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AN APPROXIMATE KINEMATICAL MZO IMPULSIVE RESPONSE

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Abstract

In this paper I describe an approximate method to evaluate the kinematics of the MZO impulsive response in inhomogeneous media. This method demands smoothness of the MZO impulsive response which, in turn, demands smoothness of the velocity field. For any fixed point on the MZO impulsive response, I show that its neighboring points can be recursively determined by simple first-order relations. Motivation for this is to save CPU resources that are usually required when ray-tracing techniques are employed. For homogeneous media, a comparison between the proposed method and a conventional one based on ray tracing showed considerable savings in computational. Choosing the initial point as corresponding to a zero temporal dip in a shallow event, the accumulated errors summed up to half the smallest period allowed by anti-(spatial)aliasing filters for usual acquisition patterns. Deeper events demand a higher spatial sampling rate of the MZO impulsive response than possible with usual CMP interval in regular seismic data.

INTRODUCTION

Pre-stack migration is generally required in geologically complex areas. For laterally varying velocity media, the usual processing sequence NMO+DMO+pos-stack migration fails since, in this case, the stacked section cannot be taken as a simulated zero-offset section. This is because NMO is supposed to work on vertically inhomogeneous media and DMO was originally designed for constant-velocity areas. For arbitrary velocity media, MZO iis designed to provide reliable zero-offset output simulated sections out of input common-offset sections in one step, thus substituting the cascaded operations of NMO and DMO. MZO can be an attractive process if it can bring significant savings in computational effort as compared to pre-stack migration. Usually, Kirchhoff MZO is accomplished upon the construction of arrival-time tables that will be scanned to determine isochrons and their zero-offset reflection times. This can become a very time-cosuming task. The method I describe here is an attempt to avoid traveltime tables and searches. This is done upon the consideration of simple, first-order relationship that exist between pairs $(t,\vec{x})$ in the MZO impulsive response. At the expense of some accuracy, starting from a single point on the MZO curve, all $(t,\vec{x})$ pairs in this curve can be recursively estimated using a first-order differential relationship. Amplitudes along the impulsive response are not discussed here. It will be the subject of another work to introduce the true amplitude formulas given in Tygel et al (1996) to the present approach.

A FIRST-ORDER RELATIONSHIP BETWEEN POINTS IN AN MZO IMPULSIVE RESPONSE

Following Deregowski et al. (1981), the kinematics of an MZO impulsive response in a common-offset section can be estimated by means of a theoretical common-offset migration followed by demigration to zero-offset. The constraints to acceptable ray trajectories comes from the non-variability of time and offset (Artley et al, 1994). These variables are functions of the ray parameter vectors $\vec{p}$ associated with the source and the receiver and the ray final position vector $(\vec{x},z)$. The position vector and traveltime of a reflection ray at the surface can be written as:

$$\vec{x} = \vec{x}_i + \int_{(\vec{x}_i,0)}^{(\vec{x},z)} \frac{\vec{p}\, v\, dz'}{\sqrt{1-p^2v^2}} \; ; \; t = \int_{(\vec{x}_i,0)}^{(\vec{x},z)} \frac{dz'}{v\sqrt{1-p^2v^2}} \quad ,$$

(1)

where v is the velocity, which is a function of the position $(\vec{x}',z')$, and $(\vec{x}_i,0)$ is the starting point of the ray. An isochron in common vector offset $2\vec{h}$ section is characterized by the constant time $t_{\vec{h}}$ given by

$$t_{\vec{h}} = \int_{(\vec{x}_i,0)}^{(\vec{x},z)} \frac{dz'}{v\sqrt{1-p_S^2v^2}} + \int_{(\vec{x}_i,0)}^{(\vec{x},z)} \frac{dz'}{v\sqrt{1-p_R^2v^2}} \, , \qquad \textbf{(2)}$$

where the subscripts S and R stand for source and receiver, respectively and the constant (half) offset vector $\vec{h}$ is

