Scattering of Electromagnetic Plane Waves by a Buried Vertical Dike

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Abstract

The complete and exact solution of the scattering of a TE mode electromagnetic plane wave by a vertical dike under a conductive overburden has been established. An integral representation composed of one-sided Fourier transforms describes the scattered electric field components in each one of the five media: air, overburden, dike, and the country rocks on both sides of the dike. The determination of the terms of the series that represents the spectral components of the Fourier integrals requires the numerical inversion of a 24×24 sparse matrix, and the method of successive approaches. The zero-order term of the series representation for the spectral components of the overburden, for given values of the electrical and geometrical parameters of the model, have been computed. This result allowed to determine an approximate value of the variation of the electric field on the top of the overburden in the direction perpendicular to the strike of the dike. The results demonstrate the efficiency of this forward electromagnetic modeling, and are fundamental for the interpretation of VLF and Magnetotelluric data.

INTRODUCTION

The investigation of electromagnetic plane wave scattering caused by lateral variations of the physical properties of rocks is fundamental for exploration geophysics. Sommerfield (1896) and Wiener & Hopf (1931) have employed different techniques to solve the problem of the scattering of electromagnetic plane waves by a perfectly conductive half-plane.

Several numerical modeling algorithms, recently developed, reduce the computational cost in the investigation of 3-D problems. Some of them have the flexibility of non-uniform interval sampling or of evaluation of the derivatives with different accuracies in the computation of the wave field. Others excite the system from different functions or restrict the information along the boundaries of the model. All these modifications, however, decrease the accuracy of the final result.

There are few analytical solutions of electromagnetic modeling of the Earth, except for the case of simple geological structures like: horizontal layers, vertical faults, and spheres. Sampaio & Fokkema (1992) established the analytical solution of the scattering of an electromagnetic plane wave by a vertical fault in the form of a series expansion, for the TE mode in the frequency domain. Sampaio & Popov (1997) analysed the solution of this same model in the time domain, and computed the fields employing the first term (zero-order term) of the series expansion.

This paper presents the basic formulation of the analytical solution of the scattering of TE mode electromagnetic plane waves for the case of a vertical dike between two quarter-spaces and overlaid by a horizontal layer. The analytical tools employed in the solution of this problem will be useful to check the efficiency of the numerical techniques of forward and inverse modeling.

FORMULATION AND SOLUTION OF THE PROBLEM

The primary electromagnetic plane wave propagates in the positive z direction and is modified by the properties of the geological structure of the dike depicted in Figure 1. For this TE mode problem the electric field vector is always along the y direction, and each medium has a constant, finite, and distinct value of the electrical conductivity. Therefore, the problem consists of finding the solution of the two-dimensional Helmholtz homogeneous differential equation for each medium:

$$\frac{\partial^2 E_n}{\partial x^2} + \frac{\partial^2 E_n}{\partial z^2} + k_n^2 E_n = 0, \qquad n = 0, 1, 2, 3, 4. \tag{1}$$

In (1): $k_n^2 = k_0^2 - i\omega\mu_0\sigma_n$, is the propagation constant of each medium; ω is the angular frequency; μ_0 is the free-air magnetic permeability; σ_n is the conductivity of each medium; and E_n represents the electric field component in the y direction.

$$E_n = E_y(x, z, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e_y(x, z, t) e^{i\omega t} dt.$$

The solutions of the differential equation in each medium described in Figure 1 are of the following form:

For $-\infty < z < 0$:

$$\begin{aligned}
E_{0,2} &= E^I + E^R_{0,2} + \int_0^\infty [F_{0,2}e^{u_0x}cos(\alpha z) + G_{0,2}e^{u_0z}cos(\alpha x)]d\alpha,\ x < -a; \\
E_{0,3} &= E^I + E^R_{0,3} + \int_0^\infty [(F_{0,3A}e^{-u_0x} + F_{0,3B}e^{u_0x})cos(\alpha z) + G_{0,3}e^{u_0z}cos(\alpha x)]d\alpha,\ -a < x < +a; \\
E_{0,4} &= E^I + E^R_{0,4} + \int_0^\infty [F_{0,4}e^{-u_0x}cos(\alpha z) + G_{0,4}e^{u_0z}cos(\alpha x)]d\alpha,\ x > +a. \qquad (2)
\end{aligned}$$

