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THREE-DIMENSIONAL KINEMATIC MIGRATION IN INHOMOGENEOUS ISOTROPIC MEDIA

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Abstract

A three-dimensional (3-D) kinematic migration method is developed for layered media with variable velocities in layers and curved interfaces. Zero-offset traveltimes $t_0(x,y)$ and interval velocities obtained from CMP and borehole data are assumed to be known. The method is based on the integration of the suggested ray tracing system. This system consist of five ordinary differential equations of the first order. Vertical coordinate z has been taken as independent variable. The formulas obtained make it possible to derive the necessary initial conditions for the solution of the initial-value problem for ray tracing system. To obtain the interfaces one should take into account the wave transmission at upper interfaces where raypath satisfy Snell's low. To adjust Snell's low for integrating of ray tracing system variational calculation has been used. The migration method comprises the two-dimensional fitting of traveltimes, interval velocities and interfaces obtained in downward sequence. For local fitting a polynomial of given order n of two variables is used. The suggested method has been successfully applied to interpretation of CMP data obtained in north-west Syria. The depth maps of five reflecting horizons are constructed.

INTRODUCTION

Complete 3-D dynamic migration of wavefields is rarely performed because it involves large data volumes and requires weeks to moths of computation time, even on vector computers. 3-D kinematic migration was suggested and developed instead of full dynamic wavefield imaging (Kleyn, 1977; Gjoystdal, 1981; Jakucowicz, 1983; Robinson. 1983; Wen, 1984). Presently all available kinematic migration algorithms are based on the assumption that the velocity in the layers is ether constant or depends on only one vertical coordinate z. Traveltimes are assumed to be independent of azimuth.
The purpose of this study is to develop the 3-D kinematic migration method and algorithm for layered media with variable velocities and curved interfaces and to apply this method to real CMP data.

THEORY

We will assume that a medium under study is layered one and the interfaces between layers are curved. Zero-offset traveltimes $t_0(x,y)$ for each of the interfaces $z_j(x,y)$, j=1, 2, ..., n, are assumed to be known from CMP-data. Interval velocities are also known from CMP and log borehole data. Within each of layers with fixed number j the interval velocity $v_j(x,y)$ is the arbitrary continuous function of horizontal coordinates x and y. At the interfaces $z_j(x,y)$ the function $v_j(x,y)$ has the jump discontinuities. The problem in question is the one of constructing the reflecting interfaces $z_j(x,y)$ using the zero-offset traveltimes $t_{0i}(x,y)$ and interval velocities $v_j(x,y)$.
The problem under consideration can be formulated as one of integrating the following system of ordinary differential equations of rays

$$\frac{dx}{dz}=\frac{\sin\gamma\cos\delta}{\cos\gamma};\qquad \frac{dy}{dz}=\frac{\sin\gamma\sin\delta}{\cos\gamma};\qquad \frac{dt}{dz}=\frac{1}{v_i\cos\gamma};$$
$$\frac{d\gamma}{dz}=\frac{-\frac{\partial v_i}{\partial x}\cos\delta-\frac{\partial v_i}{\partial y}\sin\delta}{v_i(x,y)};\qquad \frac{d\delta}{dz}=\frac{-\frac{\partial v_i}{\partial x}\sin\delta-\frac{\partial v_i}{\partial y}\cos\delta}{v_i(x,y)\sin\gamma\cos\gamma};\tag{1}$$

where $v_j(x,y)$ is a function of the layer number j (j=0, 1, 2, ..., n) and horizontal coordinates x,y. Equations (1) define the ray trajectory x=x(z), y=y(z), the traveltime t along the rays; γ is the angle between the vector of the tangent to the ray at the point (x,y,z) and z-axis and δ is the angle between the projection of this vector on the plane xy and x-axis.
It is necessary to determine the initial conditions for the system (1), to find the solution of the initial value problem for equations (1) and to determine from this solution the values x,y,z corresponding to the value $\tau=t_0(x,y)/2$, where $t_0(x,y)$ is the zero-offset traveltime from the reflecting interface. Locus of reflection for all zero-offset rays is the migrated reflector surface.

