Developments in a model to describe low-frequency electrical polarization of rocks - (II): A comparative overview

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Abstract

The main existing models proposed to describe IP are analyzed and their equivalent circuit analogs drawn. Most are grouped together as belonging to a same family under a common circuit analog representation and a respective "generating function". A circuit analog assigned to the "multi Cole-Cole" model reveales intrinsic constraints involving the values of its circuit elements. Such constraint relations plus the associated relaxation times ratio-usually many orders of magnitude different from unity-imply that this model must be limited to describe multi-phase material systems (in the sense of the polarization setting), in order to make any physical sense.

INTRODUCTION

A general overview of the existing models proposed to describe low-frequency electrical polarization of rocks is convenient in order to situate our model in the development chronology of the subject and in terms of its new concepts. This paper is a continuation of part I (Dias, 1999a).

CONSIDERATIONS ON THE EXISTING MODELS

We will now discuss the most common models proposed to describe this effect. When a particular model does not bring an equivalent circuit analog from its origin, we will provide it and also reduce them all to a standard representation in terms of complex resistivity using, whenever possible, similar parameters. Table 1 summarizes them.

Models belonging to a same family

Models 3 to 10 (see Table 1) can be described as a parallel combination of a pure resistor with a series combination of a resistor and an impedance Z', the latter given as a frequency function, as shown in Figure 1 and Table 2. In such cases, resistivity can be expressed by a single function as

$$\rho=\rho_o\left[1-m\left(1-\frac{1}{1+\left((R+R_1)/Z^{'}\right)}\right)\left[1-\left(Z^{'}/Z_o^{'}\right)\right]\right], \tag{1}$$

where Z_o' is the dc value of Z'.

To keep general the representation of ρ in equation (1), it is assumed that the point ω = 0 is excluded from ω dominion of variation. This introduces no limitation in this representation and its practical use.

The particular cases are:

(1) Debye model, with $Z'=1/(i\omega C)$, and then $Z'/Z_o'=0$; (2) Madden & Cantwell model, with $Z' = a/(i\omega)^{1/4}$, and then $Z'/Z'_o=0$.

$$\frac{R+R_1}{Z^{'}}=\left[(R+R_1)C\right]i\omega=i\omega\tau, \quad (2) \qquad\qquad \frac{R+R_1}{Z^{'}}=\left(\frac{R+R_1}{a}\right)(i\omega)^{1/4}=(i\omega\tau)^{1/4}, \quad (3)$$

where $\tau = (R + R_1)C$ and $\tau = [(R + R_1)/a]^4$, respectively.

3) Warburg model, *with* $Z' = a/(i\omega)^{1/2}$, and then $Z'/Z_o' = 0$; (4) Cole-Cole model, with $Z' = a/(i\omega)^c$, and then $Z'/Z_o' = 0$.

$$\frac{R+R_1}{Z^{'}}=\left(\frac{R+R_1}{a}\right)(i\omega)^{1/2}=(i\omega\tau)^{1/2}, \quad (4) \qquad\qquad \frac{R+R_1}{Z^{'}}=\left(\frac{R+R_1}{a}\right)(i\omega)^{C}=(i\omega\tau)^{C}, \quad (5)$$

where $\tau = [(R + R_1)/a]^2$ and $\tau = [(R + R_1)/a]^{1/c}$, respectively.

(5) Zonge model, with $Z'=(R+ R_1)/(\theta£(\theta))$, where $£(\theta) = \coth\theta - 1/\theta$ is the Langevin function, and then $Z'/Z'_o=0$.

$$\frac{R+R_1}{Z^{'}}=\theta£(\theta), \tag{6}$$

where $\theta=(i\omega\tau)^{c/2}$, $\tau = ((R+R_1)/a)^{1/c}$ and $0 \le c \le 1$, the parameter *a* having the physical meaning shown in the corresponding circuits in Table 1.

