SBGf21499

Ambiguity in AVO/AVA analysis for Anisotropic Media

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ABSTRACT

Inversion of reflection coefficients of P waves, Rpp, at an interface between an isotropic over a monoclinic medium with vertical plane of symmetry can not recover all the elastic constants of the bottom medium. Numerical experiments with multiple azimuths and large offsets synthetic data at pre-critical incidence show that there is an orthorhombic medium that fits the Rpp coefficients in this case. As a practical consequence Rpp inversion in fractured medium can hope only to recover fractures strike but no information about the fractures dip. This results holds for high contrast and large aperture data well beyond the range of validity of linearized approximations for Rpp.

INTRODUCTION

Reservoir fracture characterization through AVO/AVA analysis of P-wave reflections using anisotropic models has been receiving attention lately. Rüger (1997) obtained linearized approximations for P-waves reflection coefficient at an interface between isotropic and a transversely isotropic medium. Vavrycuk & Psencick (1998) obtained linearized approximations for an interface between arbitrarily anisotropic media. Both results assume week contrast between the elastic properties across the interface. These approximations are in good agreement with exact results for near normal incidence and weak impedance contrasts. The results of Vavrycuk & Psencik (1998) when applied to an interface between an isotropic and triclinic medium shows that only the elastic constants of a monoclinic medium with a horizontal plane of symmetry can be recovered from small apperture reflection data. Simões-Filho et al. (1999) shows that multiple azimuth and large offset data can resolve all the elastic constants of an transversely isotropic medium with a horizontal axis of symmetry underlying an isotropic medium. This results used the exact expression for the P-wave reflection coefficient and nonlinear inversion and suggested the potential use of AVO analysis for fracture characterization beyond the linearized approach. We investigate the inversion of a P-wave reflection coefficients at an interface between an isotropic over transversaly isotropic(TI) medium with a tilted axis. High impedance and a low impedance contrast model were used to generate two synthetic data sets. The first model has the isotropic top medium with elastic properties characteristic of shales and the anisotropic bottom medium with elastic properties associated with a fractured limestone. The contrast of elastic properties at the interface is high and, though anisotropy is weak, linearized approximations should not hold for the large offsets part of the synthetic data. The low impedance constrast model has the same top medium as the previous one and the anisotropic medium has elastic properties close to the ones expected for a fractured sandstone. We found that both synthetic data sets could be fitted under the accuracy of any experimental data using an isotropic over an orthorhombic model. This result shows that no information about the dipping of the symmetry axis,which is determined by the fratures dip angle, can be recovered from Rpp only.

FORWARD MODELING

The reflection coefficents for plane waves at an interface between two anisotropic media is computed using a slight modification of Schoenberg & Protazio (1992), where up and down symmetry is not assumed. Considering a incident, reflected and transmited plane waves in the form

$$\mathbf{u}_\alpha(\mathbf{x},t) = A_\alpha \mathbf{n}_\alpha \exp i\omega(\mathbf{s}_\alpha.\mathbf{x}-t) \ , \qquad (1)$$

where $\mathbf{n}_\alpha$ is the polarization vector and $\mathbf{s}_\alpha$ is the slowness vector for each wave (α=P, S_1, S_2) computed through the Christoffel equation(Schoenberg & Protazio,1992), A_α is the amplitude and ω the angular frequency. Considering the continuity of displacement and traction across the interface, the problem of computing R/T coefficents is then recast in the matrix form

$$\begin{aligned} \mathbf{N}_I\mathbf{i} + \mathbf{N}_R\mathbf{r} &= \mathbf{N}_T\mathbf{t} \\ \mathbf{Z}_I\mathbf{i} + \mathbf{Z}_R\mathbf{r} &= \mathbf{Z}_T\mathbf{t} \end{aligned} \qquad (2)$$

where,

$$N_{i\alpha} = n_{i\alpha} \quad ,$$
$$Z_{i\alpha} = c_{3ijk} s_k n_{j\alpha} , \tag{3}$$