$$2\vec{h} = \vec{x}_R - \vec{x}_S = \int_{(\vec{x}_i,0)}^{(\vec{x},z)} \frac{\vec{p}_S\, v\, dz'}{\sqrt{1-p_S^2 v^2}} - \int_{(\vec{x}_i,0)}^{(\vec{x},z)} \frac{\vec{p}_R\, v\, dz'}{\sqrt{1-p_R^2 v^2}}\,, \tag{3}$$

with $\vec{x}_S$ and $\vec{x}_R$ respectively the source and receiver positions. In equations (2) and (3) the independent variables are p_{SX}, p_{SY}, p_{RX}, p_{RY} (for laterally varying velocity media this are initial ray parameter components) and z. However, constraining the points $(\vec{x},z)$ to lie at the isochron in which $t_{\vec{h}}$ and $\vec{h}$ are constants, the number of inpendent variables is two. In differential form, we may, for instance, write dp_{RX}, dp_{RY} and dz as functions of dp_{SX} and dp_{SY}. This can be done by inverting the system of equations

$$dt_{\vec{h}} = 0 \Rightarrow \frac{\partial t_{\vec{h}}}{\partial p_{RX}} dp_{RX} + \frac{\partial t_{\vec{h}}}{\partial p_{RY}} dp_{RY} + \frac{\partial t_{\vec{h}}}{\partial z} dz = -\frac{\partial t_{\vec{h}}}{\partial p_{SX}} dp_{SX} - \frac{\partial t_{\vec{h}}}{\partial p_{SY}} dp_{SY}\,; \tag{4}$$

$$dh_X = 0 \Rightarrow \frac{\partial h_X}{\partial p_{RX}} dp_{RX} + \frac{\partial h_X}{\partial p_{RY}} dp_{RY} + \frac{\partial h_X}{\partial z} dz = -\frac{\partial h_X}{\partial p_{SX}} dp_{SX} - \frac{\partial h_X}{\partial p_{SY}} dp_{SY}\,; \tag{5}$$

$$dh_Y = 0 \Rightarrow \frac{\partial h_Y}{\partial p_{RX}} dp_{RX} + \frac{\partial h_Y}{\partial p_{RY}} dp_{RY} + \frac{\partial h_Y}{\partial z} dz = -\frac{\partial h_Y}{\partial p_{SX}} dp_{SX} - \frac{\partial h_Y}{\partial p_{SY}} dp_{SY}\,. \tag{6}$$

The system formed by equations (4), (5) and (6) is ill posed in caustic regions. Additional equations can also be found if we consider the MZO distance vector from the CMP position vector, namely

$$\vec{x}_{MZO} = \vec{x}_0 - \frac{\vec{x}_S + \vec{x}_R}{2} = \frac{1}{2}\int_{(\vec{x}_i,0)}^{(\vec{x},z)} \frac{\vec{p}_S\, v\, dz'}{v\sqrt{1-p_S^2 v^2}} + \frac{1}{2}\int_{(\vec{x}_i,0)}^{(\vec{x},z)} \frac{\vec{p}_R\, v\, dz'}{v\sqrt{1-p_R^2 v^2}} - \int_{(\vec{x}_i,0)}^{(\vec{x},z)} \frac{\vec{p}_0\, v\, dz'}{v\sqrt{1-p_0^2 v^2}}\,, \tag{7}$$

where the subscript 0 stands for zero offset. As a result of the application of Snell´s law, the zero-offset ray parameter vector at the isochron can be written as

$$\vec{p}_0 = \frac{1}{v(\vec{x},z)\sqrt{2}} \frac{\vec{p}_S + \vec{p}_R}{\sqrt{\frac{1}{v^2(\vec{x},z)} + \vec{p}_S \cdot \vec{p}_R + \sqrt{\left(\frac{1}{v^2(\vec{x},z)} - p_S^2\right)\left(\frac{1}{v^2(\vec{x},z)} - p_R^2\right)}}}\,, \tag{8}$$

which, in conjunction with equations (4), (5) and (6), yields the differential expressions

$$dx_{MZO} = \frac{\partial x_{MZO}}{\partial p_{SX}} dp_{SX} + \frac{\partial x_{MZO}}{\partial p_{SY}} dp_{SY}\,;$$

