Anisotropy in a vicinity of a borehole from the qP wave slowness and polarization

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Summary

We propose an algorithm for evaluating weak anisotropy parameters at receivers in a borehole in a multi-azimuthal multiple-source offset VSP experiment. The parameters can be determined from measurements of slowness vectors and/or deviations of measured qP wave particle motions from the slowness vectors at individual receivers. The formulae presented are applicable under assumption of arbitrary but weak anisotropy and vertical inhomogeneity of the medium. The formulae represent, in several respects, generalization of an algorithm proposed by Gaiser (1990) and an alternative to the algorithm proposed by Horne et al. (1998). Basic formulae and a synthetic example of their application are presented.

Introduction

Let us consider a multi-azimuthal multiple-source offset VSP experiment with three-component recordings not affected by a free surface. Let us assume that the studied medium is vertically inhomogeneous and weakly anisotropic (WA) with arbitrary symmetry. For this medium, we find a background isotropic medium, from which the WA medium differs only slightly. Then we can express the qP wave slowness and the polarization vector in the WA medium using the formulae derived by Pšenčík and Gajewski (1998). The formulae relate linearly the slowness and the polarization vector to the so-called weak anisotropy (WA) parameters specifying anisotropy of the medium. The formulae thus represent a system of linear algebraic equations for the determination of the WA parameters. The system can be solved if the qP wave slowness and the polarization vector are known. They can be determined from observations. The determination of the polarization vector is straightforward. The determination of the slowness vector (from which slowness can be determined) is more complicated. We use a generalization of the approach proposed by Gaiser (1990). The vertical component of the slowness vector of the studied wave is determined from its travel times recorded in the borehole. As Gaiser (1990), we assume that the medium is laterally homogeneous. This assumption allows us to determine the horizontal inline component of the slowness vector from travel times between sources and a given receiver. Instead of a VTI medium considered by Gaiser(1990) we consider a weakly anisotropic medium of an arbitrary symmetry. Thus, in contrast to Gaiser's laterally homogeneous VTI medium, in which the slowness vector always remains confined to the vertical plane containing the source and the borehole, the slowness vector in our model can deviate from that plane. We must, therefore, use the eikonal equation to determine the third component of the slowness vector. When the slowness vector and polarization vector are known, an overdetermined system of linear equations for WA parameters at each receiver can be constructed.

Basic formulae

We consider a right-handed Cartesian coordinate system with $x-$ and $y-$axes horizontal and $z-$axis vertical, positive downwards. We can express P wave slowness c^{-1} and the polarization vector g_i in the WA medium using the formulae derived by Pšenčík and Zheng (1998), see also Pšenčík and Gajewski (1998)

$$c^{-1} \sim \alpha^{-1}(1 - \frac{B_{33}}{2\alpha^2}), \tag{1}$$

$$g_i \sim n_i + \alpha p_k e_k^{(1)} e_i^{(1)} + \alpha p_k e_k^{(2)} e_i^{(2)} + \frac{B_{13} e_i^{(1)}}{\alpha^2 - \beta^2} + \frac{B_{23} e_i^{(2)}}{\alpha^2 - \beta^2}. \tag{2}$$

The symbol p_i denotes the slowness vector in the WA medium, $p_k p_k = c^{-2}$. The vectors $e_i^{(1)}$, $e_i^{(2)}$ and $n_i = e_i^{(3)}$ are three mutually perpendicular unit vectors defined at a receiver in the background isotropic medium. The isotropic medium is characterized by the P and S wave velocities α and β. The vector n_i can be chosen parallel to, for example, p_i or g_i. The vectors $e_i^{(1)}$ and $e_i^{(2)}$ can be chosen arbitrarily in the plane perpendicular to n_i. We choose them so that all three vectors read

$$\vec{e}^{(1)} = D^{-1}(n_1 n_3, n_2 n_3, n_3^2 - 1), \quad \vec{e}^{(2)} = D^{-1}(-n_2, n_1, 0), \quad \vec{e}^{(3)} = \vec{n} = (n_1, n_2, n_3), \tag{3}$$