and c_{ijkl} are the stifiness tensor components. The polarization matrices for incident, reflected and transmitted waves are indicated by $\mathbf{N}_I$, $\mathbf{N}_R$ and $\mathbf{N}_T$, respectively. Similarly the impedance matrices are labeled by $\mathbf{Z}_I$, $\mathbf{Z}_R$ and $\mathbf{Z}_T$. The amplitudes of incident, reflected and transmited waves are represent by the vectors $\mathbf{i} = i_\alpha$, $\mathbf{r} = r_\alpha$ and $\mathbf{t} = t_\alpha$, respectively. We compute R_{PP} by solving this linear system for,

$$\mathbf{r} = \mathbf{Ri} \quad \therefore \quad \mathbf{t} = \mathbf{Ti} \tag{4}$$

with,

$$\mathbf{R} = [R_{\alpha\beta}] = \begin{bmatrix} R_{PP} & R_{PS_1} & R_{PS_2} \\ R_{S_1P} & R_{S_1S_1} & R_{S_1S_2} \\ R_{S_2P} & R_{S_2S_1} & R_{S_2S_2} \end{bmatrix} \quad \therefore \quad \mathbf{T} = [T_{\alpha\beta}] = \begin{bmatrix} T_{PP} & T_{PS_1} & T_{PS_2} \\ T_{S_1P} & T_{S_1S_1} & T_{S_1S_2} \\ T_{S_2P} & T_{S_2S_1} & T_{S_2S_2} \end{bmatrix} . \tag{5}$$

RPP INVERSION

In order to investigate the potential of nonlinear inversion of *R*pp to detect fractures orientation we generated two synthetic data sets of reflection coefficents at the interface between an isotropic and a TI medium with a tilted axis of symmetry. The rotated elastic tensor is has the same structure of a monoclinic media with a vertical plane of simmetry, in this case the YZ plane. The isotropic top medium has parameters at the range of shales $\rho = 2.38\, g/cm^3$, $V_P = 2.743\, km/s$ and $V_S = 1.509\, km/s$. Table 1 shows the anisotropic media parameters. The elastic parameters for the first anisotropic medium are at the range of a fractured limestone rock simulating a high impedance constrast interface. The second anisotropic model has parameters at the range of a fracture sandstone and simulates a interface with low impedance contrast. Figure 1 shows the stereogram of Rpp for this model. The main feature of this figure is its orthorhombic symmetry. This leads us to try fitting an orthorhombic medium to the synthetic data, assuming that the parameters of the top medium were known. In order to perform inversion we computed eight *Rpp* profiles at equally spaced azimuths from 0^o to 90^o. The incidence angle varied from 0^o to 35^o for the first model and from 0^o to 40^o for the second model, which are close to the critical incidence for these models.

Linearized approximation for *Rpp* (Psencik et al., 1998) across two weakly anisotropic triclinic media shows that only a subset of elastic parameters contrasts corresponding to monoclinic media with a horizontal plane of symmetry can be recovered from *Rpp* data. When applied to our model this implies that only some parameters of an orthorhombic medium could be recovered. Our purpose in this work was to investigate if using the wide aperture data the linearized results still hold.

The two synthetic data sets were inverted using the simplex algorithm of Nelder&Mead (Press at al, 1992). The algorithm was modified to include constraints. The elastic parameters were allowed to vary in a wide range of geologicaly plausible values as indicated in Table 2. Besides, the tensor positive definiteness was checked every iteration. The initial simplex was generated sampling the feasible domain randomly with a uniforn distribuition. The algorithm was restarted until convergence was achieved or a maximum number of restarts, 100, were reached. Convergence is assumed if after 5 restarts the cost function varied less then 0.1%. Every new iteration starts with a simplex containing besides randomly generated models the previous iteration's best solution.

Table I - Model parameters before counterclokwise rotation around the X axis.

