# A New Algorithm for Traveltime Multiparameter Estimation*

**SBGF301**

Ernesto G. Birgin, Ricardo Biloti,
Martin Tygel, and Lúcio T. Santos

Department of Applied Mathematics
IMECC – UNICAMP

**Abstract**

**For a fixed, central ray in an isotropic elastic or acoustic media, traveltime moveouts of rays in its vicinity can be described in terms of a certain number of parameters that refer to the central ray only. The determination of these parameters out of multi-coverage data leads to very powerful algorithms that can be used for several imaging and inversion processes. Assuming two-dimensional propagation, the traveltime expressions depend on three parameters. We present a new method to extract these parameters out of coherency analysis applied directly on the data. It uses one-dimensional searches on different sections extracted from the multi-coverage data, followed by a recently developed spectral projected gradient optimization algorithm. Aplication of the method on a synthetic example shows an excellent performance of the algorithm both in accuracy and efficiency. The results obtained so far indicate that the algorithm may be a feasible option to solve the corresponding, harder, full three-dimensional problem, in which eight parameters, instead of three, are required.**

## INTRODUCTION

In the framework of zero-order ray theory, traveltimes of rays in the vicinity (paraxial) of a fixed (central) ray can be described by a certain number of parameters which refer to the central ray only. The approximations are correct up to the second order of the distances between the paraxial and central ray at the corresponding initial and end points. They are, thus, valid independently of any seismic configuration.

Assuming the central ray to be the primary zero-offset ray, the number of parameters (emergence angles and curvatures of certain wavefronts) are three and eight, for two- and three-dimensional propagation, respectively. For two-dimensional propagation, the parameters are the emergence angle of the normal ray and the wavefront curvatures of the normal and normal-incident-point eigenwaves as introduced in Hubral (1983). All parameters are defined at the point of emergence of the central ray, called the central point. This point is, in general, a CMP point where the simulated zero-offset trace is to be constructed.

The use of multiparametric traveltime approximations for imaging purposes is a well-investigated subject. Main contributions to the subject are the *Method of Multifocus* (see, e.g., Gelchinsky (1997) for a recent description), the *Method of Shifted Hyperbolas* (see, e.g., de Bazelaire (1994)) and the very recent *Common-Reflection-Surface (CRS) Method* (see, e.g, Hubral *et al.*. (1998) and Perroud *et al.*. (1999)). These methods vary in general on two aspects, namely (a) the multiparamentric traveltime moveout formula that is used and (b) the strategy employed to extract the traveltime parameters from coherency analysis applied on the multi-covered data.

In this work, we present a new method for the estimation of the parameters for two-dimensional multi-coverage data. It is based on (a) initial one-dimensional searches performed on different sections (CMP and CMP-stacked) and (b) a recently introduced spectral projected gradient (SPG) optimization algorithm (Birgin *et al.*, 1997) applied on common-source sections. The method will be illustrated by its application on a simple synthetic example, where the various aspects of the algorithm can be better understood.

## HYPERBOLIC TRAVELTIME EXPANSION

Let us assume a fixed target reflector $\Sigma$ in depth, as well as a fixed *central* point $X_0$ on the seismic line, considered to be the location of a coincident source- and -receiver pair $S_0 = G_0 = X_0$. We also assume the two-way normal, zero-offset reflection ray, $X_0 R_0 X_0$, called from now on the *central ray*. It hits the reflector at point $R_0$, known as the normal-incident-point (NIP). For a source- and receiver pair $(S, G)$ in the vicinity of the central point, we consider the primary reflected ray $SRG$ relative to the same reflector $\Sigma$. We use the horizontal coordinates $x_0$, $x_S$ and $x_G$ to specify the location of the central point $X_0$, the source $S$ and the receiver $G$. It is convenient to introduce the midpoint and half-offset coordinates $x_m = (x_G + x_S)/2 - x_0$ and $h = (x_G - x_S)/2$. We consider the hyperbolic traveltime expression as in Tygel *et al.* (1997)

$$T^2(x_m, h, \beta_0, K_N, K_{NIP}) = \left(t_0 + \frac{2x_m \sin\beta_0}{v_0}\right)^2 + \frac{2t_0 \cos^2\beta_0}{v_0}\,(K_N\, x_m^2 + K_{NIP}\, h^2)\;, \tag{1}$$

*This work was supported by FAPESP (Grants 98/07704-1 and 97/12125-8), CNPq, Pronex and WIT (Germany).