Initial conditions: $x(0)=x_0$, $y(0)=y_0$, $t(0)=t_0$, $\gamma(0)=\gamma_0$, $\delta(0)=\delta_0$ for the system (1) have the following meaning: x_0, y_0 are the coordinates of the zero-offset ray at the plane z=0, t(0)=0 is the initial time and γ_0, δ_0 determine the initial direction of the zero-offset ray. Formulae for the determination γ_0 and δ_0 can be obtain using eikonal equation, that gives

$$\frac{\partial t}{\partial x}=\frac{\cos\alpha}{v(x,y,z)};\quad \frac{\partial t}{\partial y}=\frac{\cos\beta}{v(x,y,z)};\quad \frac{\partial t}{\partial z}=\frac{\cos\gamma}{v(x,y,z)};\qquad (2)$$

where cosα, cosβ and cosγ are the direction cosines of the unit vector of the tangent to the ray on the point (x,y,z). Introducing the notations: $\partial t(x,y,z)/\partial x\big|_{z=0}=t'_{0x}$, $\partial t(x,y,z)/\partial y\big|_{z=0}=t'_{0y}$ we obtain from (2)

$$t'_{0x}=\frac{\sin\gamma_0\cos\delta_0}{v_0};\quad t'_{0y}=\frac{\sin\gamma_0\sin\delta_0}{v_0};\qquad (3)$$

and final formulae

$$\gamma_0=\arcsin\ v_0\sqrt{t'^2_{0x}+t'^2_{0y}};\quad \delta_0=arcctg\ \frac{t'_{0x}}{t'_{0y}};\qquad (4)$$

It should be emphasized that the right sides of equalities (6) can be determine from the experimental data and thus the angles γ_0 and δ_0 are calculated.

In order to construct the second interface and the other more depth interfaces one should take into account wave transmission at the upper interface according to Snell's low. For expressing Snell's low in three-dimensional case we use the variational calculation. Consider Fermat's functional

$$I=\int_A^B\frac{\sqrt{x'^2(\sigma)+y'^2(\sigma)+z'^2(\sigma)}}{v_u(x,y,z)}d\sigma+\int_B^C\frac{\sqrt{x'^2(\sigma)+y'^2(\sigma)+z'^2(\sigma)}}{v_b(x,y,z)}d\sigma;\qquad (5)$$

where σ is a parameter, v_u is a velocity in the upper medium, v_b is a velocity in the bellow medium, the points A, B, C are shown on the Figure 1.

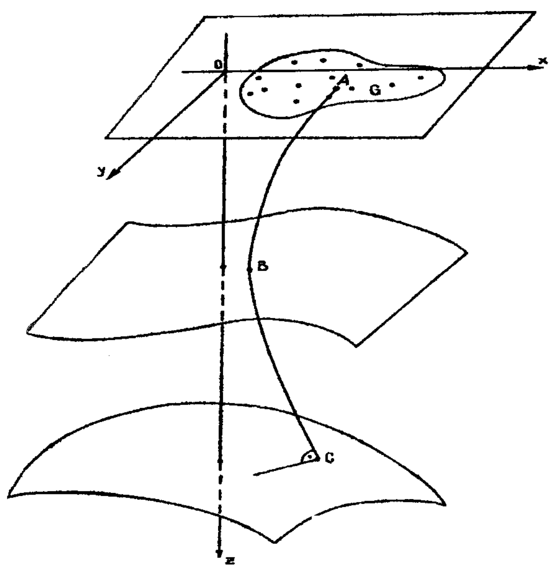

Figure 1 - Zero-offset ray ABC in the medium with variable velocity and curved reflecting and refracting interfaces; B is a point of refraction; C is a point of reflection from the curved surface. G is a domain of definition of the zero-offset traveltime function f(x,y,z)

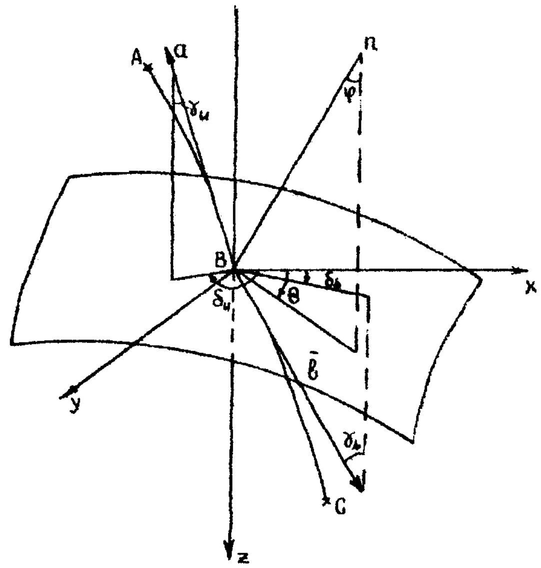

Figure 2 - Schematic illustration of generalization of Snell's low in 3-D case for the curved interface and variable velocities in upper and below media.
$\overline{n}$ is a unit vector of normal to the refracting surface;
$\overline{a}$ is a unit vector of the tangent to the incident ray;
θ, φ; γ_u, δ_u; γ_b, δ_b are angle coordinates of vectors $\overline{n}$, $\overline{a}$, $\overline{b}$ accordingly.

