Common converting elements (CCE) ray tracing for the calculation of traveltimes for reflected converted waves

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ABSTRACT

In the present paper are carried out numerical calculations of a new approximation to obtain the traveltime for P-SV reflected converted waves. A set of sources emit P-waves and they travel to a given interface. These waves are converted in S-waves after the reflection and they are recorded at the corresponding receivers as P-SV converted waves. Common converting elements (CCE) have been simulated by fans of rays emmitted upward by fictitious sources (elements) located on the reflecting interface. The sources emit P-and S-waves upward. The corresponding true traveltimes are calculated for the pairs of rays within the fans. The true-and approximated traveltimes are compared and one can see a fairly good approximation of the new formula for the traveltime of P-SV reflected converted waves, for small offsets. This means that the new formulas can be used for stacking of P-SV reflection data. The accuracy of the new formulas is investigated for two models.

INTRODUCTION

P-S reflected converted waves can be used to obtain the velocity of the shear waves (Stewart, R. R. and Ferguson, R. J., 1996), the ratio v_s/v_p or the poisson's number (Kähler, S. and Meissner, R., 1983). The problems associated with the sorting of traces with a common depth point, the time delay correction and stacking of multifold P-SV data have been discussed widely by several authors (Nedlin, G., 1986, Kennett, B. L. N., 1993). The selection of the common converting points (CCP) depends on the offset, the velocity ratio v_s/v_p (or poisson's number), the depth and dip of the reflector (Tessmer and Behle 1988, 1991; Tessmer et al., 1990). They used the second order approximation for the NMO correction (Taner and Koehler, 1969, Brown, R.J.S., 1969, Al-Chalabi, M., 1973, Hubral, P. and Krey, Th., 1980). Den Rooijen (1991) calculated a set of P-SV dip moveout operators for a homogeneus, isotropic medium and applied them to stack reflection P-SV data.

In this work is presented the theory and numerical calculations of the new so-called CCE-formulas for obtaining the traveltime of P-SV reflected converted waves and the distribution of the source-receiver pairs. For this purpose, a different system of observation is chosen, simulated by fans of rays emitted upward by fictitious sources (elements) located on the reflecting interface. This system of rays is the basis of the common converting elements (CCE) system of observation (the word "element" is used instead of "point" since the directions of the rays are determined by Snell's law). In the CCE-formulas, it is shown how the distribution of the surface locations of the shot-offset pairs connected to a CCE and the corresponding traveltimes can be obtained by tracing only the central (normal incidence) rays, for the P-and S-waves. It is based on two succesive types of approximation: 1) a binomial distribution for calculating the selected shot-offset pairs and 2) a dynamic correction for the central wave front, called an oblique spherical correction (OSC), for calculating the corresponding traveltimes. This allows an efficient way for the calculation of the corresponding traveltimes for P-SV converted waves.

THE CCE-FORMULAS

Let us consider the seismic line formed by the locations $A_k{}^p A_0 A_k{}^s$. The points $A_k{}^p$ and $A_k{}^s$ correspond to the P-waves sources and the S-waves receivers, respectively and A_0 is the "central point" between sources and receivers. Let's consider a fictitious source at the point C_0 of the reflector S. All the rays $\mathrm{A}_k^p C_0 A_k{}^s$ have the common converting point C_0, k denotes the *kth* ray from the reflecting point. The source radiates P-and S-waves at different moments of time. Assumption: The P-and S-waves reach the point A_0 at the same time. With the combination of the P-and S-waves a fictitious front will be formed, the radius of curvature r of the wavefront has a discontinuity at the point A_0. The calculation of arrival times $t(A_k{}^p C_0 A_k{}^s)$, (k=*0,1,2,3,...,K*) for a reflected converted waves, is based on the calculation of the traveltimes pairs, $t(C_0 A_k{}^p)$ and $t(C_0 A_k{}^s)$ of the P- and S-waves, emitted by a fictitious source, located at the CCE-the point C_0 on the reflector S. The rule defining two connected points $A_k{}^p$ and $A_k{}^s$ is determined by the normal **N** to the interface S at the CCE and by Snell's law. The CCE system of observation is obtained by repeating the present procedure for several CCE. For laterally smooth media, this system of observation can be obtained by tracing only the central (normal incidence) rays (for the P-and S-waves) using two types of approximations (Gelchinsky, 1989): 1) "binomial asymmetrical ditribution" for the surface location of the sources and receivers and 2) "oblique spherical correction" for the central wave fronts, for calculating the corresponding traveltimes.