where $t_0$ is the zero-offset traveltime and $\beta_0$ is the emergence angle the zero-offset ray makes with the surface normal at the central point. The quantities $K_N$ and $K_{NIP}$ are the wavefront curvatures of the *normal* $N$- and *normal-incident-point* $NIP$-waves, respectively, measured at the central point. For particular source-receiver gathers, the above traveltime formula can be simplified. Of interest here are three configurations: the CMP ($x_m = 0$), the zero-offset ($h = 0$) and the common-shot ($x_m = h - x_0$) configurations. The CMP traveltime depends on the combined one parameter $q = \cos^2\beta_0\, K_{NIP}$. The zero-offset traveltime depends on the two parameters $\beta_0$ and $K_N$. Finally, the common-shot traveltime depends on the two parameters $\beta_0$ and $\mu = K_N + K_{NIP}$.
The strategy of using particular configurations to reduce the number of parameters to be estimated has advantages and disadvantages. The main advantage is the sometimes significant reduction of computational effort. As a disadvantage, less redundancy is made use for, as many traces that do not conform to the selected configuration have to be left out.

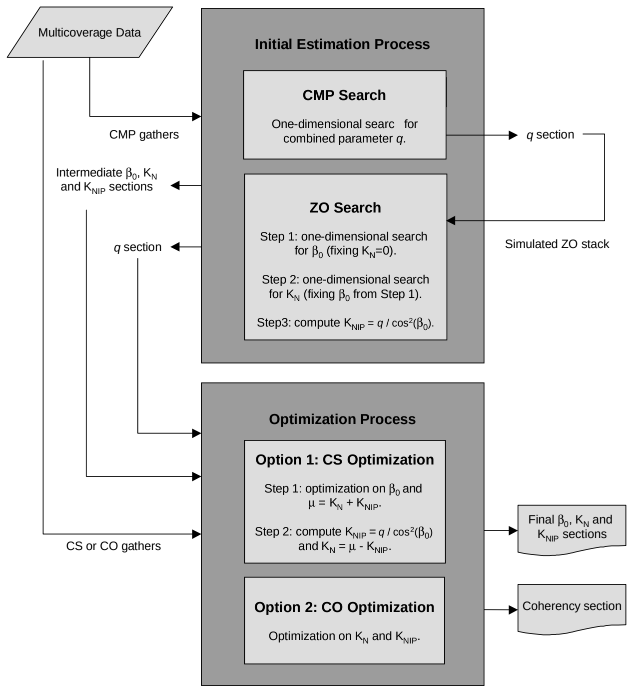


Figure 1: Optimization strategy.

## FORMULATION OF THE PROBLEM AND SOLUTION

The data obtained by a multi-coverage seismic experiment performed on a given seismic line consists of a multitude of seismic traces $U(x_m, h, t)$ corresponding to source-receiver pairs located by varying coordinate pairs $(x_m, h)$ and recording time $0 < t < T$. The basic problem we have to solve is the following:

> Consider a dense grid of points $(x_0, t_0)$, where $x_0$ locates a central point $X_0$ on the seismic line and $t_0$ is the zero-offset traveltime. For each central point $X_0$ let the medium velocity $v_0 = v(x_0)$ be known. From the given multi-coverage data, determine, for any given point $(x_0, t_0)$ and velocity $v_0$, the corresponding parameters $\beta_0$, $K_N$ and $K_{NIP}$.

The general approach to solve this problem is to apply a multiparameter coherency analysis (semblance) to the data, using the traveltime formula (1) to a number of traces $U(x_m, h, t)$ in the vicinity of the central ray $X_0$ and for a suitable time window around the time $t_0$. The desired values of sought-for parameters will be the ones for which one achieves maximum coherence when applying the traveltime (1) to the data.
Given the seismic traces $U(x_m, h, t)$, and the vector of parameters $P = (\beta_0, K_N, K_{NIP})$, the *semblance function* $S$ is

$$S = \frac{\sum[\sum U(x_m, h, T(P))]^2}{M\sum\sum[U(x_m, h, T(P))]^2} \tag{2}$$

where $T(x_m, h, P)$ is given by equation (1), $M$ is the total number of traces, the inner summing is performed over all traces, and the outer one is performed over the time window around $t_0$. For each pair $(x_0, t_0)$ the objective is to find the global maximum of the semblance function (2) with respect to the parameters $\beta_0$, $K_N$ and $K_{NIP}$. These parameters are restricted to the ranges $-\pi/2 < \beta_0 < \pi/2$ and $-\infty < K_N, K_{NIP} < \infty$.

