

# Equivalent offset migration: additional applications

John C. Bancroft

CREWES Project, University of Calgary

## ABSTRACT

The concepts of equivalent offset migration are extended to rugged topographies, prestack migration of converted-wave (P-S) data, computation of residual statics before NMO correction, perform anisotropic prestack depth migration, and prestack migration of vertical receiver array data from VSP's or vertical marine cables. These applications are all based on common scatter point (CSP) gathers, which are formed rapidly by summing input data before any moveout computations.

## INTRODUCTION

The equivalent offset method of prestack migration separates the Kirchhoff method into two processing steps (Bancroft et al., 1998, and the previous paper in this volume), i.e.

all input traces are summed into common scatter point (CSP) gathers, and

the CSP gathers are Kirchhoff NMO corrected and stacked to produce the prestack migrated section.

Each input trace that is within the migration aperture contributes energy to each migrated trace. These input traces are summed into a CSP gather at an equivalent offset  $h_e$  before any moveout correction is applied. This method assumes the source and receiver are co-located on the surface at an equivalent offset distance from a defined scatter point. The traveltimes of the source and receiver raypaths are now equal, and allow their total travel time be defined by a hyperbolic equation. These hyperbolic traveltimes are then equated with the double square-root equation that defines the traveltimes of the offset source and receiver, giving an exact solution for the equivalent offset  $h_e$ , i.e.,

$$T = \left[ \left( \frac{T_0}{2} \right)^2 + \frac{(x+h)^2}{V_{mig}^2} \right]^{1/2} + \left[ \left( \frac{T_0}{2} \right)^2 + \frac{(x-h)^2}{V_{mig}^2} \right]^{1/2} = 2T_e = 2 \left[ \left( \frac{T_0}{2} \right)^2 + \frac{h_e^2}{V_{mig}^2} \right]^{1/2} \quad (1) \quad \text{GIVING} \quad h_e^2 = x^2 + h^2 - \left( \frac{2xh}{TV_{mig}} \right)^2$$

(2)

The CSP gathers may be formed with inaccurate velocities, but then analyzed with conventional methods to obtain accurate velocities for Kirchhoff NMO.

After the CSP gathers have been formed, velocity analysis provides accurate velocities that may be used for the moveout portion of the process. The NMO correction that is applied to CMP gathers becomes a Kirchhoff NMO correction for the CSP gathers and requires the antialias filtering, scaling, and phase filtering, that is required in Kirchhoff migrations. Stacking of the CSP gathers completes the prestack migration process.

## RUGGED TOPOGRAPHY

Prestack migration by the Kirchhoff method allows the source and receiver raypaths to be considered independently. The migration travel times are calculated from the surface elevation as indicated in Figure 1, and equated to the two-way travel time to the equivalent offset, which is located on the datum (E). The RMS velocity for the scatter point is defined at the datum, but can be modified (using Dix formula) to accommodate the elevations and interval velocities of the source and receiver. The total travel times for the original ray paths are computed from the zero offset time for the source  $T_0/2 + t_s$ , the zero offset time for the receiver

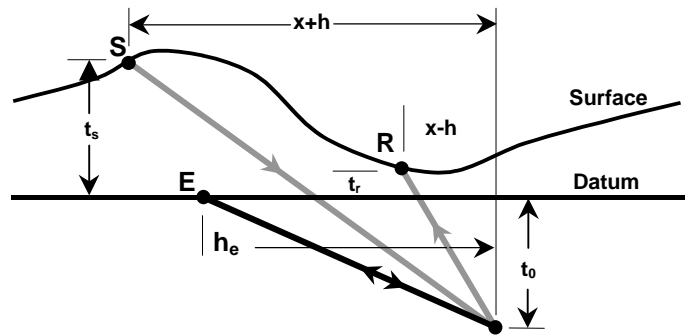


Figure 1. Ray for rugged topography

$T_0/2 + t_r$ , and appropriate velocities for the source and receiver defined from the surface, i.e.

$$T = \left( \left( \frac{T_0}{2} + t_s \right)^2 + \frac{h_s^2}{V_{rms}^2 (T_0 + 2t_s)} \right)^{1/2} + \left( \left( \frac{T_0}{2} + t_r \right)^2 + \frac{h_r^2}{V_{rms}^2 (T_0 + 2t_r)} \right)^{1/2} = 2 \left( \frac{T_0^2}{4} + \frac{h_e^2}{V_{rms}^2 (T_0)} \right)^{1/2},$$

(3)

WHERE  $T_S$  AND  $T_R$  ARE THE VERTICAL TRAVEL TIMES FROM SOURCE AND RECEIVER TO THE DATUM. USING EQUATION (3), INPUT TRACES MAY NOW BE SUMMED INTO THE CSP GATHERS SIMILAR TO THE PREVIOUS METHOD (GEIGER AND BANCROFT 1996).

