# Stability and Accuracy Analysis of 2.5-D True Amplitude Diffraction Stack Migration

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## ABSTRACT

The weighted modified diffraction stack operator provides the necessary correction to remove the geometrical spreading in the compressional primary reflection wavefield. This is reached by applying a proper weight function to the input data during the migration process. The theoretical development is based on describing the seismic wavefield by the ray asymptotic zero-order approximation and using a Kirchhoff type migration operator. In this paper we present a true-amplitude migration algorithm that is able to be applied to two-and-one-half (2.5-D) seismic data. Through example of application to a set of synthetic seismic data with noise enviroment, it is possible to confirm the high efficiency of the algorithm for obtaining the reflection coefficient, which is of great interest for the amplitude versus offset (angle) analysis.

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## INTRODUCTION

In several recent papers, we can find migration algorithms which intend to obtain the reflectivity properties of the subsurface from reflection seismic data. One of the main publications is given by Bleistein (1987), that based on the Born approximation obtains a weight function necessary for determining the reflection coefficient by migration process. In another way, Schleicher et al. (1993) presented a three-dimensional (3-D) true amplitude diffraction stack migration integral based on ray theoretical assumptions. This paper follows the latter proposed research line.

The main concern in this theoretical development is to determine a weight function to be used in the diffraction stack integral operator. After this operator is applied to the input data, it is expected that the migrated seismic data does not suffer from geometrical spreading, providing a measure of the reflection coefficient. Based on this theoretical result, we have developed a migration algorithm and tested it for two-and-one-half-dimensional (2.5-D) synthetic data, in a common-shot configuration, although it is able to be used for any seismic data configuration. The 2.5-D synthetic data is generated by using the ray software SEIS88, considering a bi-dimensional (2-D) seismic model with a point source.

### 2.5-D True Amplitude Migration

Starting from the three-dimensional (3-D) diffraction stack migration as presented by Schleicher et al. (1993), and after applying the stationary phase analysis on relation to the parameter coordinate along which the velocity is constant, we can derive the 2.5-D version of the migration integral (Urban, 1999)

$$V(M,t) = \frac{1}{\sqrt{2\pi}} \int_A d\xi\, w_{2.5}(\xi, M)\, \partial_{t-}^{1/2} U(\xi, t+\tau_D). \qquad (1)$$

In the equation (1) the diffraction in-plane traveltime curve $\tau_D$ is defined for all points of parameter $\xi$ on the earth surface, and each point $M$ within a specified volume of the seismic model. The source and geophone pair (*S,G*) is then defined by the surface parameter $\xi$. The symbol $\partial_{t-}^{1/2}$ is the anti-causal half-time derivative operator that corresponds in the frequency domain to the filter $\sqrt{-i\omega}$. The weight function $w_{2.5}$ must be chosen so that the result of migration process has no effect of the geometrical spreading. As part of the stationary phase solution of the 3-D diffraction stack migration integral we can write the weight function by

$$w_{2.5}(\xi, M) = w_{2.0}(\xi, M)\left(\frac{1}{\sigma_S} + \frac{1}{\sigma_G}\right), \qquad (2)$$

where the quantity $\sigma$ in the equation (2) is an in-plane parameter, related with each ray branch by $d\sigma = v(s)ds$, beeing the velocity $v(s)$ a function of the ray arc length $s$ on the considered ray path, *S-M* or *G-M*. The function $w_{2.0}(\xi, M)$ is the in-plane weight calculated for the bi-dimensional seismic experiment (Urban, 1999)

$$w_{2.0} = \frac{\sqrt{\cos\alpha_S \cos\alpha_G}}{v_S} \frac{|\Gamma_S N_{SM} + \Gamma_G N_{GM}|}{\sqrt{|N_{SM}|}\sqrt{|N_{GM}|}} \exp[-\frac{i\pi}{2}(\kappa_1 + \kappa_2)], \qquad (3)$$

where $N_{SM}$, $N_{GM}$ are second derivatives of the traveltime functions for the two ray branchs, the first starting at *S* reaching the point *M* within the seismic model and from *M* up to the point *G* in the seismic line. The $\Gamma_S$ and $\Gamma_G$ parameters are related with the data configuration, $\kappa_1, \kappa_2$ are the KMAH index (number of caustics), $\alpha_S$ and $\alpha_G$ are the start and emergence angles of the ray at *S* and *G*, respectively. By stacking the data with the integral operator (1) weighted by (2), we have an estimate of the reflection coefficients, that could be even complex, preserving the form of the source wavelet as presented by Hanitzsch (1995).