where

$$D = (n_1^2 + n_2^2)^{1/2}, \quad n_1^2 + n_2^2 + n_3^2 = 1. \tag{4}$$

The symbols B_{mn} in Eqs.(1) and (2) denote elements of the weak anisotropy matrix, see Pšenčík & Gajewski (1998). The explicit expressions for B_{33}, B_{13} and B_{23} read:

$$B_{33} = 2\alpha^2[\epsilon_z n_3^4 + 2n_3^3(\epsilon_{34} n_2 + \epsilon_{35} n_1) + n_3^2(\delta_x n_1^2 + \delta_y n_2^2 + 2\chi_z n_1 n_2)$$
$$+2n_3(\chi_x n_1^2 n_2 + \chi_y n_1 n_2^2 + \epsilon_{15} n_1^3 + \epsilon_{24} n_2^3) + \epsilon_x n_1^4 + \delta_z n_1^2 n_2^2 + \epsilon_y n_2^4 + 2\epsilon_{16} n_1^3 n_2 + 2\epsilon_{26} n_1 n_2^3], \tag{5a}$$

$$B_{13} = \alpha^2 D^{-1}\{2\epsilon_z n_3^5 + n_3^4(\epsilon_{34} n_2 + \epsilon_{35} n_1) + n_3^3(\delta_x n_1^2 + \delta_y n_2^2 + 2\chi_z n_1 n_2 - 2\epsilon_z)$$
$$+n_3^2[(4\chi_x - 3\epsilon_{34}) n_1^2 n_2 + (4\chi_y - 3\epsilon_{35}) n_1 n_2^2 + (4\epsilon_{15} - 3\epsilon_{35}) n_1^3 + (4\epsilon_{24} - 3\epsilon_{34}) n_2^3]$$
$$+n_3[(2\delta_z - \delta_x - \delta_y) n_1^2 n_2^2 + 2(2\epsilon_{16} - \chi_z) n_1^3 n_2 + 2(2\epsilon_{26} - \chi_z) n_1 n_2^3 + (2\epsilon_x - \delta_x) n_1^4 + (2\epsilon_y - \delta_y) n_2^4]$$
$$-\chi_x n_1^2 n_2 - \chi_y n_1 n_2^2 - \epsilon_{15} n_1^3 - \epsilon_{24} n_2^3\}, \tag{5b}$$

$$B_{23} = \alpha^2 D^{-1}\{n_3^3(\epsilon_{34} n_1 - \epsilon_{35} n_2) + n_3^2[(\delta_y - \delta_x) n_1 n_2 + \chi_z n_1^2 - \chi_z n_2^2]$$
$$+n_3[(2\chi_y - 3\epsilon_{15}) n_1^2 n_2 - (2\chi_x - 3\epsilon_{24}) n_1 n_2^2 + \chi_x n_1^3 - \chi_y n_2^3]$$
$$+(\delta_z - 2\epsilon_x) n_1^3 n_2 + (2\epsilon_y - \delta_z) n_1 n_2^3 + 3(\epsilon_{26} - \epsilon_{16}) n_1^2 n_2^2 + \epsilon_{16} n_1^4 - \epsilon_{26} n_2^4\}. \tag{5c}$$

The coefficients of the directional cosines n_i in Eqs.(5) are the qP wave weak anisotropy parameters. They represent a generalization of Thomsen's (1986) parameters for anisotropy of arbitrary symmetry. They are defined as follows

$$\epsilon_x = \frac{A_{11} - \alpha^2}{2\alpha^2}, \quad \epsilon_y = \frac{A_{22} - \alpha^2}{2\alpha^2}, \quad \epsilon_z = \frac{A_{33} - \alpha^2}{2\alpha^2}, \quad \delta_x = \frac{A_{13} + 2A_{55} - \alpha^2}{\alpha^2},$$
$$\delta_y = \frac{A_{23} + 2A_{44} - \alpha^2}{\alpha^2}, \quad \delta_z = \frac{A_{12} + 2A_{66} - \alpha^2}{\alpha^2}, \quad \chi_x = \frac{A_{14} + 2A_{56}}{\alpha^2}, \quad \chi_y = \frac{A_{25} + 2A_{46}}{\alpha^2},$$
$$\chi_z = \frac{A_{36} + 2A_{45}}{\alpha^2}, \quad \epsilon_{15} = \frac{A_{15}}{\alpha^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha^2}, \quad \epsilon_{26} = \frac{A_{26}}{\alpha^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha^2}. \tag{6}$$

