



# Developments in a model to describe low-frequency electrical polarization of rocks - (III): Applications

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## Abstract

The performance and the analysis of the various models to describe a few carefully selected experimental data show that only two out of the entire set of models, the "multi Cole-Cole" and Dias models, can provide a function with structure capable of fitting these data. This fact in addition to some basic characteristics of these two models, such as the way they were derived (empirically the former, phenomenologically the latter one) and the number of coefficients in the function (directly related to the degree of ambiguity of their determination), 7 coefficients in the former versus 5 in the latter model, make the author's model attractive and promising.

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## INTRODUCTION

A given model to be considered as a good one, when compared to the existing models, is expected to show advantages (like a better description of experimental data with less ambiguity in its coefficients determination) and more consistency in the petrophysical interpretation derived from such parameters. This paper is a continuation of Part I and II (Dias, 1999ab) and is devoted to demonstrate Dias model potentialities while providing a comparative analysis with the most common polarization models.

## MODELS COMPARATIVE PERFORMANCE

Each model owes its origin to the obvious fact that, to a greater or smaller extent and precision, it was verified that the model was good to describe some reliable experimental data. It is not our intention here to make a survey of all cases in which a given model is a good model; but instead, to select a few representative data samples that, by their peculiar complexity, can be used to test the capacity of the various models to fit them.

Consider three samples of data:

- 1) the case consisting of phase spectrum experimental data, in the frequency range  $10^{-2}$  to  $10^3$  Hz, originally studied by Nelson et al. (1982), shown on Figure 1. This case simply shows that more general models, such as the "multi Cole-Cole" and Dias models (Dias, 1999ab), can describe the same experimental data even better than the model (Warburg) originally used (see Figure 1). In the order cited, the percent rms error for each model is, respectively, 4.6 %, 7.8 % and 12.8 %. For coefficients see Table 1.

According to Nelson et al. (1982), these data relate to the core sample NW-1-584 in a drill hole sequence, consisting of a "spotted" siltstone, very fine-grained dark gray rock, 25 - 30 % quartz, 30 - 35 % plagioclase, 5 - 7 % muscovite, 7 - 10 % chlorite, 7 - 10 % graphite and 5 - 7 % pyrite. The pyrite is 0.01 - 1.0 mm, disseminated and vein; the veins occur in the most pyrite-rich parts of the bed; pyrite concentration varies from less than 1 % to 20 % from top to bottom (or bottom to top) of beds 1 to 2 cm thick;

- 2) the case consisting of amplitude and phase spectra experimental data, in the frequency range  $10^{-3}$  to  $10^3$  Hz, originally studied by Klein and Sill (1982), shown on Figure 1. This case shows the failure of the Generalized Cole-Cole model to describe the phase data in the section of the spectrum from about 30 Hz to  $10^3$  Hz. One must keep in mind that the Generalized Cole-Cole model includes Cole-Cole and Davidson-Cole models (Dias, 1999b) as particular cases. Figure 2 shows also that with this data, both amplitude and phase can be totally described by "multi Cole-Cole" and Dias models, with percent rms errors (amplitude and phase) given, respectively, by (1.0 %, 20 %) and (0.68 %, 14 %). For coefficients see Table 1.

According to Klein and Sill (1982), these data relate to a synthetic mixture of 3 % dry weight Ca-montmorillonite (84.4 meq/100g), plus fine-grain size glass beads (0.040 to 0.125 mm diameter), saturated with  $10^{-2}$  molar NaCl electrolyte;

- 3) the case consisting of amplitude and phase spectra experimental data, in the frequency range  $10^{-2}$  to  $10^6$  Hz, originally studied by Mahan et al. (1986), shown on Figures 3 and 4. This case shows the failure of Debye, Warburg and Wong models in describing both amplitude and phase in the portion of the spectrum above  $10^3$  Hz (see Figure 3). It is a well-known fact that simple Cole-Cole type functions possess only one maximum for phase and amplitude; this is true also for Zonge model curves, since each of these curves is bounded asymptotically by two Cole-Cole type curves (Dias, 1999b). Consequently, the Zonge model also is unable to describe the experimental data of Figure 3. Instead, Figure 4 shows the same data being properly described, for both amplitude and phase in the whole spectrum, by "multi Cole-Cole" and Dias models, with percent rms errors (amplitude and phase) given, respectively, by (1.4 %, 22 %) and (0.8 %, 24 %). For coefficients see Table 1.

