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# Prestack equivalent offset migration: the basics

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**Abstract**

**A method of prestack time migration is presented that is simpler, faster, and provides better velocity information than conventional Kirchhoff methods. It is based on prestack Kirchhoff time migration and can be applied to both 2-D and 3-D data. The method is divided into two steps, a gathering process that forms common scatterpoint (CSP) gathers, followed by an imaging process using a Kirchhoff moveout correction that is performed independently on each CSP gather. The CSP gathering process sums input traces into equivalent offset bins in each CSP gather with *no* time shifting. The *equivalent offset* is defined by an exact hyperbolic simplification of the double square root (DSR) equation of prestack time migration. A CSP gather is similar to a CMP gather as both contain offset traces, and both represent a vertical array of scatterpoints. CSP gathers can be formed at any arbitrary location, have high fold in their offset bins, and have a much larger offset range due to the gathering of all input traces within the migration aperture.**

**After the CSP gathers have been formed, conventional velocity analysis estimates accurate prestack migration velocities. The high fold and large offset of the CSP gathers provide better focusing for improved velocity analysis. The imaging process collapses each CSP gather into a single migrated output trace. It is performed as a Kirchhoff process, which consists of scaling, filtering, normal moveout (NMO) correction, and stacking. Significant computational savings result from delaying arithmetic operations on the input samples until after a CSP gather has been formed. This space-time domain method is suitable for uneven geometries, may be adapted to migrate from topography, enables velocity analysis at random locations, and permits prestack migration of a 3-D volume into an arbitrary 2-D line. An accompanying paper describes how the equivalent offset method is extended to rugged topography, prestack migration of converted-wave (P-S) data, compute residual statics before NMO correction, perform anisotropic prestack depth migration, and prestack migrate vertical receiver array data from VSP's or vertical marine cables.**

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## INTRODUCTION

Migration is a process that reconstruct an image of the earth's reflecting structure from elastic wavefield energy recorded at the surface in seismic traces. Since the invention of the CMP processing method, conventional processing has concentrated on producing a stacked section from common midpoint (CMP) gathers, followed by a poststack migration. Stacking velocities generally differ from those required for poststack migration, and some form of migration velocity estimation is usually required. Further processing advances recognized that dip dependent stacking velocities and reflection point smearing could be corrected by the inclusion of dip moveout (DMO) (Hale 1984, Deregowski 1986) or prestack migration (Shultz and Sherwood 1980, Sattlegger et al. 1980). The use of these prestack processes in velocity analysis loops enabled a more accurate estimate of the subsurface velocities and improved subsurface images.

DMO and poststack migration are currently more economical than typical methods of prestack migration; consequently, in areas with smooth velocities, DMO tends to be the current processing standard. In areas where the smooth velocity criterion fails, prestack migration is the preferred processing method. Prestack migration methods include migration of source records (Schultz and Sherwood 1980, Reshef and Kosloff 1986), migration of constant (or limited) offset sections (Sattlegger et al. 1980, Deregowski 1990), and migration by alternating downward continuation between shot gathers and geophone gathers (Denelle et al. 1986, Diet et al. 1993). I will refer to *full* prestack Kirchhoff migration (Lumley 1989, Lumley and Claerbout 1993) as the scaled and filtered summation of input samples along a defined trajectory (similar to a diffraction of poststack migration) for each output migrated sample. The common use of full prestack migration continues to be limited by computer hardware and long run times.

The method presented in this paper is based on the principles of full prestack Kirchhoff time migration. However, an intermediate step in the process, forms prestack migration gathers at each migration output location. All traces within the migration aperture are added to each gather with *no time shifting*. The offset of the samples in each trace is defined by a new offset that is called *equivalent offset*. Energy from one scatter point will be contained in all the input traces, and, at the appropriate gather, will align along a hyperbolic moveout curve. We refer to these prestack migration gathers as common scatter point (CSP) gathers. After the CSP gathers have been formed, Kirchhoff NMO (time shifting, scaling, and filtering) and stacking of each CSP gather completes the prestack migration. This new method is called *equivalent offset migration* (EOM) with initial results reported by Bancroft and Geiger (1994) and Bancroft et al. (1998).

## PRESTACK MIGRATION MODEL

The full Kirchhoff approach to prestack migration is based on a model of scatterpoints, which scatter energy from any source to all receivers, (in contrast to specular reflections assumed by the CMP method). The surface position of a vertical array of scatterpoints is referred to as the *common scatterpoint* (CSP) location. The objective of prestack migration is to gather all of the scattered energy and relocate it to the position of the scatterpoints. The traditional concept of full prestack Kirchhoff migration assumes an output scatterpoint, and then gathers the appropriate energy from all available input traces. This procedure is repeated for every output scatter point (or migration sample).

