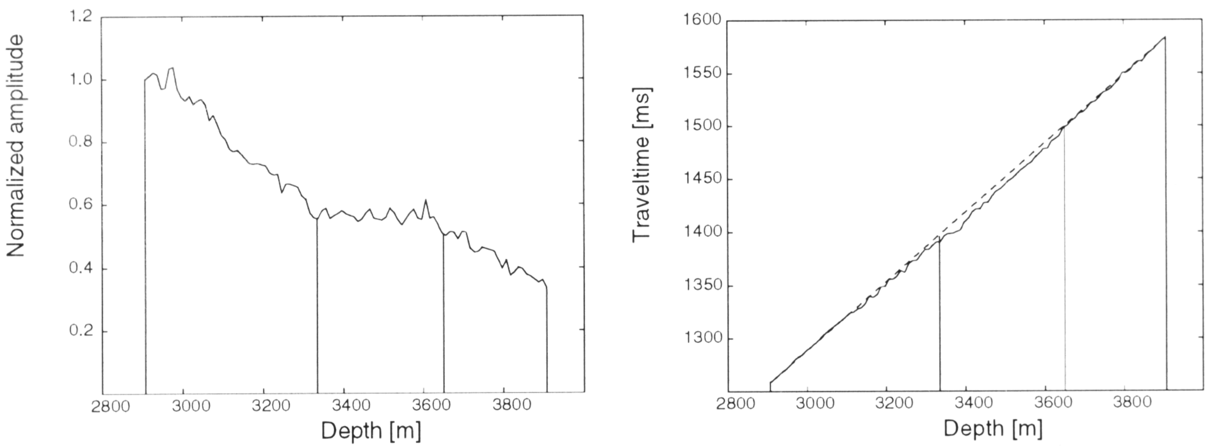

Figure 2: The normalized amplitudes of the first arrival (left). The arrival times of the first arrival (right). The dashed line corresponds to $c = 3000$ m/s.

Model	Layer 1	Layer 2	Layer3
Kolsky-Futterman	0.045	0.069	0.040
Standard linear solid	0.046	0.066	0.042
Power law	0.045	0.067	0.040
Azimi's second model	0.045	0.066	0.039
Azimi's third model	0.045	0.067	0.039
Cole-Cole model	0.046	0.066	0.041
Müller's model	0.047	0.070	0.040
Kjartansson's model	0.045	0.066	0.040

Table 1: The normalized error energy, $\Delta E/E$.