According to Mahan et al. (1986), these data relate to a synthetic sample with the following characteristics: 6.5 % vol. chalcopyrite 0.150 - 0.125 mm grain radii, disseminated in a matrix consisting of quartz sand, grain size smaller

than 53  $\mu\text{m}$  (without any cementing agent), 34,3 % porosity, saturated with a  $10^{-3}$  molar NaCl electrolyte. This mixture was held in place within a plastic tube whose end faces were covered with filter paper.

#### PETROPHYSICAL INTERPRETATION THROUGH THE COEFFICIENTS

The purpose, in this section is to show how reasonable the values of the coefficients are, when generated by our model, and make their associated petrophysical meanings clearer.

##### a) $\rho_0$ and $m$

The agreement in the values of  $\rho_0$  is evident among the various models tested for the cases analysed (see Table 1). Something applies to the magnitude of  $m$  (if one considers for Multi Cole-Cole model only that value closer to the ones given by the other models).

Concerning the petrophysical information provided by these coefficients, the core samples analysed by Nelson et al. (1982) and Mahan et al. (1986) are expected to correspond to strong magnitudes of the dispersion effect, as actually shown by the large values of  $m$  obtained from the models. The same kind of agreement exists concerning the small values of  $m$  obtained from the models for the core sample analysed by Klein and Sill (1982).

##### b) $\tau$

Quite different values of  $\tau$  were generated by the various models. The best possible way to check their validity is to look for the average particle sizes which these values of  $\tau$  correspond to in the curve constructed by Olhoeft (1985) and to compare them with experimental information. Table 2 shows that Dias' model exhibits a far better agreement with the experimental data than the other models in that table.

##### c) $\eta$ and $\delta$

These parameters pertain just to our model,  $\eta$  having a strong dependence on the separation between particles (sources of IP) and a weak dependence on the ionic concentration in solution. Since  $\eta$  must increase to the limit when the volume distribution of the particles gets tight, it is supposed to increase with the number of particles per unit volume. This is verified from Table 3.

As to the petrophysical property associated to  $\delta$ , namely the ratio of the size of the "affected zone" (nearly the thickness of the double layer) to the size of this same zone added of the "homogeneous zone" next to it, there is no experimental information in the data analysed for a true comparison. Nonetheless, the sample of Klein and Sill (1982) permits to compute the value for a coefficient  $A = (1 - \delta)/\delta$  using the value of  $\delta$  given by the model, obtaining  $A = 4.52$ , which is in the range ascribed by these authors to  $A$  (0.1 to 10).

#### DISCUSSION

Based on the selected cases presented here, two models out of those analysed to describe induced polarization can survive confronting the experimental data shown: "multi Cole-Cole" and Dias models. This is also supported in a more extensive survey underway, being conducted by the author.

When comparing models, two aspects also must be kept in mind: (1) the non-uniqueness (ambiguity) in the determination of the coefficients of the main function when inverted, searching for the petrophysical inherent informations possibly existing in them. This ambiguity increases with the number of these coefficients. In this sense, when two models equally describe the same experimental data, the one having the smaller number of coefficients is expected to be less ambiguous if not unique; and (2) the existence or not of a physical meaning interpretable from such coefficients. This aspect favors a model constructed on a phenomenological reasoning and possessing coefficients with petrophysical meaning defined from origin, when compared to another built empirically. In this respect, it is known that the "multi Cole-Cole" model, although good to superimpose experimental data, often has not a clear and consistent physical meaning interpretable through its coefficients. This is clearly shown, for instance, by the **single phase** electrochemical medium analyzed as case number 3, in the previous section, described here by a **double** Cole-Cole model.

Despite the small number of cases analysed, the petrophysical interpretation shows a better overall consistency for the coefficients provided by the Dias model.

#### CONCLUSIONS

The experimental data displayed, while limited to a few specially selected cases, permitted a comparison of performance and discussion of the various existing models, as well as to discuss shortly the consistency of the petrophysical meaning associated to their coefficients. These results suggest that only the "multi Cole-Cole" and Dias models have adequate structure to cope with the most intricate experimental data curves. However, since the "multi Cole-Cole" model is expressed through a function of seven coefficients and the Dias model by a function of five, the latter has intrinsically less ambiguity in the coefficients determination. Also, since the first model is empirical while the second is phenomenological, it seems reasonable that a full analysis of cases using Dias model is worthwhile for checking: (a) its capacity to describe all the reliable experimental data available, and (b) the petrophysical meaning of its coefficients and the corresponding interpretation consistency.

