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Fast 3-D Modeling and Inversion of Surface EM Data

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ABSTRACT

This work presents an algorithm for three-dimensional modeling and inversion of electromagnetic field data. In recent years, there has been an increase in use of electromagnetic techniques for mineral, oil and geothermal exploration. Since most of the targets are of three-dimensional nature, it is important to consider the full dimensionality of the problem in the interpretation. The forward code is based on a finite difference approximation while the sensitivities needed in the inversion are obtained from the forward problem based on the reciprocity principle. A parametric functional is constructed using the data misfit and a model misfit to an *a priori* earth model, weighted by a regularization parameter. This functional is then minimized using the conjugate gradient method. The most time consuming parts of an inversion algorithm are the forward problem and calculation of the sensitivities. Also, in the 3-D problem, storage becomes another important issue. First results will be presented, along with methods that will help improve both speed and storage on the way to make the 3-D problem tractable.

INTRODUCTION

The use of electromagnetic (EM) methods in mineral, oil and geothermal exploration has increased considerably in the past years. This is due mostly to developments in hardware which made possible the acquisition of high-quality data (Zerilli and Botta, 1998). From the interpretation point of view, increase in computer speed and storage capability makes the development of higher dimensionality codes possible. Since most targets are three-dimesional (3-D) in nature, it is important that the interpretation takes into account all the possible features.
This work extends to 3-D and controlled sources, the work previously done by de Lugão et al. (1997) on two-dimensional (2-D) modeling and inversion of plane-wave (magnetotelluric, MT) data. As before, the forward problem is discretized by a finite difference scheme, and the linearized inversion is based on reciprocity calculation of sensitivities. However, due to storage and speed limitations of the 3-D problem, an iterative scheme will be used for solution of the matrix system in the forward problem.

THE 3-D FORWARD PROBLEM

The electric **E** and magnetic **H** fields generated by a source current density **J** satisfy Maxwell's equations (Hohmann, 1991). The problem is treated in cartesian coordinates with the *x*-coordinate towards the North, *y* to the East and *z* down. Time dependency $e^{i\omega t}$ is assumed throughout and displacement currents are neglected. We start with the first two Maxwell's equations and follow Hohmann (1991) to arrive at the equation that will be solved for **E**:

$$\nabla^2 \mathbf{E} + \nabla (\mathbf{E} \cdot \nabla \sigma / \sigma) + \kappa^2 \mathbf{E} = i \omega \mu_0 \mathbf{J} - 1/ \sigma \nabla (\nabla \cdot \mathbf{J}). \qquad (1)$$

where

$$\mathbf{J} = I \delta (x) \delta (y) \delta (z),$$

and

$$\kappa^2 = - i \omega\mu_0 \sigma.$$

The advantage of using equation (1) above, stressed by Zhdanov et al. (1982) and Hohmann (1991) is that the finite difference discretization leads to a 7-point scheme, decreasing the storage. Also, in regions where the gradient of the conductivity is zero, the second term vanishes.

For numerical solution we use the finite difference method proposed in Zhdanov et al. (1982) and only demonstrated for the 2-D plane-wave (MT) case. Here, we derived the coefficients and consider also controlled sources.

We subdivide the region **V** to be modeled in a 3-D mesh Σ of rectangular prisms. A second 3-D mesh Σ' is introduced with nodes located in the center of mesh Σ (Zhdanov et al., 1982). If we take the integral of (1) over the volume $\mathbf{V}_{ijk}$ of mesh Σ', and apply Gauss' theorem, we obtain:

$$\iint_{S_{ijk}} \partial \mathbf{E}/\partial n \, dS + \iint_{S_{ijk}} (\mathbf{E} \cdot \nabla \sigma / \sigma) + \mathbf{E} \iiint_{V_{ijk}} \kappa^2 \, dV = RHS_{source}. \qquad (2)$$