BINOMIAL ASYMMETRICAL DISTRIBUTION

Instead of shooting a fan of rays from the common converting elements, one can obtain formulas for a desired distribution of the *kth* shot-offset pairs $(A_k{}^p, A_k{}^s)$ using a polynomial distribution (the binomial distribution is in this case a good approximation).
The offset from the location of the central ray are denoted $\Delta X_k{}^p$ and $\Delta X_k{}^s$, where the asymmetrical distribution is given by

$$\Delta x_k{}^p = Y_k + \alpha_p Y_k{}^2 \tag{1}$$

$$\Delta x_k{}^s = -\frac{v_s}{v_p} Y_k + \alpha_s \frac{v_s{}^2}{v_p{}^2} Y_k{}^2 \tag{2}$$

when $V_s = V_p$, following equation is obtained from equation (2)

$$\mathit{\Delta} X_k{}^{pp} = -Y_k + \alpha_p Y_k{}^2, \tag{3}$$

where Y is the half distance between $A_k{}^p$ and $A_k{}^{pp}$ (the location $A_k{}^{pp}$ correspond to detector of the PP-reflection).

The coefficients α_p and α_s determine the degree of asymmetry of a spread with respect to the central point A_0 and are called factors of asymmetry. If the inhomogeneity of the overburden is not very strong, then we can obtain following equations:

$$\alpha_p = \frac{sin\beta_p}{R_p} \tag{4}$$

$$\alpha_s = \frac{sin\beta_s}{R_s} \tag{5}$$

(For $\alpha_p = 0$ and $\alpha_s = 0$ is obtained the so-called asymptotic approximation for the selection of the common converting points). β_p is an acute angle between the normal to the seismic line and the normal incidence ray for the P-wave and β_s is the acute angle between the normal to the seismic line and the normal incidence ray for the S-wave, R_p is the P-wavefront radius of curvature of the normal incidence ray, R_s is the S-wavefront radius of curvature of the normal incidence ray and V_p and V_s represent the velocities of P-and S-waves near the seismic line, respectively.

OBLIQUE SPHERICAL CORRECTION

The wave fronts can be approximately considered as spherical in the vicinity of the central ray point A_0. The calculation of the corresponding traveltimes $t(A_k{}^p C_0 A_k{}^s)$ for the binomial asymetrical distribution can be obtained applying the cosine rule at the transformed image. It is expressed by

$$t(A_k{}^p C_0 A_k{}^s) = t(A_0 C_0 A_0) + \Delta t_k \tag{6}$$

with

$$\Delta t_k = \frac{\sqrt{R_p{}^2 + 2R_p \Delta X_k{}^p sin\beta_p + (\Delta X_k{}^p)^2} - R_p}{V_p} + \frac{\sqrt{R_s{}^2 - 2R_s \Delta X_k{}^s sin\beta_s + (\Delta X_k{}^s)^2} - R_s}{V_s} \tag{7}$$

The accuracy of the present approximation is investigated with the aid of a special CCE ray tracing. Two models are considered for the numerical calculation of exact and approximated traveltimes for P-SV reflected converted waves. Figure 1 shows the second model (he has a dipping and a horizontal layer). For this model, 13 CCE-gathers have been simulated and the corresponding normal moveout traveltime curves are calculated (fig. 2: bottom). A good agreement between the exact and approximated traveltimes is obtained (fig. 2: top). Figure 3 and figure 4 show the corresponding parameter of the central rays for each CCE-gather. The sensitivity of the approximated traveltime formula on the ratio V_s/V_p has been analysed and demonstrated.

CONCLUSIONS

The results of this work show that the new CCE-Formulas can be used to carry out the sort of traces with a common reflecting point (for P-SV reflected converted waves) and the corresponding time delay correction. After the application of these formulas one can carry out the stacking of P-SV reflection data. The parameter of wavefronts (radius of curvature, β and the asymmetry Factor α) represent the new stacking parameter. In a forthcoming work will be demonstrated the viability of this new CCE-Method to stack P-SV reflection data, with its application on synthetic data.