The strategy for computing the global maximum of the semblance function is described in Figure 1. It consists of two main parts: At first, we aim at obtaining good initial values of the parameters; these will be used in the second part, which is an optimization process to produce the final values. The first part consists of two steps (a) a one-parameter search of the combined parameter $q$, performed on the CMP sections and (b) two one-parameter searches for $\beta_0$ and $K_N$, performed on the CMP-stacked section realized using the previous $q$-parameter. The CMP-stacked section is considered as an approximate zero-offset section in these computations.

The optimization process of the second part determines the two parameters $\beta_0$ and $\mu$. For this purpose, we use the recently introduced Spectral Projected Gradient method described in Birgin *et al.* (1997). Finally, using the relationships $K_{NIP} = q/\cos^2\beta_0$ and $K_N = \mu - K_{NIP}$ all the desired parameters can be determined.

## A SYNTHETIC EXAMPLE

We consider the model of a single smooth reflector between two homogeneous acoustic half-spaces. Assuming unit density, the constant velocities above and below the reflector are $v_1$ =2.5 km/s and $v_2$ =2.6 km/s, respectively. The input data for our experiment is an ensemble of 61 common-shot (CS) and 61 common-mid-point seismic sections. Figure 2 shows the model and one of the common-shot experiments. The common-shot seismic sections have 30 traces each. The sources ($x_0$) lie in the range from 0 km to 0.6 km. The CMP seismic sections have 25 traces each, the mid points lying in the range from 0 km to 0.6 km. In both cases, the time window is $0.4s \leq t \leq 9.11s$. We have added a colored noise of 20%. This was obtained by the convolution of white noise with the wavelet used to construct the seismograms.

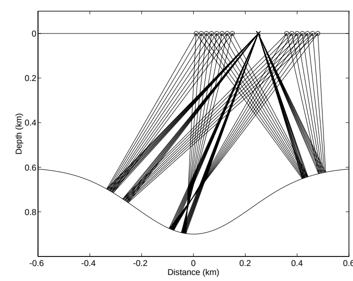


Figure 2: Model and CS acquisition geometry.

Figures 3a-c show the theoretical and estimated parameters after the *Initial Estimation Process*. Figures 3d-f show the theoretical and optimized parameters. Comparisons between theoretical and optimized parameters are depicted in Figures 3g-i. Figure 4a,b show the semblance sections before and after the *Optimization Process*. These sections can be looked upon as simulated zero-offset images of the reflector. They are called in Gelchinsky *et al.* (1997) *semblancegrams*. Finally, Figure 4c shows the maximum semblance function values on the upper branch of the simulated zero-offset image of the reflector before and after the Optimization Process.

## CONCLUSIONS

We have proposed a new algorithm to determine the traveltime parameters out of coherency analysis applied on 2-D multi-coverage seismic data. Using the hyperbolic traveltime approximation and a sequential application of simpler one-dimensional searches followed by a two-dimensional optimization scheme, we were able to derive a very fast and accurate method for the estimation of all sought-for parameters. We applied the algorithm to a synthetic example. Although the application involves a simple one-reflector model, the obtained results were very encouraging. Next steps will be to test the new algorithm to more realistic situations and to extend it to 3-D multi-coverage seismic data. These topics are under current investigation.

## REFERENCES

[1] Birgin, E. G., Martínez, J. M. and Raydan, M., 1997, Nonmonotone spectral projected gradient methods on convex sets. Technical Report 92/97, IMECC-UNICAMP, Brazil.

[2] de Bazelaire, E. and Viallix, J. R., 1994, Normal moveout in focus: *Geophys. Prosp.*, **42**, 477–499.

[3] Gelchinsky, B., Berkovitch, A. and Keydar, S., 1997, Multifocusing homeomorphic imaging: Parts I and II. Presented at the special course on Homeomorphic Imaging by B. Gelchinsky, Seeheim, Germany.

[4] Hubral, P., 1983, Computing true amplitude reflections in a laterally inhomogeneous earth. *Geophysics*, **48**, 1051–1062.

[5] Hubral, P., Höcht, G., and Jäger, R., 1998, An Introduction to the Common Reflection Surface Stack, EAGE 60th meeting and technical exhibition, Leipzig, *Extended Abstracts* 1–19.

[6] Perroud, H., Hubral, P., and Höcht, G., 1999, Common reflection point stacking in laterally inhomogeneous media: *Geophysical Prospecting*, **47**, 1–24.

[7] Tygel, M., Müller, T., Hubral, P. and Schleicher, J., 1997, Eigenwave based multiparameter traveltime expansions. Ann. Internat. Mtg., Soc. Expl. Geoph., *Expanded Abstracts*, 1770–1773.