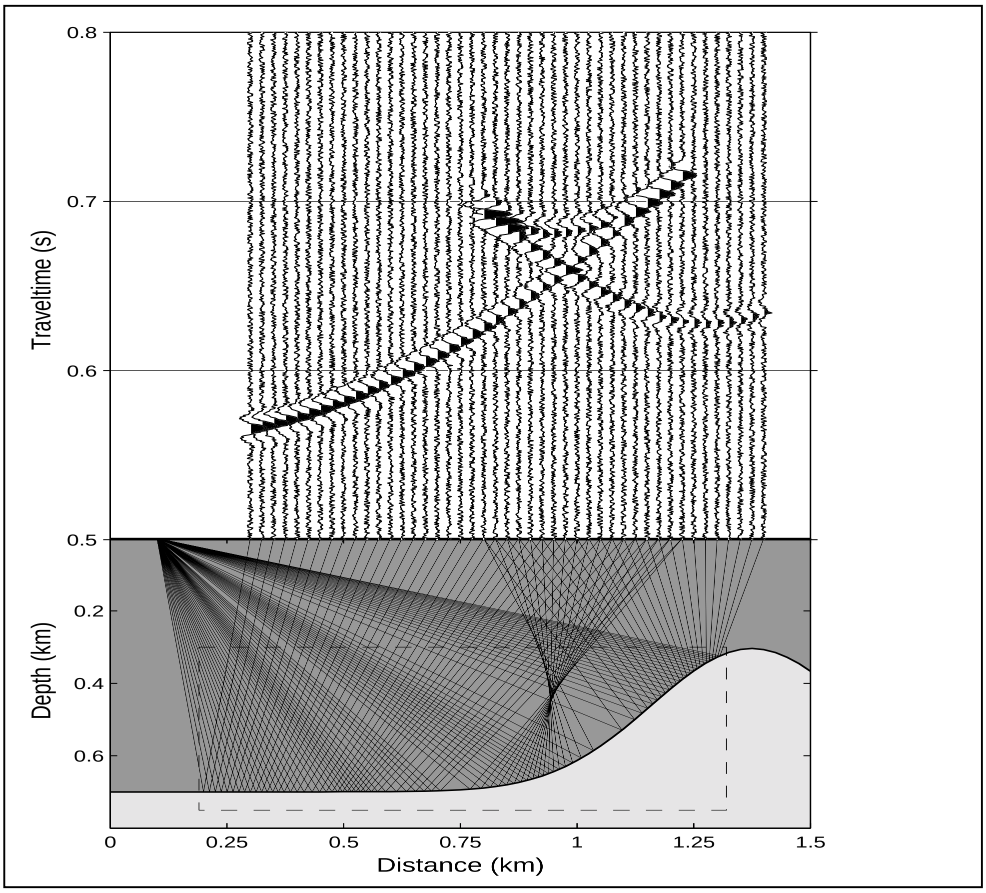

Figure 1 – *Lower-half*: Seismic model with ray trayectories corresponding to a common shot gather. *Upper-half*: Common shot section with noise. The wavefield corrensponds to the principal component of the displacement of the primary compressional seismic wave.

## EXAMPLE

The true-amplitude migration algorithm was tested on synthetic data obtained from the SEIS88 ray tracing software. The seismic model is constituted by a layer above an arbitrary curved reflector. The interval velocity of the reflected P-wave at the overburden is 2.5 Km/s, and 3.0 Km/s in the half-space. The seismic data was generated tinto a common-shot configuration, with the source at x=0.1 km in the earth surface and 177 geophones positionated between 0.3 and 1.4 Km, beeing the geophone interval distance 6.25 m. The source pulse is represented by a Gabor wavelet with frequency 80 Hz, while the seismic trace has the sample interval of 0.5 ms. In the seismic data was added a random noise with uniform distribution, which maximum value is 10% of the maximum amplitude.of the seismic data The macrovelocity model and the seismogram with noise is presented in the Figure 1.

The seismic data were migrated by using the true constant velocity model, having the target zone $0.19 \le x \le 1.32$ Km; $0.3 \le z \le 0.75$ Km, with $\Delta x = 5$m and $\Delta z = 1$m . The migrated seismic image (real part) is presented in the Figure 2. In the Figure 3 we have the reflection coefficients, where the dotted line is the exact value of the reflection coefficients, while the cross line is obtained from the amplitude picking of the migrated sections – real and imaginary.