## PRESTACK MIGRATION OF CONVERTED-WAVE (P-S) DATA

EOM may be applied to converted wave data where the scattered energy is recorded on three component receivers to

separate the compressional P-wave and the shear S-wave. Converted waves have a P-wave velocity  $V_p$  from the source to the scatterpoint (or conversion point), and an S-wave velocity  $V_s$  from the scatterpoint to the surface. Conventional processing of the P-S data has been quite difficult, but Wang et al. (1996) extended the concept of equivalent offset to include the appropriate P and S wave velocities. The co-located source and receiver traveltimes to the scatterpoint are now different as indicated in equation (4) where the zero offset traveltimes and velocities must be modified to account for the changing velocities.

$$T = \left[ \left( \frac{T_{0p}}{2} \right)^2 + \frac{(x+h)^2}{V_p^2} \right]^{1/2} + \left[ \left( \frac{T_{0s}}{2} \right)^2 + \frac{(x-h)^2}{V_s^2} \right]^{1/2} = \left[ \left( \frac{T_{0p}}{2} \right)^2 + \frac{h_e^2}{V_p^2} \right]^{1/2} + \left[ \left( \frac{T_{0s}}{2} \right)^2 + \frac{h_e^2}{V_s^2} \right]^{1/2} \quad (4)$$

$$T = \frac{1}{V_p} [\hat{z}_{0p}^2 + h_e^2]^{1/2} + \frac{1}{V_s} [\hat{z}_{0s}^2 + h_e^2]^{1/2} = \frac{1}{V_p} [\hat{z}_{0p}^2 + h_e^2]^{1/2} + \frac{\gamma}{V_p} [\hat{z}_{0s}^2 + h_e^2]^{1/2} = \frac{(1+\gamma)}{V_p} [\hat{z}_0^2 + h_e^2]^{1/2} \quad (5)$$

The zero offset traveltimes may be replaced with pseudo depths ( $\hat{z}$ ) in equation (5). The P and S pseudo depths  $\hat{z}_{0p}$  and  $\hat{z}_{0s}$  are almost equal, and with  $\gamma = V_p/V_s$  we get the final expression in equation (5), which is a hyperbola. Again, the complex double square-root equation is simplified to a single square root, which will allow prestack migration gathers to be formed for converted wave data. Forming these common-conversion-scatterpoint (CCSP) gathers is identical to the conventional EOM method with the scattered energy aligned on hyperbolic paths. Velocity analysis of the CCSP gathers will produce velocities and traveltimes that are related to P-wave velocities and traveltimes by  $(1 + \gamma)$ , which, when compared with P-wave data, will produce the S-wave velocities. Examples may be found in Bancroft and Wang (1996).

### COMPUTATION OF RESIDUAL STATICS BEFORE NMO CORRECTION

Conventional residual-statics methods require the application of NMO to the input traces followed by correlation with a model trace that has usually been formed from a filtered brute stack. Li and Bancroft (1997) demonstrated an algorithm for the computation of residual-static corrections *before* the application of NMO. The method uses model traces extracted from CSP gathers.