In the variational problem under consideration B is the moving point on refracting surface f(x,y,z)=0. Using known conditions of transversality at the point B one can obtain the connection between normal vector to the refracting surface and vectors of the tangents to incident and refracted rays at point B (Figure 2). Conditions of transversality give the following expressions of Snell's low

$$\frac{\sin\gamma_u\sin(\theta-\delta_u)}{v_u}=\frac{\sin\gamma_b\sin(\theta-\delta_b)}{v_b};$$

$$\frac{\sin\varphi\cos\theta\cos\gamma_u - \cos\delta_u\sin\gamma_u\cos\varphi}{v_u} = \frac{\sin\varphi\cos\theta\cos\gamma_b - \cos\sigma_b\sin\gamma_b\cos\varphi}{v_b}; \tag{6}$$

$$\frac{\sin\delta_u\sin\gamma_u\cos\varphi - \cos\gamma_u\sin\theta\sin\varphi}{v_u} = \frac{\sin\delta_b\sin\gamma_b\cos\varphi - \cos\gamma_b\sin\theta\sin\varphi}{v_b};$$

The meaning of angles φ, θ; γ_u, δ_u; γ_b, δ_b are shown on the Figure 2. On refracting interface values γ and δ changed from values $\gamma_u=\gamma_j$, $\delta_u=\delta_j$ to values $\gamma_b=\gamma_{j+1}$, $\delta_u=\delta_{j+1}$ and these new initial values that are need for integrating the ray tracing system (1) can be found in the explicit form from the Snell's low (6).
For integrating the equations (1) with the above considered initial conditions we must calculate values of functions $t_0=t_0(x,y)$, $v=v(x,y)$, $z=z(x,y)$ and their partial derivatives of x and y. Functions $t_0=t_0(x,y)$ and $v=v(x,y)$ are given in a tabular form using experimental data. A function z(x,y) is to be determined by calculating the coordinates of the reflection points, i.e. in a tabular form as well. Values of all these functions may contain errors, and we have to fit these data.
The problem can be formulated as the following one. There is a set of values of the function f(x,y) obtained experimentally inside the domain G(x,y). It is necessary to fit the values of function f(x,y) and to solve the interpolation problem for any inner point (x_0,y_0) of domain G(x,y). For the solving the given problem the following known approach is used. It is significant that the solution of the equations (1) must be obtained only in the local vicinities of the points where values $t_0(x,y)$ are given. Therefore for the point (x_0,y_0) one can take the m nearest points where we have got experimental data about the function f(x,y). The polynomial of given order n which passing through the m points is used. The given approach allows to unify the processes of the fitting, interpolation and numerical differentiation into one process. Coefficients a_{ij} of the polynomial

$$P(x,y) = \sum_{i=0}^{n}\sum_{j=0}^{n-i} a_{ij}x^i y^j; \tag{7}$$

are found as a solution of a problem of minimization of the expression

$$\sum_{k=1}^{m}\mu_k\left[P(x,y) - f(x_k,y_k)\right]^2 \to \min; \tag{8}$$

where n is the given order of a fitting polynomial; $f(x_k,y_k)$ are experimental data about the function f(x,y); m is the number of data points (m>n); μ_k are the weighted coefficients for the points (x_k,y_k), where the values of function f(x,y) are given.
For the quantitative determination of μ_k the following approach is used. The point (x_0,y_0) receives the maximal value of the coefficient $\mu=\mu_{max}$ and the points (x_k,y_k) receive the value μ_k decreasing linearly from the value μ_{max} to 1 in dependence with the distance r_k between points (x_0,y_0) and (x_k,y_k). The value $\mu=\mu_{max}$ is a parameter of approximation determined from of a priori information about behavior of function f(x,y) inside the domain G(x,y).

REAL DATA EXAMPLE

The above described method of determination of the reflecting interfaces in three-dimentional media with variable velocity is applied to interpretation of CMP data obtained on the Latakia area in north-west Syria. On time sections obtained for a set profile lines reflections from five reflecting horizonts were recorded. The time their registration is the range 0-2 s. At the Latakia area there are fore deep wels where the seismic log was curried out. Using log data the interval velocities were determined. RMS velocities obtained along the profile lines at 80 observation points were transformed into interval velocities. These velocities are functions of the vertical time, but interval velocities determined using log data are depended on depth z. Therefore it is necessary to transform dependencies $v_{int}(x_i,y_i,t_{ij})$ into dependencies $v_{int}(x_i,y_i,z_{ij})$ for integrating the ray tracing system (1). Transformation is realized using the following recursive formula