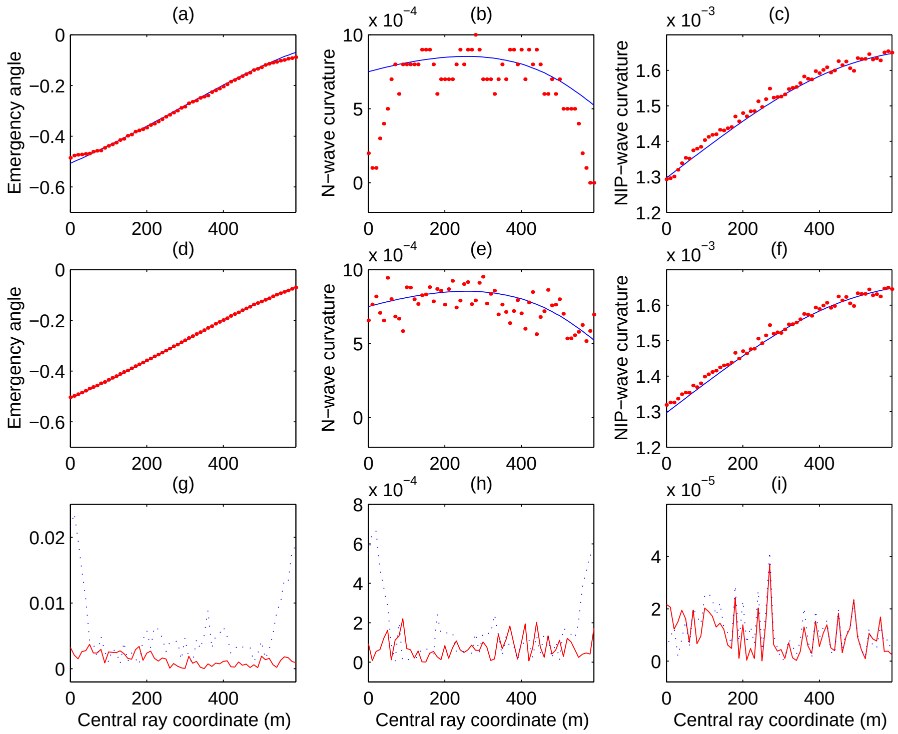


Figure 3: (a-c) theoretical (solid lines) and estimated (dots) parameters after one-dimensional searches. (d-f) theoretical (solid lines) and estimated (dots) parameters after optimization process. (g-i) absolute errors between theoretical and pre- (dots) and post-optimization (solid line) parameters.

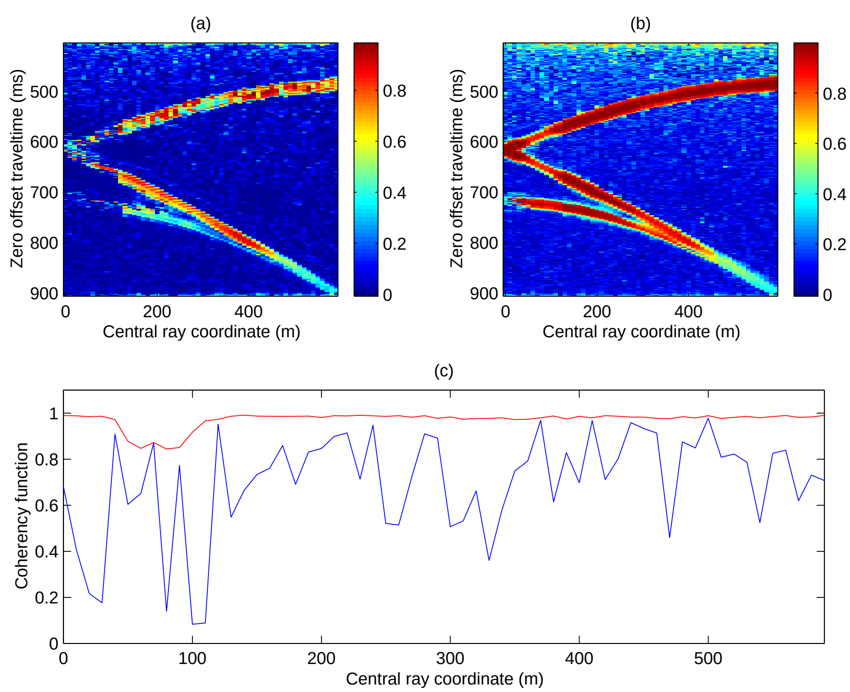


Figure 4: Coherency function evaluatead on CS gathers. (a) before optimization, (b) after optimization, and (c) pre- and post-optimization over the reflector. See the great improvement in coherency fucntion value along the reflector.