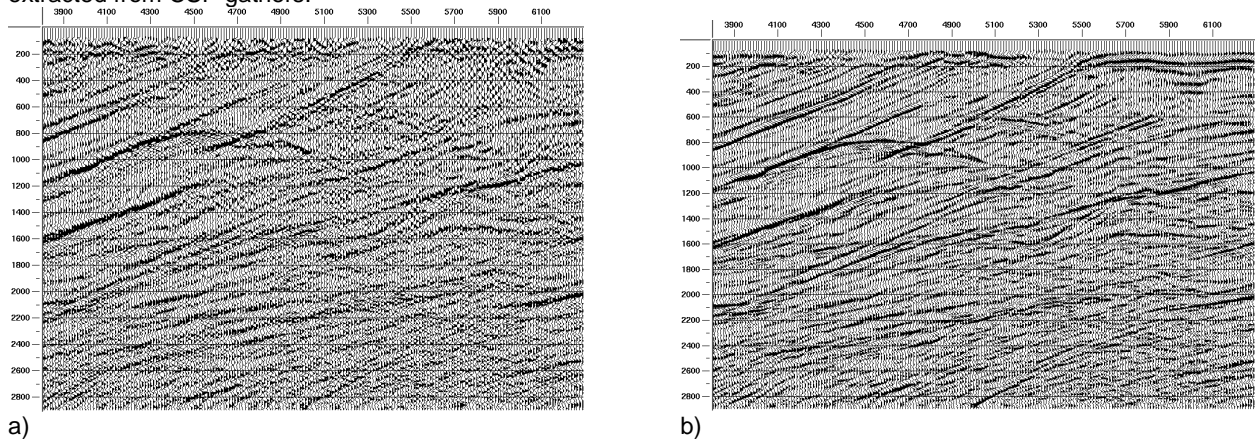


Figure 2. Marmousi stacks: a) the stack that includes random statics, and b) with EOM estimated statics.

After the CSP gathers have been formed, the mapping of an input trace to many CSP gathers can be used to extract many model traces for cross correlation with input trace. The cross-correlation provides a time shift that can be combined with the cross correlation times of all other traces to extract surface consistent residual statics. Figure 2 show two stacked section of the Marmousi data set. The first section (a) is a stack formed when random statics (• 20 ms) have been applied to each source and receiver. Section (b) shows the result of using the equivalent offset method of estimating the residual statics. It should be noted that all efforts to obtain conventional residual statics failed.

### ANISOTROPIC PRESTACK DEPTH MIGRATION

Chernis (1998) extended the equivalent offset method to include prestack depth migration. Rather than use the RMS assumptions of time migration, the traveltime  $T$  at  $(x, z_0)$  is computed from raytracing or traveltimes maps. The equivalent offset method again provides an intermediate process in which the input data is first mapped to CSP gathers. The equivalent offset is defined by a hyperbolic relationship, but now the average velocity may be used to provide an exact match between the  $T_0$  times in the CSP gathers to the depth  $z_0$  of the depth section.

The effects of anisotropy have been included in the traveltimes computations to allow the formations of CSP gathers that include the effects of anisotropy. Physically modelled data was created from anisotropic material that was inclined at 45 degrees. The data was prestack depth migrated with conventional methods that assumed isotropic velocities and compared and compared with a method that included the anisotropic effects. Two CSP gather are displayed in Figure 3 that have NMO correction applied to evaluate the linearity of the energy. The first gather (a) assumed isotropic velocities while (b) used anisotropic processing. Note the improved linearity in the anisotropic CSP gather.