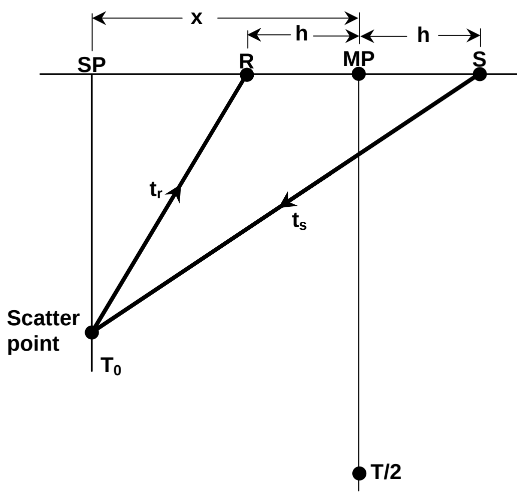


Figure 1. Geometry for prestack time migration showing the source and receiver raypaths.

The time $T$ of scattered energy in each input trace is the sum of the traveltime $t_s$ along the source ray that is between the source and the scatter point, and the traveltime $t_r$ along the receiver ray that is between the scatter point and receiver. For time migration, $T$ is estimated from the double square root (DSR) equation

$$T = t_s + t_r = \left[\frac{T_0^2}{4} + \frac{(x+h)^2}{V^2}\right]^{1/2} + \left[\frac{T_0^2}{4} + \frac{(x-h)^2}{V^2}\right]^{1/2}, \qquad (1)$$

where $T_0$ is the vertical two-way time from the scatter point to the surface, $x$ the distance from the midpoint (MP) to the scatterpoint (SP), and $h$ is the half source-receiver offset as illustrated in Figure 1. The migration velocity $V$ is the RMS approximation of Tanner and Koehler (1978) evaluated at time $T_0$.

The DSR equation may be used to compute the traveltime $T$ for one scatterpoint at $T_0$ into a continuum of 2-D $x$ and $h$ locations as shown in Figure 2. This surface is known as Cheops pyramid (Claerbout 1984). Cheops pyramid is the prestack migration equivalent to the zero offset hyperbola of 2-D poststack migration, i.e. energy on Cheops pyramid will be summed to the scatterpoint.

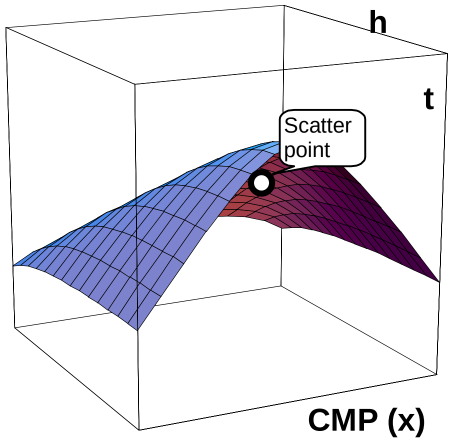


Figure 2. Cheops pyramid; the prestack location of 2-D energy from one scatter point

The migration velocity for Cheops pyramid is defined at the scatterpoint, regardless of the spatial or temporal extent of the scattered energy. Cheops pyramid is a useful visualization tool for scattered energy on a 2-D line, but is not applicable to 3-D data.

## THE EQUIVALENT OFFSET

The *equivalent offset* is defined by converting the DSR equation (1) into an equivalent single square root or hyperbolic form. This is accomplished by defining a new source and receiver *collocated* at the equivalent offset position E in Figure 3. The equivalent offset is chosen to maintain the same total traveltime $2t_e$ as the original path $T$, i.e.,

$$T = \left[\left(\frac{T_0}{2}\right)^2 + \frac{(x+h)^2}{V_{mig}^2}\right]^{1/2} + \left[\left(\frac{T_0}{2}\right)^2 + \frac{(x-h)^2}{V_{mig}^2}\right]^{1/2} = 2T_e = 2\left[\left(\frac{T_0}{2}\right)^2 + \frac{h_e^2}{V_{mig}^2}\right]^{1/2}, \qquad (2)$$

then solving for the equivalent offset $h_e$ to exactly get:

$$h_e^2 = x^2 + h^2 - \left(\frac{2xh}{TV_{mig}}\right)^2. \qquad (3)$$

This equation is roughly a quadratic sum of the distance between the CSP and the CMP $x$, and the half source-receiver offset $h$. The bracketed term in equation (3) contributes a small time and velocity dependence to the equivalent offset.