(9)

$$dy_{MZO} = \frac{\partial y_{MZO}}{\partial p_{SX}} dp_{SX} + \frac{\partial y_{MZO}}{\partial p_{SY}} dp_{SY}\,.$$

(10)

Equations (9) and (10) constitute a system that can be inverted to obtain dp_{SX} and dp_{SY} as functions of dx_{MZO} and dy_{MZO}. This completes the cumbersome computations. All partial derivatives in equations (4), (5) and (6) are integrals to be numericaly evaluated. This may become computationally demanding, especially for laterally-varying velocity media where the ray parameter vectors will vary along the ray trajectory. The partial derivatives in equations (9) and (10) are a result of a 3X3 system inversion (in the vicinity of caustics SVD analysis may be required). This does not impose much computational effort. The 2X2 equation system (9) and (10) can be inverted (for instance, in constant velociity areas it will demand SVD analysis) with even less CPU demand.

The algorithm to generate the kinematics of the impulsive response for a given input common offset sample comprises the following steps:

1) For a given pair $(t_{\vec{h}}, \vec{x}_{CMP})$ of the input common offset section, compute as accurately as needed (using ray tracing) the pair $(t_0, \vec{x}_{MZO})$ for a particular zero-offset position $\vec{x}_{MZO}$ and the corresponding source ray parameter vector $\vec{p}_S$.
2) Use the determined $\vec{p}_S$ to estimate all the partial derivatives found in equations (4), (5), (6), (9) and (10).
3) Determine $d\vec{p}_S$ from a given $d\vec{x}_{MZO}$, a chosen fraction of the CMP interval, and increment $\vec{p}_S$.
4) Determine $(t_0, \vec{x}_{MZO})$ for this new value of $\vec{p}_S$ and go back to step 2 until all points of the MZO impulsive response have been obtained.

A TEST FOR CONSTANT VELOCITY MEDIA AND COMMENTS

For the sake of simplicity, I compare the kinematic of the impulsive response as given by this method with the theoretically known for constant velocity media. In this test, the velocity is 1500 meters per second and the offset is 2.0 kilometers.The comparison is made for two different zero-offset, zero-dip time. For the first one, at 1.5 seconds in Figure 1.a, the approximated curve was generated with an interval dx_{MZO} of 20 meters. For the second one, at 5.0 seconds in figure 1.b, the approximated curve was generated with an interval dx_{MZO} of 1 meter. The variation in dx_{MZO} in figures 1.a and 1.b is a function of the curvature of the impulsive response. The greater the curvature, the lesser the necessary dx_{MZO} for a better fit to the theoretical curve. Problems are greater for greater temporal dips since the initial point was the zero dip one.