The finite difference approximation is then:

$$E_x(i,j,k)\,(\alpha^{(0)}_{ijk} + \beta^{z}_{ijk} + \beta^{y}_{ijk}) +$$

$$E_x(i+1,j,k)\,\alpha^{(1)}_{ijk} + E_x(i-1,j,k)\,\alpha^{(2)}_{ijk} + E_x(i,j+1,k)\,\alpha^{(3)}_{ijk} + E_x(i,j-1,k)\,\alpha^{(4)}_{ijk} + E_x(i,j,k+1)\,\alpha^{(5)}_{ijk} + E_x(i,j,k-1)\,\alpha^{(6)}_{ijk}$$

$$= RHS_{source}. \qquad (3)$$

In this 7-point formulation, the coefficients $\alpha^{(0)}$ and β are complex and depend on the conductivities, and the coefficients $\alpha^{(l)}$ $l=1,\ldots,6$ are real and depend only on the mesh geometry. The discretization will lead to a linear system of equations, where the matrix of coefficients is symmetric and positive definite. This is very important for the calculation of the sensitivities in the inversion. Since the matrix of coefficients is symmetric, the reciprocity principle can be applied and the calculation of sensitivities is of the order of receiver numbers (Rodi, 1976). In the 3-D case, conjugate gradient techniques will be used for solving the linear system due to their speed and minimum storage requirements. Also, once one forward problem is solved, it can be used as a starting point for other calculations of the same system with small perturbations (Chan and Wan, 1997), as is the case for the sensitivities.

PRELIMINARY RESULTS

Only very preliminary results are presented here. If the forward modeling code is properly implemented, all other inversion steps, such as calculation of the sensitivities, construction of the regularized parametric functional and its minimization will follow since they were already proven in the 2-D counterpart of this work (de Lugão et al., 1997) .

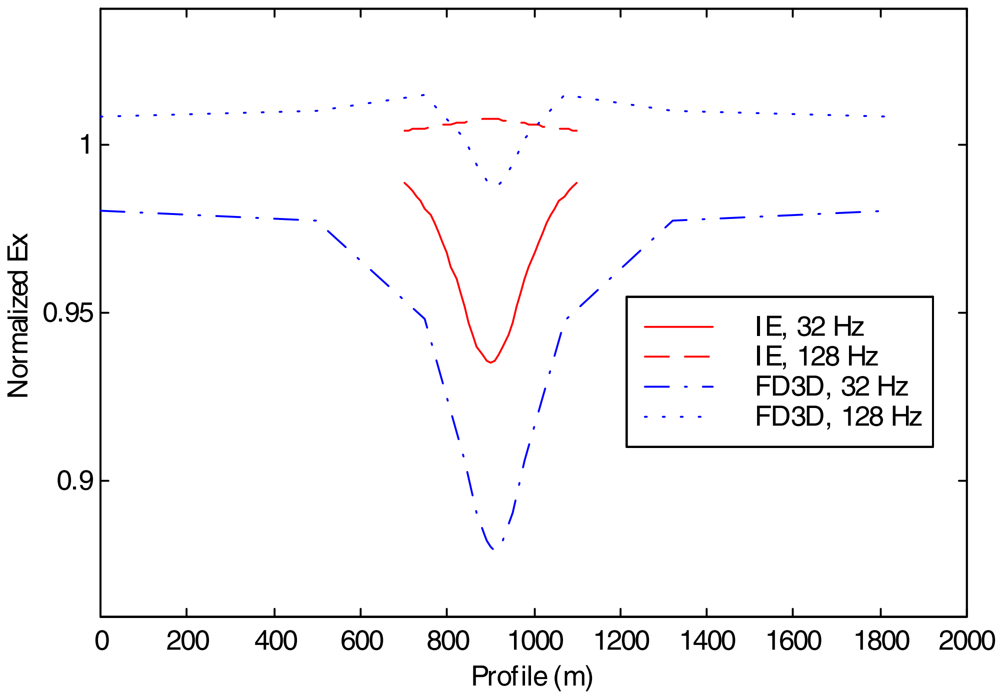

Figure 1 shows a comparison between the finite difference code presented here and the integral equation code SYSEM of Xiong (1992) for a 1ohm-m square (50 · 50 · 50 meters) body in a 10 ohm-m half-space for frequencies 32 and 128 Hz. The comparison here is for the Ex field component only and presents a discrepancy due to the finite extent of the mesh. Some improvement had already been achieved by extending the mesh to account for proper decay of the anomalous field. However, since a direct solver (LU decomposition) is being used, and the computer capabilities were small (32 Mbyte Ram, 100 MHz Pentium), the mesh could not be extended appropriately to account for field decay. Solution of the linear system by an iterative technique will minimize this problem.

CONCLUSIONS

The theory was presented along with some very preliminary results for a fast solution of the 3-D problem in electromagnetics. Further implementation of an iterative procedure for solution of the linear system in the forward modeling problem will certainly improve the accuracy of the solution, as well as the speed. Multigrid techniques (Briggs, 1987) have received much attention recently and can be tested in this work. After the forward problem is solved, the inversion will follow as in the 2-D counterpart of this work.

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