## COMMON SCATTERPOINT GATHERS

A common scatterpoint gather is composed of offset traces similar to a CMP gather. In practice, the equivalent offset is quantized to discrete bins around the offset traces and all energy, which falls into a bin, is summed to the corresponding trace. A consequence of the time-varying equivalent offset in equation (3) is that an input trace may have its samples spread over a number of offset bins.

It may appear from equation (3) that the equivalent offset needs to be computed for each input sample. However, because the CSP gather is formed by summing traces into equivalent offset bins, only times at which the input samples start in a new bin need to be computed. The initial equivalent offset is be computed and assigned to an appropriate bin. The following samples are added to this bin until the equivalent offset increases to the bin boundary at which point the input samples are added to the next bin. The time at which these transitions occur is $T_n$, where $n$ is the bin index, and may be found from rearranging equation (3) to give equation (4) where $h_{en}$ is the equivalent offset of the $n^{th}$ bin boundary. The transition times of each offset bin for a given input trace may be computed to allow efficient copying of the samples into the respective bins.

$$T_n = \frac{(2xh)}{V_{mig}\left[x^2 + h^2 - h_{en}^2\right]^{1/2}} \qquad (4)$$

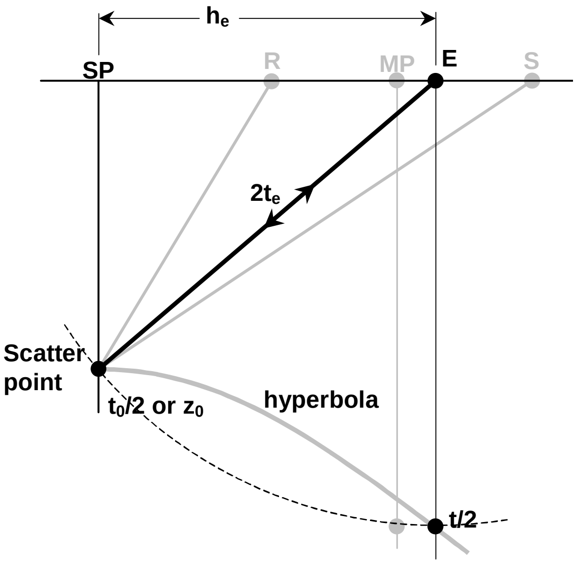


Figure 3. Illustration of prestack migration showing the equivalent offset position *E* for a collocated source and receiver. Compare to Figure 1.

A CSP gather at a different location will have a different mapping of the same input data. Consequently, each input trace will map into many CSP gathers, and each CSP gather contains all traces within the migration aperture. In contrast, a CMP gather contains only those traces within a single CMP bin. CMP gathers tend to be sparsely populated in offset and low in fold when compared to CSP gathers, which may contain thousands of input traces for 2-D data, (or hundreds of thousands of input traces for 3-D data). All these input traces are spread over, (for example), one hundred equivalent offset bins as illustrated in Figure 4. The extremely high fold and large offsets enable accurate velocity analysis at each migrated position. The formation of these CSP gathers is fast and simple. CSP gathers can be created at any arbitrary location within a 2-D line or 3-D volume for velocity analysis, or to image any sub-volume such as a 2-D line from 3-D data.

## DATA EXAMPLE

An example that compares conventional poststack migration with prestack migration using EOM is shown in Figure 5. Note the improved resolution of the anomaly at the indicated location. This improvement occurs in relatively flat data, illustrating the harm that is caused by conventional NMO processing, and the need for prestack migrations.

## CONCLUSIONS

EOM is a prestack migration that is based on the principles of Kirchhoff time migration and uses an equivalent offset to form CSP gathers. Velocity analysis of the gathers enables accurate Kirchhoff NMO to be applied, forming the prestack migrated section. EOM postpones Kirchhoff integration until after the CSP gathers have been formed, which greatly reduces the computations required for the migration.