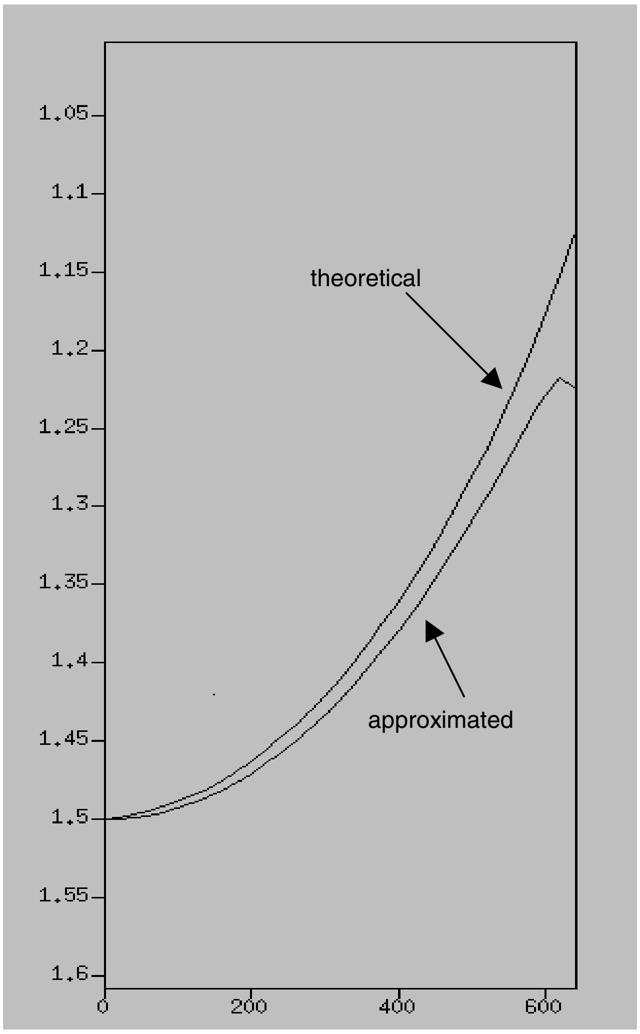

Figure 1.a – A comparison between the theoretical and the approximatted curves for a reflector at 1.5 seconds in a 2.0 km common offset section, with a dx_{MZO} of 20 meters.

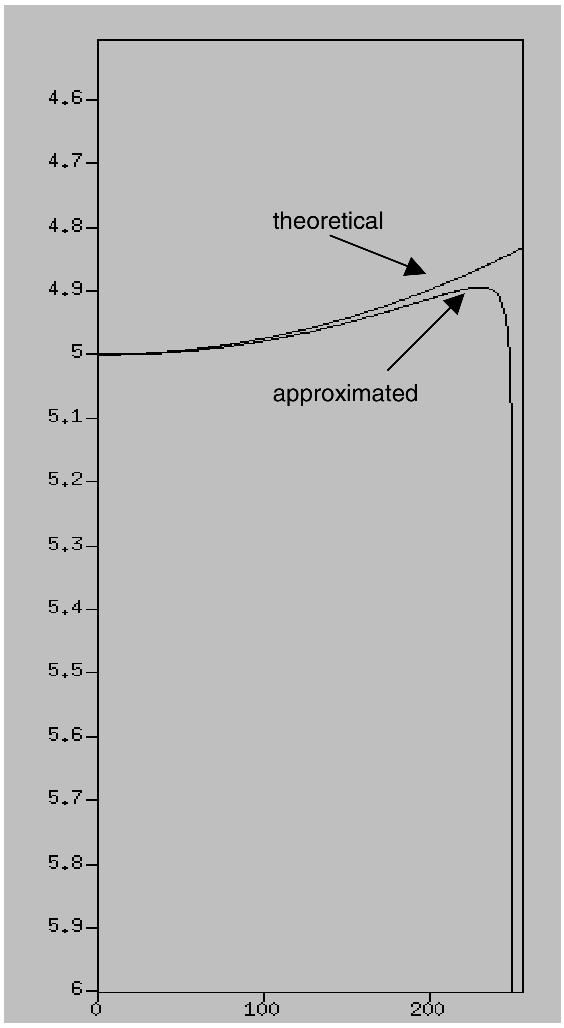

Figure 1.b – A comparison between the theoretical and the approximatted curves for a reflector at 5.0 seconds in a 2.0 km common offset section, with a dx_{MZO} of 1 meter.

I expect this process will gain in accuracy if schemes like Runge-kuta technique are employed. The main question to be answered in this case concerns to the increase in CPU resource demands. The CPU demand to generate all the impulsive response kinematics for a 5 seconds long trace was around 25% of that for the conventional approach using tables of arrival times created with ray tracing. Preliminary considerations on the application of this method to several different offsets yields to the same amount of CPU savings if tables with the values of the partial derivatives in (4), (5) and (6) are created. But this is still to be tested.

CONCLUSIONS

An approximate method for estimating the kinematics of MZO impulsive responses for heterogeneous media was described. The MZO kinematics for constant velocity media estimated by this method was compared to the theoretically determined one. Comparison shows that this method has its application range limited to a vicinity of the initial accurately calculated point. This limitation varies with the increment in CMP positions. Higher accuracy is achieved with smaller CMP position increment. This technique can be improved with more sophisticated schemes like Runge-Kuta with an acompanied increase of CPU demand.

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