



# Sensibility Analysis of the Multifocusing Traveltime Approximation

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## ABSTRACT

The zero-offset (ZO) seismic section can be simulated by properly stacking a set of common-offset seismic sections, using conventional procedures like the well know Common-Midpoint (CMP/DMO) method. In the recent past years, a new stack technique for 'simulating a ZO section was proposed, the so-called Multifocusing Stack (MFS). This technique can be used for arbitrary configuration and number of source and geophone pairs. The travelttime approximation of the stack formula depends on three wavefront parameters: (1) the radius of curvature of the *NIP* wave,  $R_{nip}$ ; (2) the radius of curvature of the normal wave,  $R_n$ ; and (3) the emergence angle of the reflection normal ray,  $\beta_o$ . All these three wavefront parameters are obtained as solution of a inverse problem. They provide the best fitting of the stack surface, the so-called Common-Reflection-Surface (CRS), on the observed multicoverage seismic data. In this paper we present a sensibility analysis of the multifocusing travelttime approximation on relation to the variation of the wavefront parameters. By analysing the first derivative of the Multifocusing Travelttime on relation to each one of the searched-for wavefront parameters, we describe the behavior of the Common Reflection Surface. This result is important to indicate if exist a region of the seismic data space where we could simultaneously do a three parameters optimization procedure.

## INTRODUCTION

In the paper of Hubral (1983) the zero-offset geometrical spreading factor is described with help of two fictitious wave, the so-called Normal-Incidence-Point Wave (*NIP* wave) and the Normal Wave (*N* wave). In recent works (Tygel et al., 1997 and Gelchinsky et al., 1997), we have seen that the same fictitious waves, *NIP* and *N* waves, can be used also to describe new paraxial travelttime approximations, that are useful for simulating zero-offset seismic sections. In this new approximations the travelttime in the paraxial vicinity of a central ray is described by certain number of parameters related with the central ray. If the central ray is the normal ray, and we assume a bi-dimensional wavefield propagation, they are three parameters: (1) The radius of curvature  $R_{nip}$ ; (2) the radius of curvature  $R_n$ ; and (3) the emergence angle  $\beta_o$ . The near surface velocity  $v_o$  is considered a priori know in the vicinity of the emergence point of the normal reflection ray. It is important to observe that there are several possibilities to express such travelttime approximation. To be known we have two second-order approximations, namely parabolic and hyperbolic (Tygel et al., 1997), and a double-square-root approximation (Gelchinsky et al., 1997). All of them make use of a stack surface defined by the paraxial travelttime of the reflection rays with arbitrary source -receiver configuration.

By using a hyperbolic approximation, Müller et al. (1998) applied the Multifocusing Method (or Common-Reflection-Surface Stack) to a set of synthetic seismic data, in a noise enviroment, by considering a heterogeneous layered medium. As result it was shown that this new technique is able to simulate zero-offset sections and as by-product gives the three wavefront parameters, that are useful for developing macrovelocity model inversion procedures (Cruz and Martins, 1998).

In order to indicate the region of the seismic data that is appropriate to apply a three parameters optimization, we describe in this paper the behavior of the CRS when varying each one of the search-for parameters.

## MULTIFOCUSING TRAVELTIME APPROXIMATION

At this point we present the multifocusing travelttime formula that was first given by Gelchinsky et al. (1997), and rewritten by Tygel et al. (1997) in the final form

$$\tau = t_o + \Delta t_S + \Delta t_G \quad (1)$$

$$\text{where } \Delta t_S = \frac{r_S}{v_o} \left[ \sqrt{1 + 2 \frac{\sin \beta_o}{r_S} \Delta x_S + \left( \frac{\Delta x_S}{r_S} \right)^2} - 1 \right], \quad \Delta t_G = \frac{r_G}{v_o} \left[ \sqrt{1 + 2 \frac{\sin \beta_o}{r_G} \Delta x_G + \left( \frac{\Delta x_G}{r_G} \right)^2} - 1 \right] \quad (2)$$

in wich  $r_j$  are wavefront radii of curvature at the source ( $j = S$ ) and receiver ( $j = G$ ), respectively. The source and

receiver separations to the central point are  $\Delta x_S = x_m - h - x_o$  and  $\Delta x_G = x_m + h - x_o$ . The expressions for the corresponding wavefront curvatures  $K_j = 1/r_j$ ; are given by

$$K_S = \frac{K_n - \gamma K_{nip}}{1 - \gamma}, \quad K_G = \frac{K_n + \gamma K_{nip}}{1 + \gamma}, \quad K_{nip} = \frac{1}{R_{nip}}, \quad K_n = \frac{1}{R_n}, \quad \text{and} \quad \gamma = \frac{h}{x_m - x_o} \quad (3)$$