Model	ρ(g/cm^3)	C_{11}(GPa)	C_{13}(GPa)	C_{33}(GPa)	C_{55}(GPa)	C_{66}(Gpa)	Rot. Angle
1	2.5	51.04	17.54	44.42	12.65	15.81	60^o
2	2.5	27.64	9.89	24.10	6.27	8.36	45^o

Table II – Parameter constraints assumed for inversion in GPa.

	C_{11}	C_{12}+2C_{66}	C_{13}+2C_{55}	C_{22}	C_{23}+2 C_{44}	C_{33}	C_{44}	C_{55}	C_{66}
Minimum	4	4	4	4	4	4	4	4	4
Maximum	70	70	70	70	70	50	50	50	50

RESULTS

Synthetic reflection coefficients for the interface between the isotropic top medium over the anisotropic model 1 was inverted for an orthorhombic medium. The density and the elastic constants of the bottom medium were assumed unknown. The density were constrained to the range 1.80 g/cm^3 to 3.00 g/cm^3. The simplex algorithm took 70 restarts until the residual rms and the estimated parameters reached the stoping criteria. Table III shows the best inversion result. The data were fitted with rms of 2.87x10^{-5}. Figure 2 shows the inversion results.

The inversion of the second data set assumed the density of the bottom medium known. After 65 iterations the rms of the residual and the estimated parameters reached the stopping criteria. Once more a orthorhombic model fitted the data with residual rms 8.415x10^{-6}, which is below the experimental accuracy. Figure 3 shows the inversion results.

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Table III. Inversion results. Density in g/cm² and elastic constants in Gpa.

Model	ρ	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{44}	C_{55}	C_{66}
1	2.39	48.80	16.16	18.59	42.23	19.54	49.54	14.32	14.56	13.28
2	2.5	27.51	4.04	15.21	24.27	8.98	24.11	6.55	10.53	10.33

The second inversion result shows that density is seems not determined unambiguosly, since we were able to invert the data assuming the density known. The normal incidence impedance contrast seems to be the resolved parameter. Besides, these numerical experiments together with the previous results of nonlinear AVO/AVA analisys of Simões-Filho et al. (1999) have two main implication for reservoir characterization. First, inversion of wide aperture and multiazimuths P-wave reflection coeficients can recover more elastic parameters than linear AVO/AVA, particularly addtional information about shear elastic parameters can be resolved. Second, since we were able to fit an orthorhombic medium to synthetic data genereted with a tilted TI media, it follows that it is not possible to recover information about the fractures dip using Rpp only.

CONCLUSIONS

The results of the numerical experiments presented together with the previous results of Simões-Filho et al. (1999) show that wide aperture data can still recover more information about the elastic parameters than the AVO/AVA analisys based just on linearized approximations. The potential estimation of additional shear wave information may contribute to infer useful reservoir properties. Regardless that, inversion of wide aperture Rpp coefficients can only recover information about fractures strike but no information about fractures dip.

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ACKNOWLEDGMENTS

We thank Prof. Ivan Psencik for suggestions and fruitful discussions. We also are thankful to Prof. Klaus Helbig for kindly editing the text. This work was supported by the program PRONEX/UNICAMP in reservoir engeneering.

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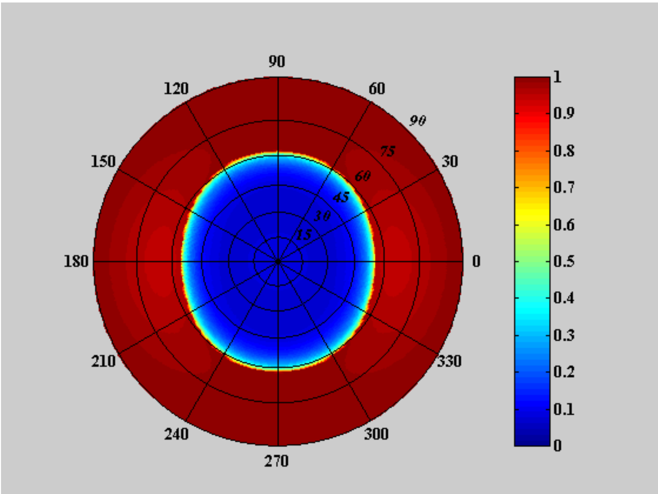

Figure 1. Rpp stereogram for model 2, although the elastic tensor is monoclinic, Rpp has two symmetry planes.