#### REFERENCES

- Dias, C.A. 1999a. *Developments in a model to describe low-frequency electrical polarization rocks-(I): Theory*. 6<sup>th</sup>. International Congress of the Brazilian Soc. of Geophysics - SBGf, 15-19 Aug. 99, Rio de Janeiro.
- Dias, C.A. 1999b. *Developments in a model to describe low-frequency electrical polarization rocks-(II): A comparative overview*. 6<sup>th</sup>. International Congress of the Brazilian Soc. of Geophysics - SBGf, 15-19 Aug. 99, Rio de Janeiro.

- Klein, J. D. and Sill, W. R., 1982, *Electrical properties of artificial clay-bearing sandstones: Geophysics*, 47, 1593-1605.
- Mahan, M. K., Redman, J. D. and Strangway, D. W., 1986, *Complex resistivity of synthetic sulphide bearing rocks: Geophys. Prosp.*, 34, 743-768.
- Nelson, P. H., Hansen, W. H. and Sweeney, M. J., 1982, *Induced polarization response of zeolitic conglomerate and carbonaceous siltstone: Geophysics*, 47, 71-88.
- Olhoeft, G. R., 1985, *Low frequency electrical properties: Geophysics*, 50, 2492-2503.

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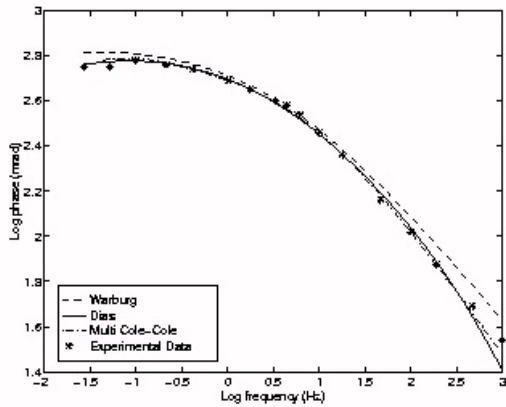


FIG.1 Phase measurements and Warburg model fitting by Nelson et al. (1982) for a core sample containing disseminated metallics. "Multi Cole-Cole" and Dias models best fitting of data by the author.

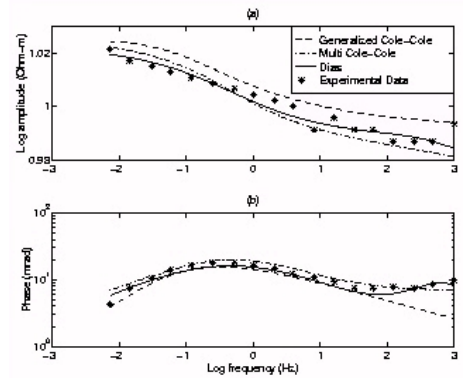


FIG. 2. Generalized Cole-Cole model fitting experimental data for a synthetic sample consisting of a mixture of 3% by weight Ca-montmorillonite plus glass beads 4.0 to  $12.5 \times 10^{-5}$  m diameter (Klein & Sill, 1982).

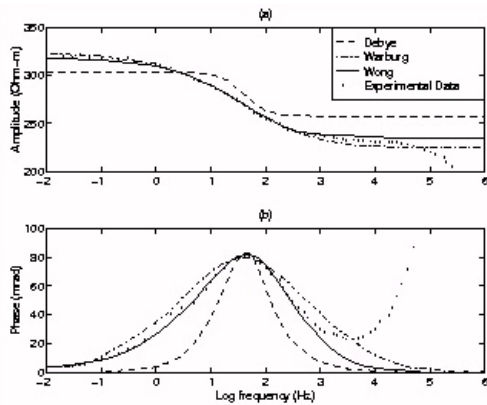


FIG.3. Wong, Warburg & Debye models best fitting of experimental data for a synthetic sample consisting of a mixture of 6.5 % chalcopyrite 150-125  $\mu$ m radii, disseminated in a matrix of quartz sand of grain size < 53  $\mu$ m saturated with NaCl  $10^{-3}$  molar electrolyte (Mahan et al., 1986).

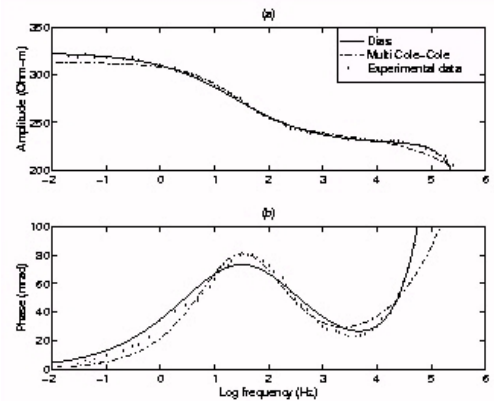


FIG.4. Same experimental data of Figure 3. Best fitting by the author for "multi Cole-Cole" and Dias models.