In the equation (1)  $t_o$  is the zero-offset reflection traveltimes.  $x_o$  is the horizontal coordinate of the emergence point of the reflection normal ray,  $x_m$  and  $h$  are the midpoint coordinate and half-offset corresponding to a source-geophone pair. In this formula  $\gamma$  is the focus parameter defined by Gelchinsky et al. (1997). For a specified point  $P_o(x_o, t_o)$  in the time section with the respective three wavefront parameters  $K_{nip}$ ,  $K_n$  and  $\beta_o$  we can calculate the paraxial traveltimes that define the Common-Reflection-Surface used to stack the seismic data (Figure No. 1).

### SENSIBILITY ANALYSIS

The most important step toward obtaining a simulated zero-offset section by the Multifocusing Traveltime Approximation, is the definition of the optimization procedure to find the best trio ( $K_{nip}$ ,  $K_n$ ,  $\beta_o$ ). In general it is necessary to expend very much computational effort and time to find out which combination of parameters is the best one. It is a basic question for any optimization procedure, how sensitive is the functional that simulated the observed data to variations in the searched-for parameters. This question is answered here after analysis of the first derivative of the referred paraxial traveltimes function (1) on relation to each one of the wavefront parameters. The three derivatives are given as follows

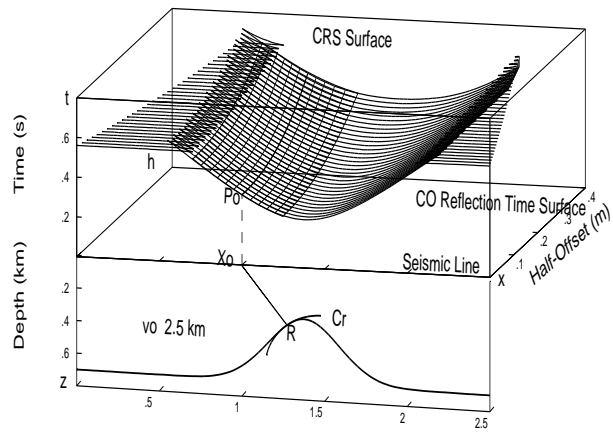


Figure No. 1.- Seismic Model and Common Reflection Surface.

Traveltime derivate on relation to  $K_n$  ( $\partial\tau / \partial k_n$ )

$$\frac{\partial\tau}{\partial K_n} = \frac{\partial\Delta t_S}{\partial K_n} + \frac{\partial\Delta t_G}{\partial K_n} \quad (4)$$

Traveltime derivate on relation to  $K_{nip}$  ( $\partial\tau / \partial k_{nip}$ )