We can see that the qP wave slowness and the polarization vector are fully specified by 15 WA parameters.

Equations (1) and (2) represent a system of linear algebraic equations for 15 unknown WA parameters. The system can be solved if the slowness c^{-1} and the polarization vector g_i are known. As described above, the determination of the polarization vector g_i and of two components of the slowness vector (vertical and inline horizontal) is straightforward. The third component of the slowness vector is determined from the linearized eikonal equation

$$a_{ijkl} p_i p_l g_j g_k = 1. \tag{7}$$

With the slowness vector and polarization vector known, we have $2N_S N_A$ equations for WA parameters at each receiver. Here N_S denotes the number of sources along a profile and N_A the number of profiles. At least 5 profiles are necessary for the determination of all qP wave WA parameters from the slowness formula (1), see Pšenčík and Gajewski (1998). Only 3 profiles are necessary for the determination of the WA parameters from the formula for the polarization vector, see Eqs.(5b) and (5c). Thus combination of both formulae can lead to a substantial reduction of measurement requirements.

Test example

To illustrate usefulness of combined use of slowness and polarization formulae, we present results of the following synthetic test. Four profiles, 0^o, 30^o, 60^o and 90^o (measured counterclockwise from the x-axis) were considered, with 9 sources along each of them. The sources were distributed evenly along the profiles with the step of 0.1 km, the closest source being situated at the distance of 0.1 km from the borehole. The