## CONCLUSIONS

From the results obtained in this paper, we can affirm that the presented 2.5-d weighted diffraction stack migration operator, when applied to 2.5-d seismic data is able to recover the reflection coeffcient even in noise enviroment. the presented algorithm is stable, so that for small changes in the input data we have only slight deviations in the output migrated data. it is to be stressed that the proposed true-amplitude migration algorithm works very good for complex situations when there are triplications on the input data due the presence of caustics.

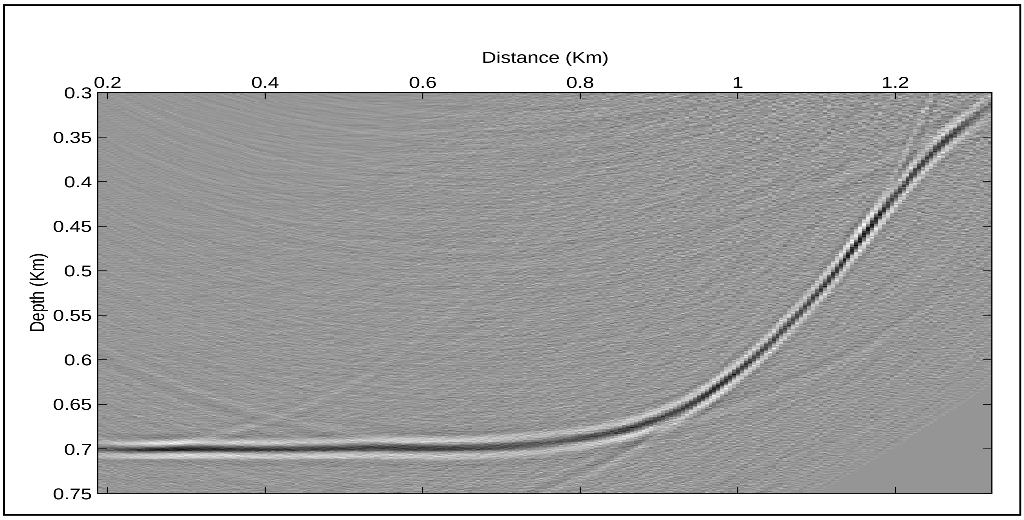

Figure 2 – Depth migrated image (real part) for the input data of the Figure 1.

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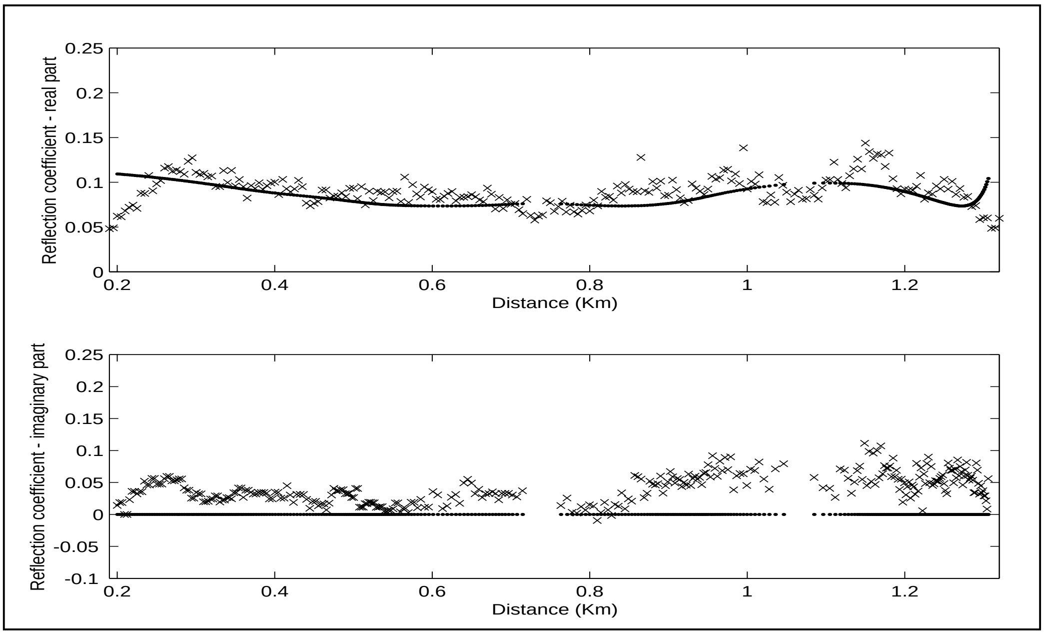

Figure 3. – Reflection coefficients obtained by the weighted diffraction stack algorithm. The dotted line corresponds to the exact values and the cross line to the true amplitude migration result.