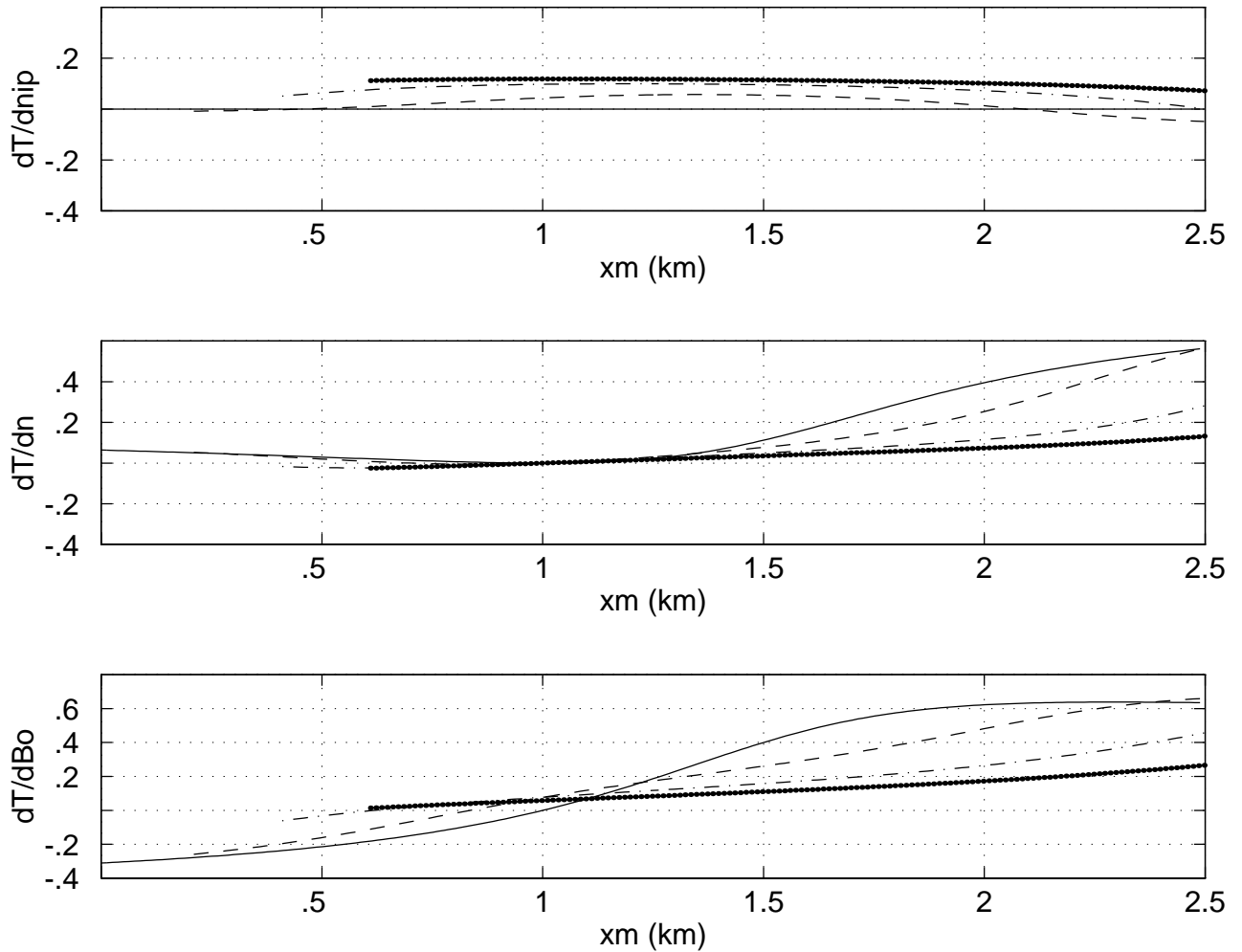
$$\frac{\partial\tau}{\partial K_{nip}} = \frac{\partial\Delta t_S}{\partial K_{nip}} + \frac{\partial\Delta t_G}{\partial K_{nip}} \quad (5)$$

Traveltime derivate on relation to  $\beta_o$  ( $\partial\tau / \partial \beta_o$ )

$$\frac{\partial\tau}{\partial \beta_o} = \frac{\partial\Delta t_S}{\partial \beta_o} + \frac{\partial\Delta t_G}{\partial \beta_o} \quad (6)$$

Each one of these derivatives, expressed by equations (4), (5), and (6), is shown in the Figure No. 2 for selected half-offsets  $h$  and as function of the midpoint coordinate  $x_m$ . We remind that in this analysis we consider a fixed point  $P_o(x_o, t_o)$  in the seismic section. As we can see in the Figures, the time derivatives on relation to  $K_n$  (4) and  $\beta_o$  (6) have higher values at smaller offsets and midpoints far from the emergence point of the normal ray. The same does not occur with the other derivative. The time derivative on relation to  $K_{nip}$  shows that only in larger offsets this last parameter is important. It is possible to indicate that only within certain region of the CRS, the three wavefront parameters concur at the same way the optimization procedure. Another view of this sensibility analysis can be found through the Figure No. 3. In that Figure, the Common-Reflection-Surface is calculated by formula (1), using the true

parameters  $K_{nip}$ ,  $K_n$  and  $\beta_o$ , related with the constant velocity model of Figure No 1, and represented by the black surface. The other two stack surfaces are calculated using values of wavefront parameters, that correspond to plus or minus fifty percent of the original values. In the upper part we have stack surfaces for  $K_{nip}$ 's, in the middle part for  $K_n$ 's and in the bottom for  $\beta_o$ 's.



**Figure No. 2.-** Sensibility of three wavefront parameters for  $h=0.0$  km, (solid line),  $h=0.4$  km, (dashed line),  $h=0.8$  km, (dash-dotted line) and  $h=1.2$  km, (dotted line).