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describe experimental data

MODEL	COEFICIENTS				
Warburg Multi Cole-Cole Dias	Fig. 1 (Experimental data: Nelson et al., 1982)				
	$m = 0.990$	$\tau = 398s$			
	$m_1 = 0.832$	$\tau_1 = 15.0s$	$c_1 = 0.671$		
	$m_2 = 0.754$	$\tau_2 = 0.345s$	$c_2 = 0.575$		
	$m = 0.977$	$\tau = 1.86 \times 10^{-4}s$	$\delta = 0.037$	$\eta = 119s^{-1/2}$	
Generalized Cole-Cole Multi Cole-Cole Dias	Fig. 2 (Experimental data: Klein & Sill, 1982)				
	$\rho_o = 10.6\Omega m$	$m = 0.075$	$\tau = 1.8s$	$c = 0.72$	$k = 0.38$
	$\rho_o = 10.8\Omega m$	$m_1 = 0.059$	$\tau_1 = 0.368s$	$c_1 = 0.6$	
		$m_2 = 0.18$	$\tau_2 = 1.0 \times 10^{-7}s$	$c_2 = 0.1$	
	$\sigma_o^{-1} = 10.5\Omega m$	$m = 0.089$	$\tau = 2.41 \times 10^{-4}s$	$\delta = 0.181$	$\eta = 9.7s^{-1/2}$
Multi Cole-Cole Dias	Fig. 4 (Experimental data: Mahan et al., 1986)				
	$\rho_o = 313\Omega m$	$m_1 = 0.988$	$\tau_1 = 2.3 \times 10^{-8}s$	$c_1 = 0.452$	
		$m_2 = 0.238$	$\tau_2 = 6.24 \times 10^{-3}s$	$c_2 = 0.670$	
	$\sigma_o^{-1} = 323\Omega m$	$m = 0.786$	$\tau = 1.02 \times 10^{-6}s$	$\delta = 0.884$	$\eta = 9.7s^{-1/2}$

Table 2. Particle size determination from the values of  $\tau$ , by using the curve of time constant versus radius of particle (Olhoeft, 1985). Symbols: py = pyrite; cpy = calcopyrite; graph = graphite; qtz = quartz.

Model	$\tau$	Particle size	
		Interpreted	Experimental
Dias Multi Cole-Cole Warburg	$1.9 \times 10^{-4}s$ $\tau_1 = 15s$ $\tau_2 = 0.35s$ $4 \times 10^2s$	$5 \times 10^{-2}mm$ 35mm 2mm $> 10^2mm$	(Source: Nelson et al., 1982) $10^{-2} - 1mm$ (5 - 7% py) size? (7 - 10% graph)
Dias Multi Cole-Cole Generalized Cole-Cole	$2.4 \times 10^{-4}s$ $\tau_1 = 3.7 \times 10^{-1}s$ $\tau_2 = 1.0 \times 10^{-7}s$ 1.8s	$6 \times 10^{-2}mm$ 2mm $0.8 \times 10^{-3} - 10^{-2}mm$ 9mm	(Source: Klein & Sill, 1982) $10^{-2} - 0.3mm$ (*) (3% clay particles) $4 \times 10^{-2} - 0.125mm$ (glass beads)
Dias Multi Cole-Cole	$1.0 \times 10^{-6}s$ $\tau_1 = 2.3 \times 10^{-8}s$ $\tau_2 = 6.2 \times 10^{-3}s$	$0.4 \times 10^{-2}mm$ (**) $3.4 \times 10^{-4}mm$ 0.7mm	(Source: Mahan et al., 1986) 0.125 - 0.150mm (6.5% cpy) $0.53 \times 10^{-1}mm >$ (qtz sand)

(\*)estimated size. (\*\*) uncertain, no experimental information supporting.

Table 3. Relationship between the values of  $\eta$  and the volume concentration of "IP sources". Symbols: py = pyrite; cpy = calcopyrite; graph = graphite; qtz = quartz.

$\eta$	Experimental data
$119s^{-1/2}$	(Source: Nelson et al., 1982) 5 - 7% py, 7 - 10% graph, conc. = ?
$19s^{-1/2}$	(Source: Mahan et al., 1986) 6.5% cpy, conc. NaCl = $10^{-3}M$
$9.7s^{-1/2}$	(Source: Klein & Sill, 1982) 3% Ca - montmorillonite, conc. NaCl = $10^{-2}M$