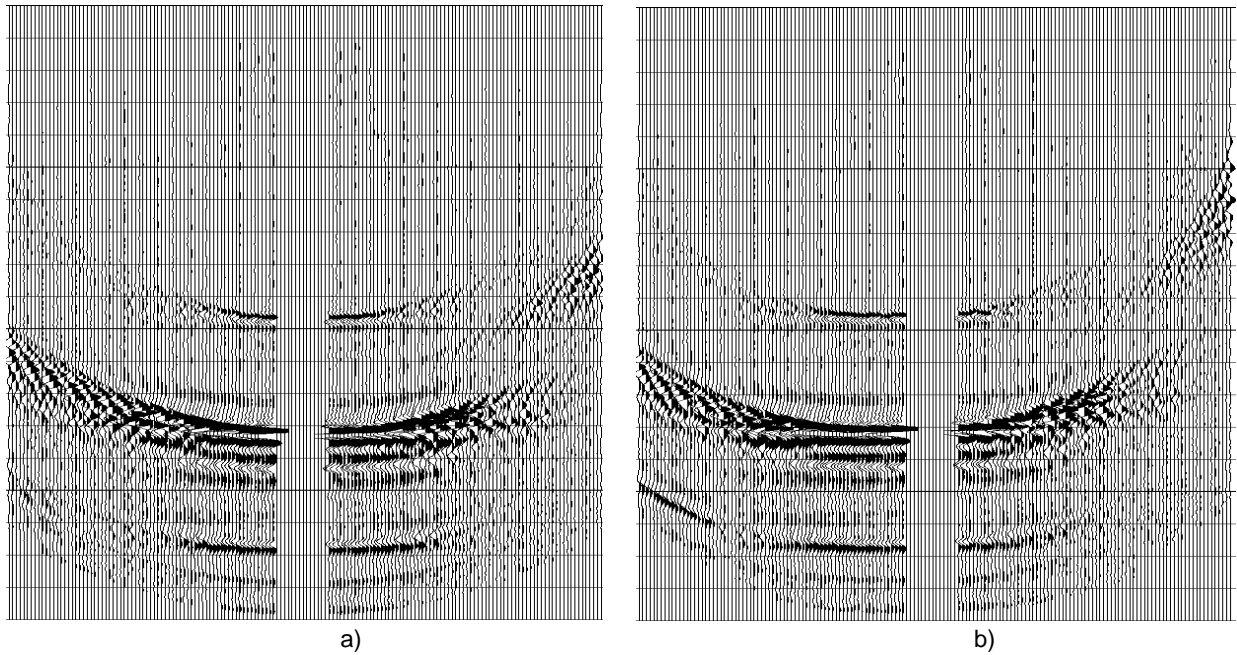


Figure 3. CSP gathers of anisotropic model: a) using an isotropic algorithm, and b) and anisotropic algorithm.

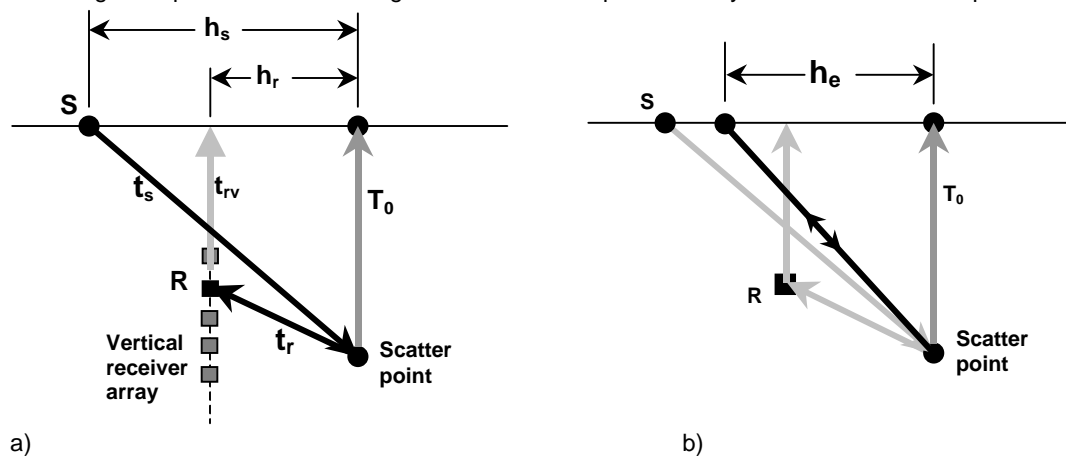
**VERTICAL RECEIVER ARRAY DATA FROM VSP'S OR VERTICAL MARINE CABLES**

The equivalent offset method of prestack migration is extended to include data that is acquired with vertical receiver arrays that may be located in a well, or on a vertical marine cable. This data may also be processed to form CSP gathers before moveout correction is applied to the data. The input traces, however, do require a single time shift to the entire trace (static) when it is summed into the CSP gathers.

A method that produces CSP gathers with velocities and times similar to conventional processing is illustrated in Figure 4a. This figure shows the source and receiver raypaths with travel-times  $t_s$  and  $t_r$  that define the total travel-time  $T$ . Figure 4b shows the co-located source and receiver raypaths. An additional vertical ray path from the receiver to the surface datum is also shown with travel time  $t_{rv}$ . A new total travel time  $T_{srv}$  is defined for vertical array EOM as,

$$T_{srv} = t_s + t_r + t_{rv} = \left[ \frac{T_0^2}{4} + \frac{(x+h)^2}{V_s^2} \right]^{1/2} + \left[ \frac{(T_0 - t_{rv})^2}{4} + \frac{(x-h)^2}{V_r^2} \right]^{1/2} + t_{rv} = 2 \left[ \frac{T_0^2}{4} + \frac{h_e^2}{V_s^2} \right]^{1/2} \quad (6)$$

The inclusion of the vertical time  $t_{rv}$  in equation (6) is the same for all scatterpoints, which has the same effect of adding a static shift of  $t_{rv}$  to the input trace when it is added to the CSP gather. The objective of maintaining the simplicity of summing the input data to the CSP gathers is therefore preserved by the inclusion of a simple static shift.



a)

b)

Figure 4. Raypath diagram in a) showing the traveltimes of  $t_s$ ,  $t_r$ , and  $t_{rv}$ , with b) showing the raypath for co-located source and receivers at the datum.