## CONCLUSIONS

By using time derivatives of the multifocusing traveltime approximation, we have analyzed the sensibility of the functional that simulates the observed data on relation to each one of the searched-for parameter. It is important to stress that the traveltime function is highly sensitive to the  $K_n$  and  $\beta_o$  parameters at smaller offsets and midpoints far from the emergence point of the normal ray. In the case of  $K_{nip}$ , it is important only when we have larger offsets.

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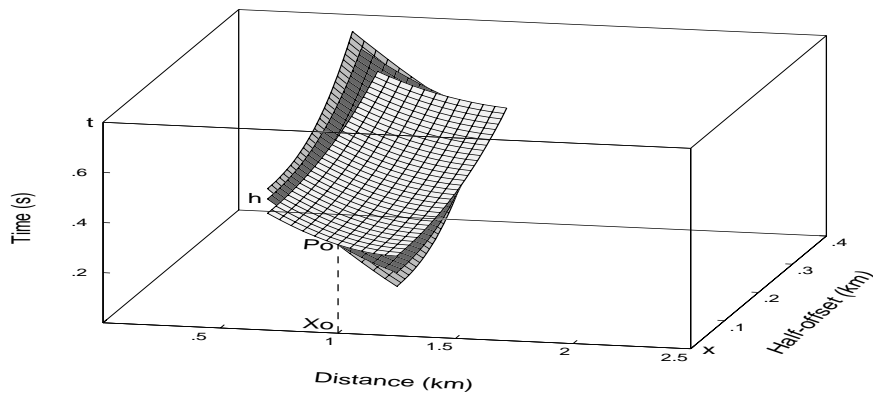
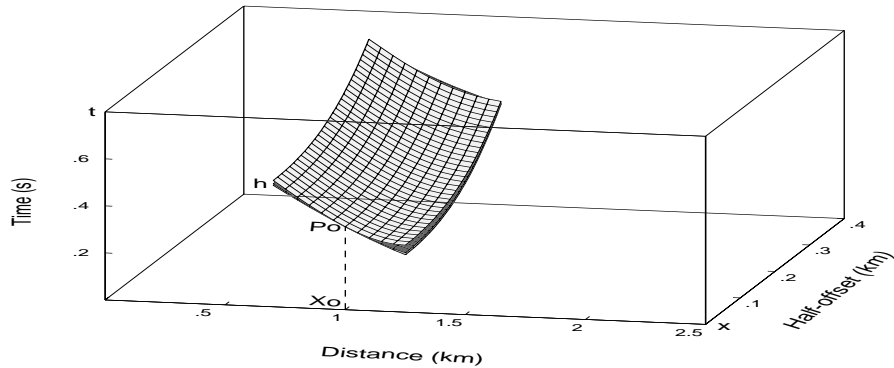
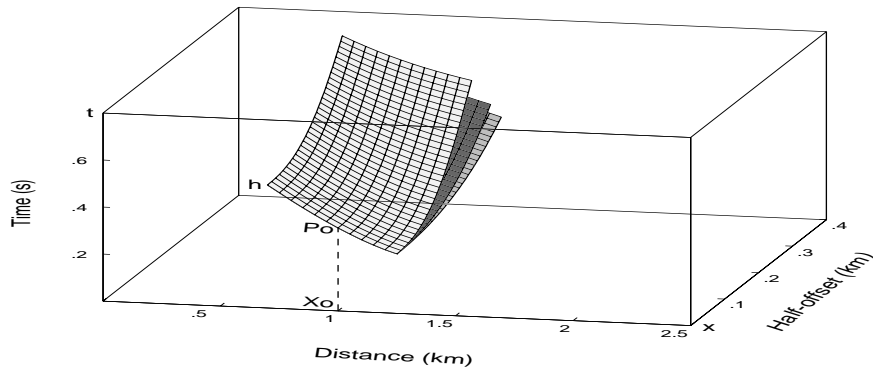
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**Figure No. 3.-** The Common-Reflections Surface for several values of  $K_{nip}$ , in the upper part, of  $K_n$ , in the middle part, and of  $\beta_o$ , in the bottom part. The black stack surface was calculated by using the true values of parameters related with the model of Figure No 1,  $K_{nip} = 2.26757 \text{ km}^{-1}$ ,  $K_n = 1.25328 \text{ km}^{-1}$ , and  $\beta_o = -38.0623^\circ$ .