For $0 < z < h$:

$$\begin{aligned}
E_{1,2} &= E^T_{0,2} + E^R_{1,2} + \int_0^\infty [F_{1,2}e^{u_1x}cos(\alpha z) + (G_{1,2A}e^{-u_1z} + G_{1,2B}e^{u_1z})cos(\alpha x)]d\alpha,\ x < -a; \\
E_{1,3} &= E^T_{0,3} + E^R_{1,3} + \int_0^\infty [(F_{1,3A}e^{-u_1x} + F_{1,3B}e^{u_1x})cos(\alpha z) + (G_{1,3A}e^{-u_1z} + G_{1,3B}e^{u_1z})cos(\alpha x)]d\alpha,\ -a < x < +a; \\
E_{1,4} &= E^T_{0,4} + E^R_{1,4} + \int_0^\infty [F_{1,4}e^{-u_1x}cos(\alpha z) + (G_{1,4A}e^{-u_1z} + G_{1,4B}e^{u_1z})cos(\alpha x)]d\alpha,\ x > +a. \qquad (3)
\end{aligned}$$

For $h < z < \infty$:

$$\begin{aligned}
E_2 &= E^T_2 + \int_0^\infty [F_2e^{u_2x}cos(\alpha z) + G_2e^{-u_2z}cos(\alpha x)]d\alpha,\ x < -a; \\
E_3 &= E^T_3 + \int_0^\infty [(F_{3A}e^{-u_3x} + F_{3B}e^{u_3x})cos(\alpha z) + G_3e^{-u_3z}cos(\alpha x)]d\alpha,\ -a < x < +a; \\
E_4 &= E^T_4 + \int_0^\infty [F_4e^{-u_4x}cos(\alpha z) + G_4e^{-u_4z}cos(\alpha x)]d\alpha,\ x > +a. \qquad (4)
\end{aligned}$$

Where: $E^I = e^{ik_0z}$ represents the incident field for $z < 0$; $E^R_{0,j} = R_{0,j}e^{ik_0z}$, $R_{0,j}$ are the reflection coefficients at the free-air boundary; $E^T_{0,j} = T_{0,j}e^{-ik_1z}$, $T_{0,j}$ are the transmission coefficients at the free-air boundary into the first layer; $E^R_{1,j} = R_{1,j}e^{ik_1(z-h)}$, $R_{1,j}$ are the reflection coefficients at the bottom of the first layer respectively related to mediums 2, 3 and 4; $E^T_j = T_{1,j}e^{-ik_j(z-h)}$, $T_{1,j}$ are the transmissions coefficients at the bottom of the first layer respectively into mediums 2, 3 and 4; and $u_n = \sqrt{\alpha^2 - k_n^2}$, represents the wave number in the α transformed space. To solve the problem it is necessary to determine the 24 spectral components: $F_{0,2}$, $G_{0,2}$, $F_{0,3A}$, $F_{0,3B}$, $G_{0,3}$, $F_{0,4}$, $G_{0,4}$, $F_{1,2}$, $G_{1,2A}$, $G_{1,2B}$, $F_{1,3A}$, $F_{1,3B}$, $G_{1,3A}$, $G_{1,3B}$, $F_{1,4}$, $G_{1,4A}$, $G_{1,4B}$, F_2, G_2, F_{3A}, F_{3B}, G_3, F_4, G_4, employing the pertinent boundary conditions:

$$l_i \lim_{x\to\lambda} \frac{\partial E_{i,j}}{\partial x} = l_m \lim_{x\to\lambda} \frac{\partial E_{m,j+1}}{\partial x} \quad \text{and} \quad E_{i,j} = E_{m,j+1}, \quad \text{in the } x \text{ direction;}$$
$$l_i \lim_{z\to\gamma} \frac{\partial E_{i,j}}{\partial z} = l_m \lim_{z\to\gamma} \frac{\partial E_{m,j+1}}{\partial z} \quad \text{and} \quad E_{i,j} = E_{m,j+1}, \quad \text{in the } z \text{ direction.}$$

The weighting factor l_n is the inverse of the impedittivity of each medium; $\lambda = \pm a$; and $\gamma = 0$ or h. Substituting equations (2) - (4) on the boundary conditions and applying Fourier Transform ($f(z) \to F(\beta)$), we obtain a sparse system of integral equations containing 24 equations and 24 unknowns (spectral components). We can write them in the following reduced form:

$$\bar{W}(\beta) \cdot \phi(\beta) = \bar{\psi}(\beta) + \int_0^\infty \bar{K}(\alpha;\beta)\phi(\alpha)d\alpha. \qquad (5)$$

Where $\bar{W}(\beta)$ is a pentadiagonal matrix that contains the coefficients; ϕ is a vector that contains the spectral components; $\bar{\psi}(\beta)$ is a constant vector; and $\bar{K}(\alpha;\beta)$ is a sparse matrix. Multiplying equation 5 by $\bar{W}^{-1}(\beta)$, one obtains:

$$\phi(\beta) = \psi(\beta) + \int_0^\infty K(\alpha;\beta)\phi(\alpha)d\alpha. \qquad (6)$$