Note the different velocities of  $V_s$  and  $V_r$  in equations (6).  $V_s$  is the conventional RMS velocity that is defined at the scatter point for rays that travel to the surface. The receiver ray, however, does not travel to the surface and requires a new RMS velocity to be calculated for the time range from  $T_0$  to  $t_{rv}$ , using Dix formula. As in conventional EOM processing, the transition times at bin boundaries of the VCSP gather are computed. The input trace is then added to the corresponding bins in the VCSP gather. This process is repeated for all input traces and for each CSP gather. Kirchhoff NMO and stacking completes the prestack migration of the VSP data.

3-D VSP data was preprocessed with the sources at a flat datum. CSP gathers were then formed for a North/South line for comparison with previously published material Bicquart (1998). A cut and paste comparison of the two prestack migrations is shown in Figure 5.

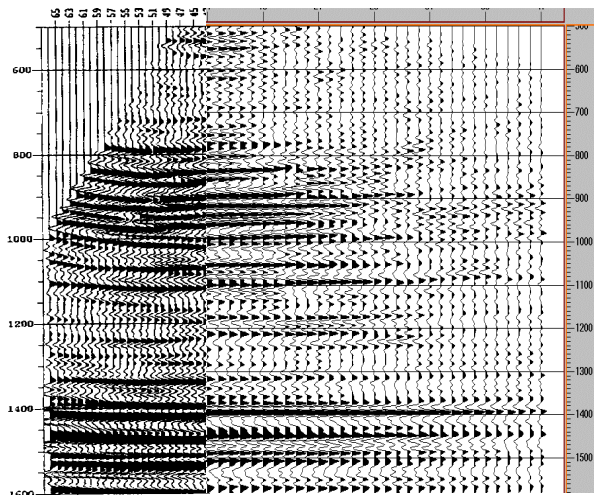


Figure 5. Comparison of prestack migrations by Bicquart and EOM on VSP data.

## CONCLUSIONS

The versatility of the EOM method of prestack migration was demonstrated by a variety of applications. Each application took advantage of the formation of CSP gathers that are formed before moveout correction.

### References

- Bancroft, J. C., and Geiger, H. D., 1994, *Equivalent offsets and CRP gathers for prestack migration: 64<sup>th</sup> Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts*, 672-675.
- Bancroft, J. C., Geiger, H. D., and Margrave, G. F., 1998, *The equivalent offset method of prestack time migration: Geophysics, Vol.63, No. 6, P. 2042-2053.*
- Bancroft, J. C., and Wang, S., 1996, *Converted-Wave (P-S) migration and velocity analysis using equivalent offset migration, Extended Abstracts, 58<sup>th</sup> Conference and Technical Exhibition*, p X-23.
- Bicquart, P., 1998, *Application of Kirchhoff Depth Migration to 3D-VSP, Exp. Abs. SEG Nat. Conv., p 390-392*
- Chernis, L., 1998, *Depth migration by generalised equivalent migration offset mapping: Exp. Abs. SEG Nat. Conv., p 1550-1553.*
- Geiger, H. D., and Bancroft, J. C., 1996, *Equivalent offset prestack migration – application to rugged topography and formation of unmigrated zero-offset section, Extended Abstracts, 58<sup>th</sup> Conference and Technical Exhibition*, p X24.
- Li, X, and Bancroft, J. C., 1997, *Integrated residual statics analysis with prestack time migration, Expanded Abstracts, SEG 67<sup>th</sup> Annual Meeting*, p. 1452-1455
- Wang, S., and Bancroft, J. C., 1996, *Converted wave (P-SV) prestack migration and velocity analysis, Expanded Abstracts, SEG International Exposition*, pp.1575-1578.