Envelope functions for this model are determined by its asymptotic forms, as follows: (a) when $|\theta| << 1$, the function $£(\theta)$ approaches $\theta/3$ and, so, equation (6) becomes a Cole-Cole type function of exponent *c* and relaxation time $\tau/3^{1/c}$; (b) when $|\theta| >> 1$, $£(\theta)$ approaches 1, and (6) becomes another Cole-Cole function with exponent *c/2* and relaxation time τ.

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This means that, when plotted on Argand plane, Zonge model type curves are reduced to a Cole-Cole arc of circle of smaller radius, for the smaller values of frequency, and will depart toward another Cole-Cole arc of greater radius for the higher values of ω; both asymptotic arcs will start and finish at the same points on the real axis, given by ρ_o at ω=0 and (1-m) ρ_o at ω = ∞. The original Zonge model was introduced (Zonge, 1972) specified for *c=1/2*, having then as asymptotes Warburg and Madden & Cantwell type curves. The modif\/ication leading to Cole-Cole general functions asymptotes was introduced by Pelton (1977).

The dual representation of Zonge model (see Table 1) either through a *network circuit system* or an equivalent *fundamental circuit* is convenient to show the reasoning behind these two procedures for generating the same function representation to describe rock polarization.

(6) Dias model, with $Z' = [r + a/i\omega)^{1/2}]/[1+(r+a/(i\omega)^{1/2})i\omega C_{dl}]$, and then $Z'/Z_o' = 0$.

$$\frac{R+R_1}{Z'} = i\omega(R+R_1)C_{dl}\left[1+\frac{1}{i\omega rC_{dl}+(i\omega)^{1/2}aC_{dl}}\right] = i\omega\tau'\left[1+\frac{1}{i\omega\tau+(i\omega\tau'')^{1/2}}\right], \tag{7}$$

where = r C_{dl}; $\tau' = (R + R_1)\, C_{dl}$ and $\tau'' = (a\, C_{dl})^2$.

(7) Davidson-Cole model, with $Z'=a/\omega_L^{c}/(1+i\omega/\omega_L)^c$ and $(R+R_1)/Z_o >> 1$.

$$\frac{Z'}{Z_o} = \frac{1}{(1+i\omega\tau)^c}, \tag{8}$$

where $\tau = 1/\omega_L$.

(8) Generalized Cole-Cole model, with $Z'=a/\omega_L^{ck}/[1+(i\omega/\omega_L)^c]^k$ and $(R+R_1)/Z_o >> 1$.

$$\frac{Z'}{Z_o} = \frac{1}{\left[1+(i\omega\tau)^c\right]^k}, \tag{9}$$

where $\tau = 1/\omega_L$.

The other models

Model number 11 in Table 1, here refered to as "multi Cole-Cole" model, can be described, as shown in Figure 2, as a parallel combination of two Cole-Cole circuits, if at least one of the following conditions is satisfied: (1) $R_1 >> R' >> R''$, (2) $R_2 >> R'' >> R'$ or (3) $R_1 >> R'$ and $R_2 >> R''$. The complex resistivity then resulting is

$$\rho = \rho_o\left[1-m_1\left(1-1/\left(1+(i\omega\tau_1)^{c_1}\right)\right)\right]\times\left[1-m_2\left(1-1/\left(1+(i\omega\tau_2)^{c_2}\right)\right)\right] \tag{10}$$

From the individual Cole-Cole circuits that make up Figure 2, one can obtain the expressions for τ_1, m_1, τ_2 and m_2, and write

$$\frac{\tau_1}{\tau_2} = \frac{\left[R'/(m_1a_1)\right]^{\frac{1}{C_1}}}{\left[R''/(m_2a_2)\right]^{\frac{1}{C_2}}}. \tag{11}$$

In practical cases, it is common that $C_1 \approx C_2$and then that

$$\frac{\tau_1}{\tau_2} \approx \left(\frac{R'/(m_1a_1)}{R''/(m_2a_2)}\right)^{\frac{1}{C}}, \tag{12}$$

where C is the average value of C_1 and C_2.