Where: $\psi(\beta) = \bar{W}^{-1}(\beta) \cdot \bar{\psi}(\beta)$ and $K(\alpha;\beta) = \bar{W}^{-1}(\beta) \cdot \bar{K}(\alpha;\beta)$. The regular part of equation 6 is an improper Riemman integral, and this kind of integral equation is called a singular Fredholm-type integral equation of the second kind. Because of the fast convergence of this improper integral the solution of the system of integral equations (6) was obtained through the method of the successive approaches (Kondo, 1991):

$$\phi_n = \psi + (K + \dot{K}^2 + ... + \dot{K}^{n-1}) \circ \psi + \dot{K}^n \circ \phi_0, \quad \phi_0 = \psi. \qquad (7)$$

NUMERICAL RESULTS

The solution of the system of integral equations will result in the complete determination of all the spectral components. As a first step toward this goal, we display in Figure 2 the variation of the real and imaginary parts of the zero-order term of those components for the overburden. They converge to a finite value in the origin, present either a maximum or a minimum value close to the origin, and decay to zero at infinity.

To demonstrate the validity and application of the algebraic expressions we substituted the spectral components in Equations (3) and computed the scattered electric field by a vertical dike with a thickness of 500 m and a conductivity of 0.1 S/m, immersed in two quarter-spaces with a conductivity respectively of 0.01 S/m and 0.005 S/m, and overlaid by a horizontal layer with a thickness of 100 m and a conductivity of 0.05 S/m. Figure 3 shows the variation of the real and the imaginary parts of the electric field on the top surface of the overburden as a function of the wavenumber ($|k_1|x$). For $|k_1|x \to \pm\infty$, $E_y \to 0$. Through a visual inspection of Figure 3 it is possible to identify the two contacts between the dike and the country rocks, as well as the center of the dike. Because the largest contrast of conductivity occurs between the dike and medium 4, the electric field presents its maximum variation at $|k_1|x = +a$.

CONCLUSION

The complete and exact algebraic solution of the scattering of a monochromatic electromagnetic plane wave, for the case of a vertical dike immersed in two quarter-spaces and overlaid by a horizontal layer, was determined. As a first step, zero-order terms of the series representation of the spectral components were selected to compute an approximate value of the electric field above the vertical dike. The results obtained in this forward electromagnetic modeling show that it can be used, in certain cases, to substitute with advantages other techniques. It also gives a physical insight of the problem and therefore leads to a better interpretation of magnetotelluric and VLF data.

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ACKNOWLEDGMENTS

We acknowledge CNPQ for the financial support.

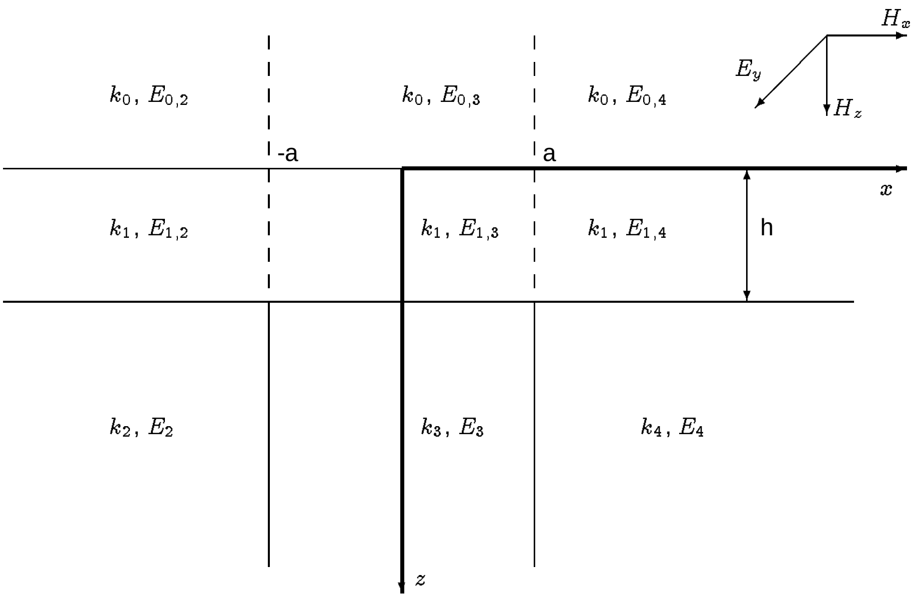

Figure 1: Configuration of the dike model and of the incident field components.