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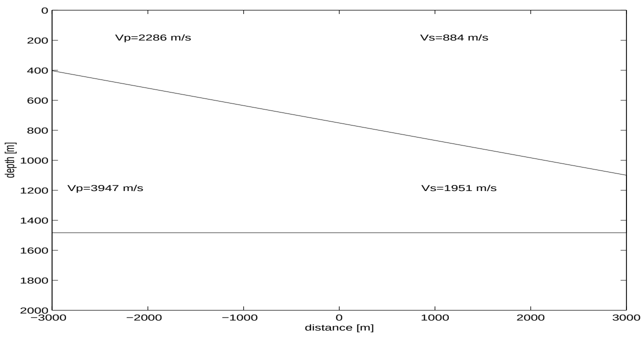

Figure 1.- The model with a dipping layer and a horizontal layer.

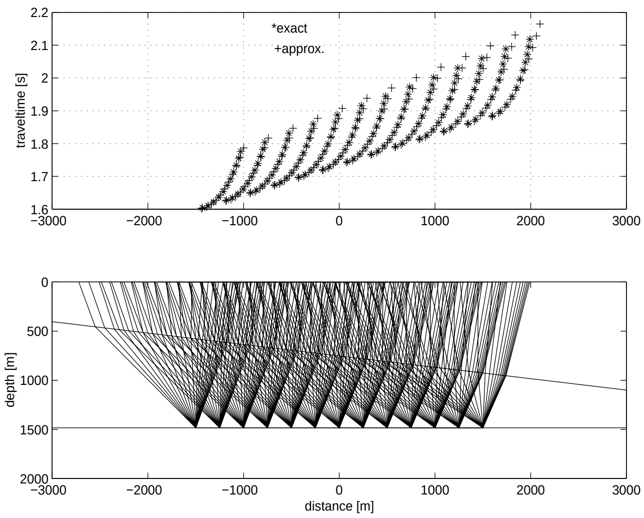

Figure 2.- 13 CCE gathers are simulated, the common converting points are on the second interface of the second model (Fig. 2) (bottom). The corresponding traveltimes of the P-SV reflected converted waves are plotted, the star-symbols (*) represent the exact calculated traveltimes and the plus-signs (+) represent the approximated traveltimes, we can see a good agreement between both (top).

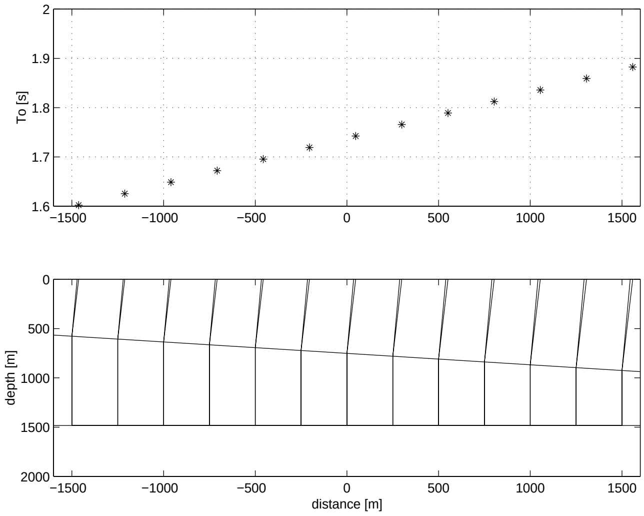

Figure 3.- The central rays for the P- and S-wavefronts (bottom) and the corresponding traveltime $t(A_0C_0A_0)$ to the central rays of each CCE gathers are plotted (top).

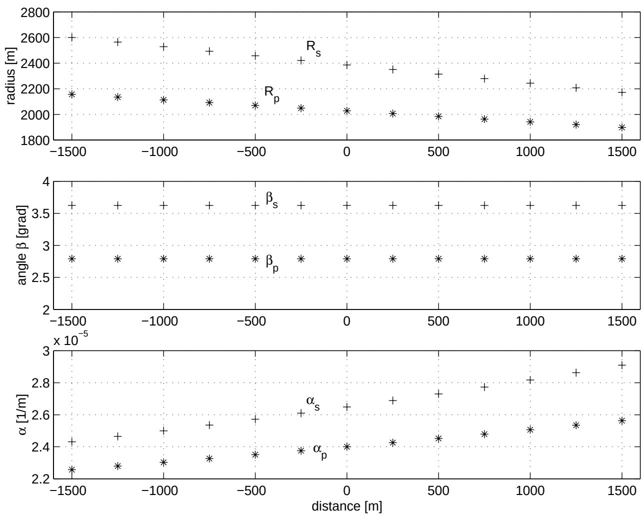

Figure 4.- The corresponding parameter Rp and Rs for each CCE gather (top), the parameter β_p and β_s (middle) and the parameter α_p and α_s (bottom) are plotted.