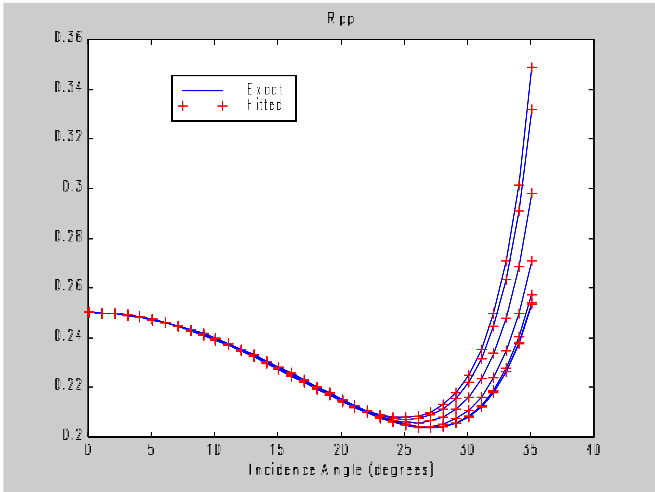

Figure 2. Rpp inversion result for data set 1.The residuals rms $2.87x10^{-5}$.

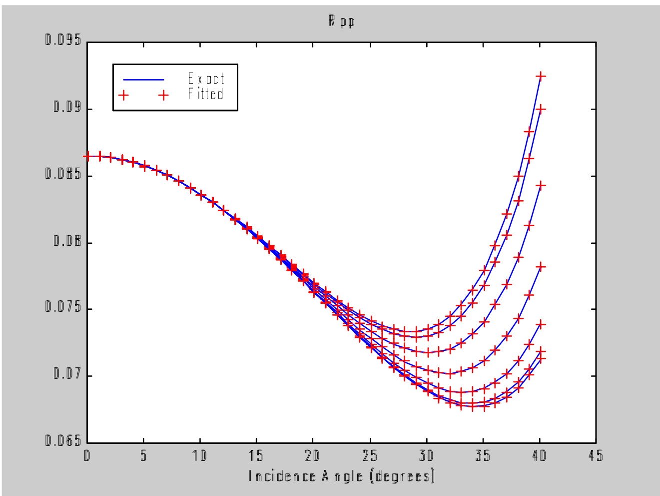

Figure 3. Rpp inversion result for data set 2.The residuals rms $8.415x10^{-6}$.

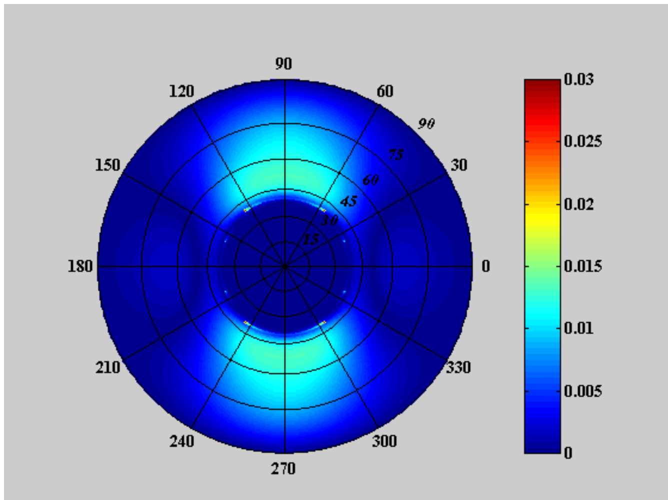

Figure 4. Stereogram of the residuals for data set 1 showing that the fitted model also macthes large part of the post-critical reflection coefficients

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