It is also very common, in practical cases, that τ_1 and τ_2 are of very dif\/ferent orders of magnitude; and that most of this contrast does not result from the m_1/m_2 ratio. So, one can write approximately

$$\frac{\tau_1}{\tau_2} \approx \left(\frac{R'/a_1}{R''/a_2}\right)^{\frac{1}{C}}. \tag{13}$$

Expression (13) shows that, if τ_1 and τ_2 exhibit different orders of magnitude, the material system must correspond necessarily to a petrophysical/electrochemical system with two distinct phases, for such time constants to be physically meaningful.

Looking it from another way, let us now consider which physical implications arise from the constraints set on the circuit components of Figure 2:

a) Conditions 1 ($R_1 >> R' >> R''$) and 2 ($R_2 >> R'' >> R'$) suggest that pore constriction is somewhat present either in one or other of the two phases, right at the branch (of the Cole-Cole individual circuits) where polarization is produced;

b) Condition 3 ($R_1 >> R'$ and $R_2 >> R''$) suggests the existence of pore constriction in each branch (of the individual cir-

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cuits) where the sources of polarization are set on both phases of the system.

So, the above subsidiary conditions 1 and 2 reinforce the point made that the "multi Cole-Cole" model is tied up to a multi-phase physical system in terms of the polarization setting. Condition 3 could still be compatible with a single phase system, if the ratio τ_1/τ_2 would not be very different from 1.

Wait (1959) and Wong (1979) models, respectively 1 and 12 in Table 1, are constructed on a similar physical structure. They consist of small spherical metallic particles, regularly spaced (Wait) or randomly spaced (Wong), distributed in a continuum and separated from each other by distances far enough to avoid mutual electric interference among the induced dipoles. In Wait's model, each particle is covered by a dielectric film to simulate the interfacial electrical mechanism and assure a perfectly polarizable sphere, using a strictly macroscopic approach. In Wong's model, the electrochemical principles were used to obtain more realistic reflection coefficients at the interfaces. In both models, these coefficients are introduced into the expression derived by Maxwell (1891) for the complex resistivity of a heterogeneous mixture. An equivalent circuit analog for such cases is hard to portray, except in a rather simple way, as shown in Table 1.

Model number 2 in Table 1 was suggested by Ward and Fraser (1967). It was very helpful for the initial insight into Dias model.

CONCLUSIONS

An analysis of all the main existing models was made in a broad context. A "generating function" has been introduced, from which an entire family of models (all the models representable by the equivalent fundamental circuit analog in Figure 1) can be derived. Also, new equivalent circuit analogs were written by us for Zonge, Davidson-Cole, Generalized Cole-Cole and "multi Cole-Cole" models. In the latter case, the introduced equivalent circuit imposed intrinsic constraints on the relative values of the circuit elements in order to generate the associated final function. The constraint relations combined with the analysis of the relaxation times ratio suggest that, for such a model to be justified, a physical association must exist connecting a multi-stage Cole-Cole function to a multi-phase electrochemical/petrophysical system. This point establishes a physical criterion to check the validity of this model.

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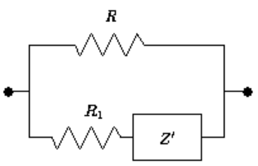


Fig. 1. Schematic equivalent fundamental circuit analog for many IP models, shown in Table 2. Z' is a characteristic frequency function.

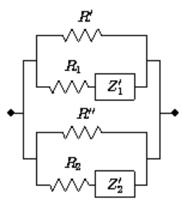


Fig. 2. Schematic equivalent fundamental circuit analog for the 'multi Cole-Cole"model, where $Z_j' = a_j/(i\omega)^{C_j}$, j=1,2, assuming at least one of the following conditions satisfied: (1) R_1>>R'>>R''; (2) R_2>>R''>>R'; (3) R_1>>R' and R_2>>R''.