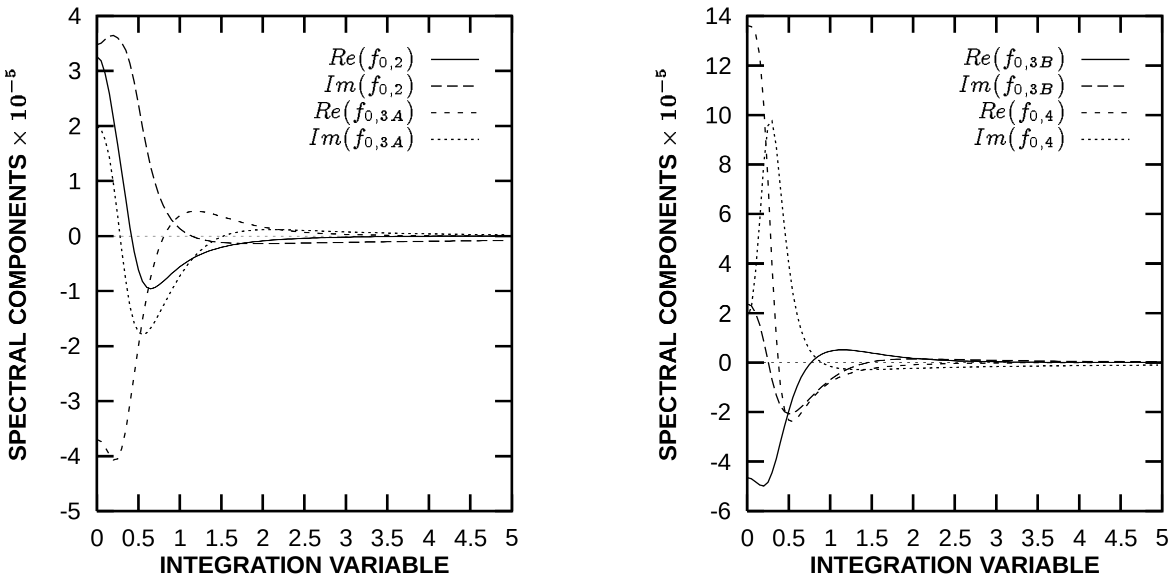

Figure 2: Variation of the real and imaginary parts of the spectral components for the overburden.

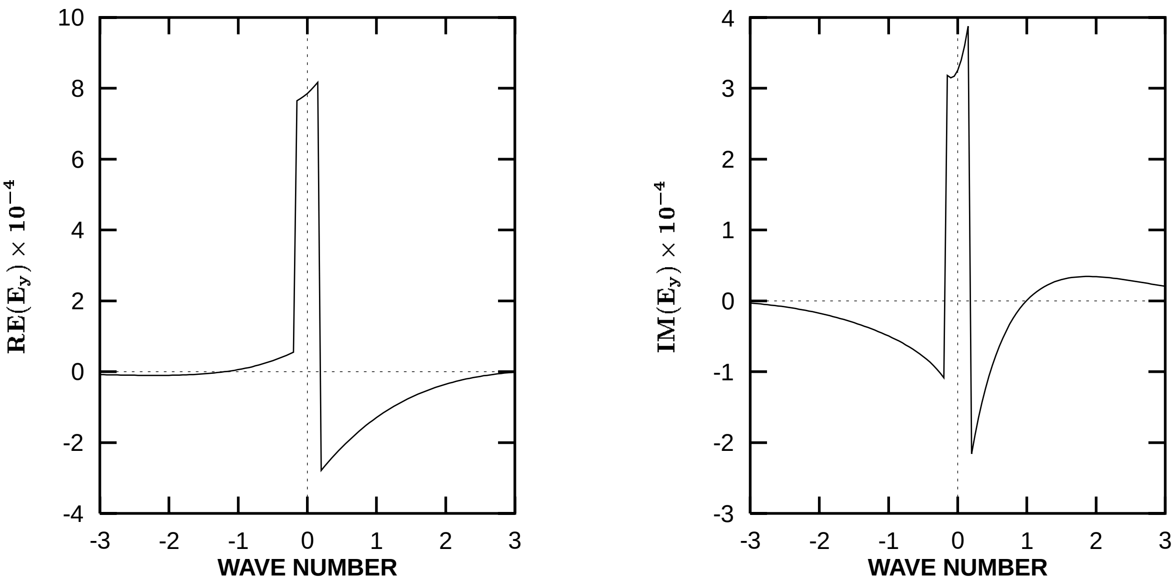

Figure 3: Variation of the real and imaginary parts of the scattered electric field with the wavenumber $|k_1|x$ at $z = 0$ for: $\omega\sigma_1 = 0.1\pi$ S/m/s, $h = 100$ m; $\sigma_2 = 0.01$ S/m; $\sigma_3 = 0.1$ S/m, $a = 250$ m; $\sigma_4 = 0.005$ S/m; $|k_1|a = 0.025\pi$.