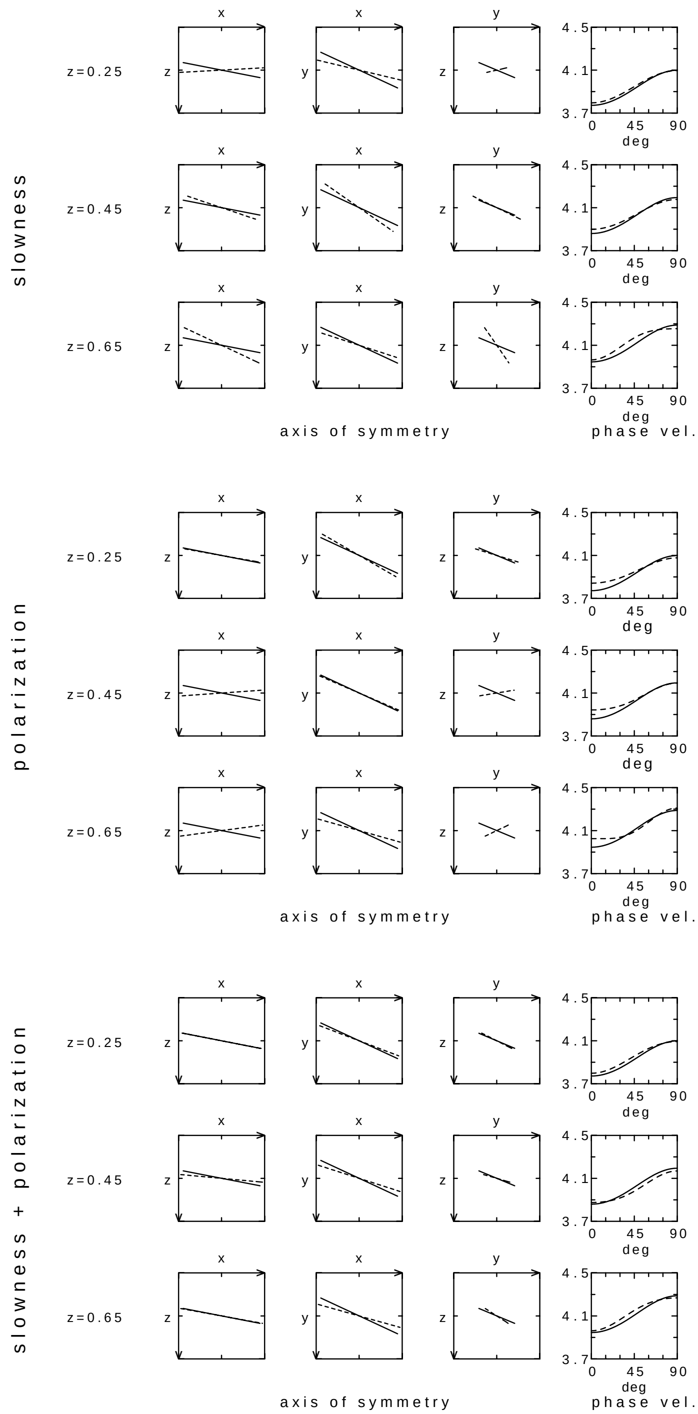

Figure 1 Comparison of exact (full lines) and inverted (dashed lines) projections of axis of symmetry (first three frames from the left) and phase velocity sections within the symmetry plane (the frames on the right).

qP wave synthetics were calculated at 12 receivers evenly distributed in the borehole starting from 0.1 km with the step of 0.05 km. The effects of the free surface were not considered. As a model, vertically inhomogeneous TI medium with the axis of symmetry inclined 10^o from a horizontal plane was considered. The horizontal projection of the axis of symmetry was outside the quadrant covered by profiles and made 25^o with the profile 0^o.

Arrival times and polarization directions were determined from noise-free synthetic seismograms. At each receiver the arrival times were first used for the determination of a background P wave velocity α (S wave velocity β was taken as $\alpha/\sqrt{3}$), second for the determination of vertical and horizontal inline components of the slowness vector. The horizontal crossline component was determined from Eq.(7). The vector n_i was chosen $n_i = g_i$. Then SVD method was used to solve Eqs.(1) and (2) for WA parameters. The results are shown in Fig.1. Three blocks, from the top to the bottom, show results using slowness formula, i.e. Eq.(1) only (top), polarization formula, i.e. Eq.(2) only (middle) and both slowness and polarization formulae (bottom). In each block, results for three receivers situated at depths $z = 0.25$, 0.45 and 0.65 km are shown. For each depth, projections of the exact axis of symmetry (full line) and the axis of symmetry found by solving (1) and/or (2) (dashed line) into (x, z), (x, y) and (y, z) planes are shown. The rightest plots show sections of phase velocity surfaces by planes containing the axis of symmetry. Although the observational system provides, intentionally, insufficient azimuthal coverage, agreement of the inverted and exact results is rather good. Results are best when both Eqs.(1) and (2) are used. Slightly worse results can be observed for the deepest receiver. This can be explained by incomplete incidence angle coverage at this receiver. The results could further improve if a complete azimuthal coverage was used.

Conclusions

A procedure for the determination of WA parameters of an anisotropic medium of arbitrary symmetry at receivers situated in a borehole is proposed. As a source of information, slowness and deviation of the polarization and slowness vectors are used. Use of the polarization vectors, in addition to the slowness, leads to more accurate results and reduces measurement requirements. Explicit approximate formulae for the polarization and slowness vectors are used. It would not be difficult to generalize the procedure for a less ideal case of non-vertical borehole, receiver twist, different source elevations, etc., see Gaiser (1990), Horne et al. (1998). We are aware that the assumption of vertical inhomogeneity of the medium may represent a limiting condition of applicability of the proposed procedure. Generalization of the procedure to laterally varying, generally but weakly anisotropic structures is under preparation.

Acknowledgements

This work was supported by the consortium project "Seismic waves in complex 3-D structures (SW3D)". XZ is greateful for the support of the National Natural Science Foundation of China, No.49774230.

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