Table 1. Existing models with respective references, equivalent circuit analogs and associated complex resistivities, listed chronologically

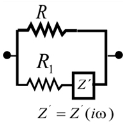
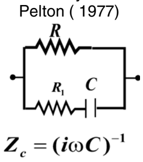
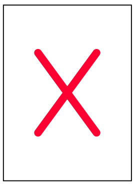
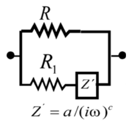
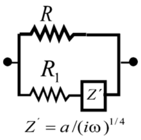
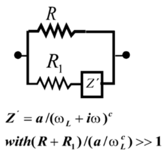
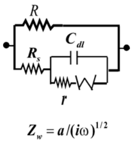
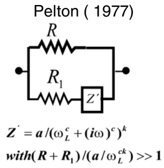
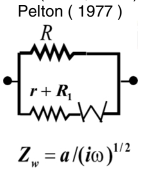
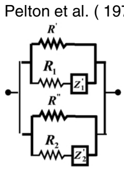
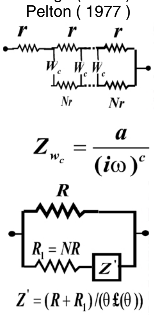
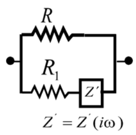
Item	Model/Reference/Circuit	Complex Resistivity	Item	Model/Reference/Circuit	Complex Resistivity
1	Wait (1959) 	$\rho = \rho_s(1-\frac{A}{1+2A/3})$ A=average polarizability per unit volume of an heterogeneous medium ρ_s= resistivity of the host medium	7	Debye Pelton (1977) 	$\rho = \rho_0[1-m(1-\frac{1}{1+i\omega\tau})]$ $m=(\rho_0-\rho_\infty)/\rho_0=\frac{R}{R+R_1}$ $\tau = C(R_1+R)$
2	Ward & Fraser (1967) 	$\rho = \rho_D[1-\frac{1}{1+(\rho_g+\gamma/i\omega)\rho_D}]$ ρ_D = resitivity fuction associated with Dias model	8	Cole-Cole Pelton (1977) 	$\rho = \rho_0[1-m(1-\frac{1}{1+(i\omega\tau)^c})]$ $m=\frac{R}{R+R_1}; \quad 0\le c\le 1$ $\tau = (\frac{R_1+R}{a})^{1/c}$
3	Madden & Cantwell (1967) 	$\rho = \rho_0[1-m(1-\frac{1}{1+(i\omega)^{1/4}})]$ $m=(\rho_0-\rho_\infty)/\rho_0=\frac{R}{R+R_1}$ $\tau = ((R+R_1)/a)^4$	9	Davidson-Cole Pelton (1977) 	$\rho = \rho_0[1-m(1-\frac{1}{(1+i\omega\tau)^c})]$ $m=\frac{R}{R+R_1}\frac{a/\omega_L^c}{R_1+a/\omega_L^c}; 0\le c\le 1$ $\tau = 1/\omega_L$
4	Dias (1968, 1972) 	$\rho = \rho_0[1-m(1-\frac{1}{1+i\omega\tau'(1+\mu^{-1})})]$ $m=(\rho_0-\rho_\infty)/\rho_0=\frac{R}{R+R_s}$ $\mu = i\omega\tau+(i\omega\tau'')^{1/2}$ $\tau = rC_{dl}; \quad \tau' = (R+R_s)C_{dl}$ $\tau'' = (aC_{dl})^2$	10	Generalized Cole-Cole Pelton (1977) 	$\rho = \rho_0[1-m(1-\frac{1}{[1+(i\omega\tau)^c]^k})]$ $m=\frac{R}{R+R_1}\frac{a/\omega_L^{ck}}{R_1+a/\omega_L^{ck}}$ $0\le(c;k)\le 1$ $\tau = 1/\omega_L$
5	Warburg Dias (1968, 1972) Pelton (1977) 	$\rho = \rho_0[1-m(1-\frac{1}{1+(i\omega\tau)^{1/2}})]$ $m=(\rho_0-\rho_\infty)/\rho_0=\frac{R}{(r+R_s)+R}$ $\tau = (\frac{r+R_s+R}{a})^2$	11	Multi Cole-Cole Pelton et al. (1978)  $Z'_j = a_j/(i\omega)^{c_j}; j=1,2$ satisfied at least one of the conditions : 1) $R_1 >> R' >> R''$ 2) $R_2 >> R'' >> R'$ 3) $R_1 >> R'$; $R_2 >> R''$	$\rho=\rho_0[1-m_1(1-\frac{1}{1+(i\omega\tau_1)^{c_1}})].[1-m_2(1-\frac{1}{1+(i\omega\tau_2)^{c_2}})]$ $m_1=\frac{R'}{R'+R_1}; 0\le(c_1;c_2)\le 1$ $\tau_1=(\frac{R'+R_1}{a_1})^{1/c_1}$ $m_2=\frac{R''}{R''+R_2}$ $\tau_2=(\frac{R''+R_2}{a_2})^{1/c_2}$
6	Zonge (1972) Pelton (1977) 	$\rho = \rho_0[1-m(1-\frac{1}{1+\theta£(\theta)})]$ $\theta = (i\omega\tau)^{c/2}$ $£(\theta) = \coth\theta - \frac{1}{\theta}$ $m=(\rho_0-\rho_\infty)/\rho_0=\frac{R}{R+R_1}$ $\tau = (\frac{R_1+R}{a})^{1/c}; \quad 0\le c\le 1$ $0 < N \le 1$	12	Wong (1979) 	$\rho = \rho_s(1-\frac{3\sum_j N_j f_j}{1+2\sum_j N_j f_j})$ N_j = number of spherical particles j per unit volume f_j = reflection coefficient at the interface between an homogeneous infinite ionic solution and the medium j ρ_s = resistivity of the homogeneous solution