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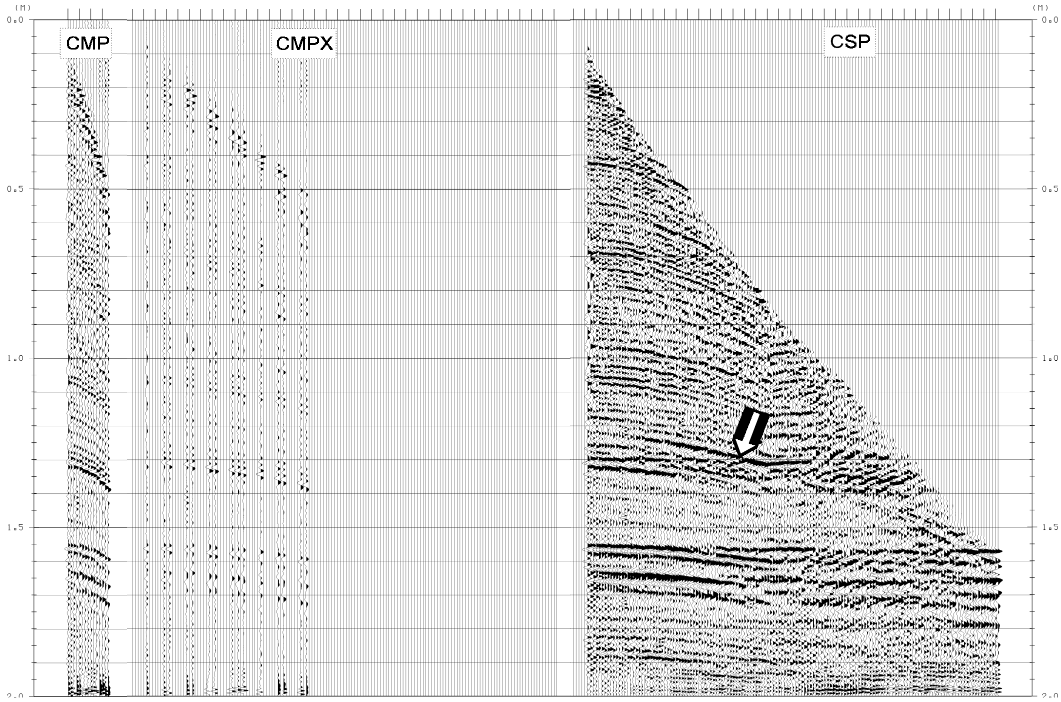
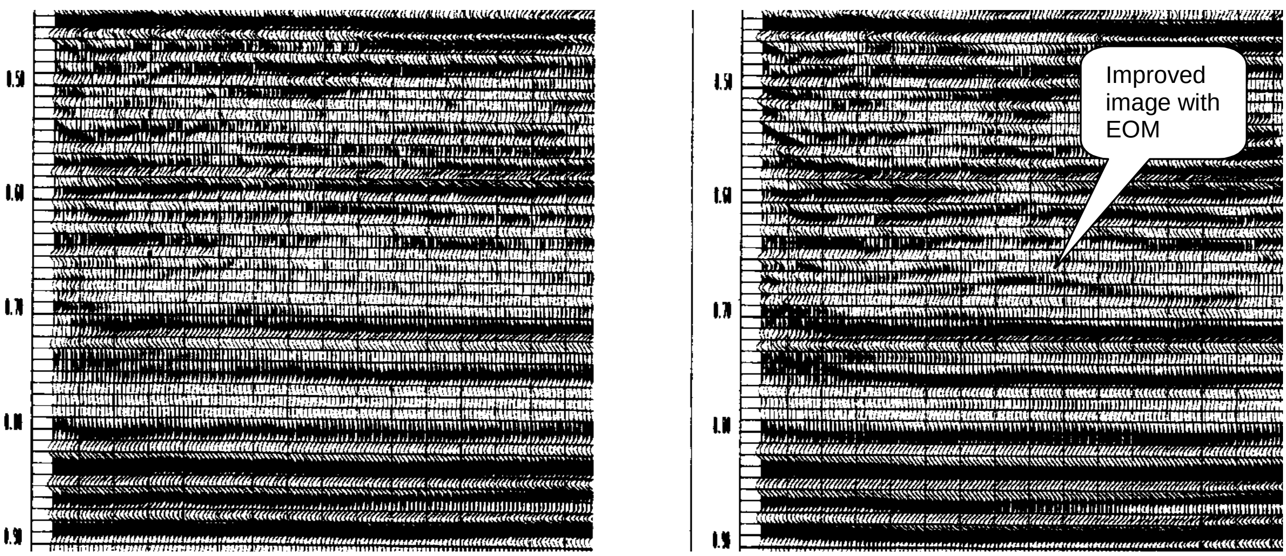



a) b) c)

Figure 4 a) a CMP gather, b) a CMP gather with traces plotted at offset locations, and c) a CSP gather.




a) b)

Figure 5 a) Conventional migration of horizontal data, and b) prestack time migration using the EOM method.