$$z_{ij} = \begin{cases} z_{i,j-1} + v(t_{ij} - t_{i,j-1}),\ j>0 \\ v_{i0}t_{i0},\ j=0 \end{cases} \tag{11}$$

where j is the layer number and values v_{ij} are not changed.
For each of layers with number j the interval velocity $v_j(x,y)$, partial derivatives $v'_{ij}(x,y)$, $v'_{jy}(x,y)$ and module of velocity gradient are calculated. For layer 4, for example, within the area 9×9 km the minimal velocity values are equal to 3600-3800 m/s and the maximal values 4400-4600 m/s accordingly. Horizontal gradient of the interval velocity is changed in the range of 0,2-0,8 s^{-1}.
On the Figure 3 the zero-offset traveltime map for the horizon 4 is shown and on the Figure 4 the migrated reflector surface 4 is presented.

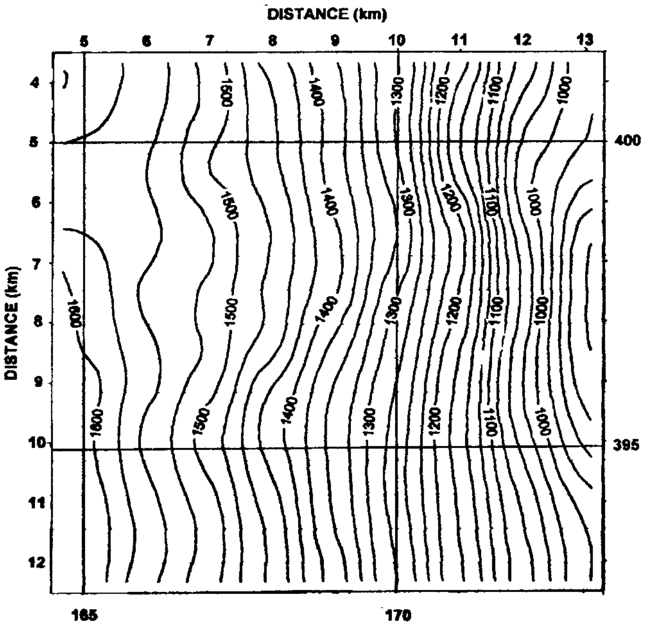

Figure 3 - Zero-offset traveltime map for 4 of the 5 interpreted reflectors. Times are in milliseconds.

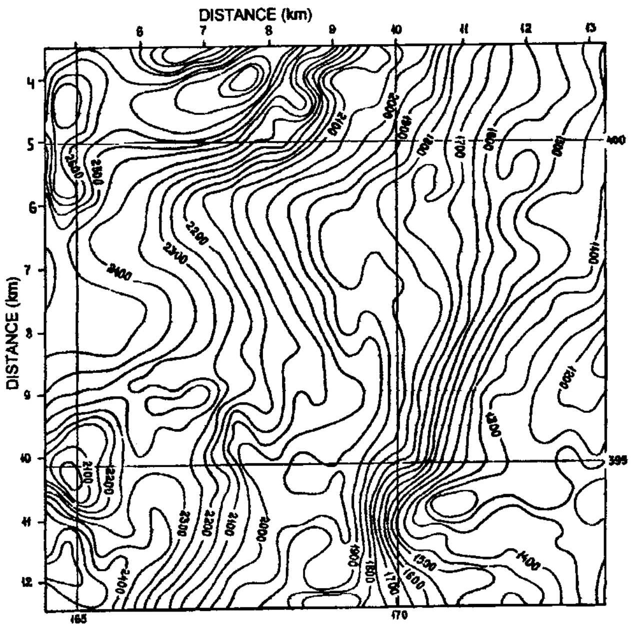

Figure 4 - Reflector depth map for interface 4 corresponding to the time map of figure 3. Depth are im meters.

CONCLUSION

A 3-D kinematic migration method for layered media with variable velocities and curved interfaces is presented. We use all available information about the distribution of velocity in a medium under study just interval velocities from CMP data along all survey lines and log borehole data. In each of layers the interval velocity depends on two horizontal coordinates.

The problem of 3-D kinematic migration is formulated as a initial-value one of integrating the suggested ray tracing system. The initial conditions for this system are obtained using eiconal equation. Obtained on the basis of variational calculation the generalization of Snell's low for 3-D medium with curved interfaces and variable velocities in upper and below media allows to take into account refraction of zero-offset ray on all interfaces obtained in downward sequence.

The proposed migration method can be used with any desired survey configuration. The input is data presented in zero-offset traveltime form for each reflector to be migrated. The output is a depth map of the migrated reflector surface. The method has been successfully applied to interpretation of CMP data obtained in north-west Syria.

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