Table 2. Family of models having Figure 1 for circuit analog: respective associated function Z' and impedances at frequencies 0 and ∞, respectively Z_o and Z_∞. The parameter "chargeability" m is defined as $(Z_o - Z_\infty) / Z_o$. Items 7 and 8 have $(R + R_1) / Z_o' >> 1$ as constraint. In item 5, Function $£ = \coth\theta - 1/\theta$, where $\theta = (i\omega\tau)^{c/2}$

Item	Model	Z'	Z_o	Z_∞	m
1	Debye	$C^{-1} / (i\omega)$	R	$RR_1 / (R + R_1)$	$R / (R + R_1)$
2	Madden & Cantwell	$a / (i\omega)^{1/4}$	R	$RR_1 / (R + R_1)$	$R / (R + R_1)$
3	Warburg	$a / (i\omega)^{1/2}$	R	$RR_1 / (R + R_1)$	$R / (R + R_1)$
4	Cole-Cole	$a / (i\omega)^c$	R	$RR_1 / (R + R_1)$	$R / (R + R_1)$
5	Zonge	$(R + R_1) / (\theta £(\theta))$	R	$RR_1 / (R + R_1)$	$R / (R + R_1)$
6	Dias	$\frac{r + a/(i\omega)^{1/2}}{1 + i\omega C_{dl}[r + a/(i\omega)^{1/2}]}$	R	$RR_1 / (R + R_1)$	$R / (R + R_1)$
7	Davidson-Cole	$\frac{a}{(\omega_L + i\omega)^c}$	$\frac{R}{1 + R/(R_1 + Z_0')}$	$RR_1 / (R + R_1)$	$\frac{R/(R + R_1)}{1 + R_1/Z_0'}$
8	Generalized Cole-Cole	$\frac{a}{[\omega_L^c + (i\omega)^c]^k}$	$\frac{R}{1 + R/(R_1 + Z_0')}$	$RR_1 / (R + R_1)$	$\frac{R/(R + R_1)}{1 + R_1/Z_0'}$