Dispersion and Attenuation of Plane Waves in Porous or Cracked Media

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Abstract

In this work are considered the finiteness of microstructure dimensions and the occurrence of stress and deformation concentrating points and their influence on the process of wave propagation in porous and cracked media.

INTRODUCTION

Frequency difference between P and S waves is a well-known fact in seismology and in multi-wave seismic prospecting. A great deal of work has been done to explain this phenomenon [1]. There are two independent quantities in the elastic theory, namely, density and stress or density and velocity. A specific time (and frequency) may occur when geometric dimensions are present, such as finite source dimension, or mean thickness of some layered structures. Finite dimensions of the source are responsible in forming more the lower frequency of S waves then of P waves. The physical reason for this is that is impossibly to create a spherically symmetric field of shear waves. Flow of shear deformations from higher stress areas to low stress areas may not be sufficiently faster than the shear wave velocity of layered structures caused dispersion of both P and S waves which can degenerate in limited cases. If the structures are periodic and the period is small relative to the wavelength, the dispersion degenerates into hexagonal anisotropy. For relatively thick layers, all waves are divided into physical interpretable sums in accordance with the ray method. Shallow layers with periodic thickness of 0.1-0.2 wavelength produce shear waves with lower frequency than the analogous compression waves.

The aforementioned phenomenon is not completely understood. First, the frequencies of both P and S waves are one or two orders lower than those generated from the real source geometry. For example, steel tubes of 20 cm in diameter dogged into the ground have been used. The tube was filled with porous absorber. It provided asymmetric effect of the charge of one detonator or several inside the tube. Good quality seismograms and phase inversion of SH waves were achieved by the use of steel cover to exclude inelastic impact. Similar methods of exclusion of strong impacts were used in cases of striking a blow into the end-wall of a massive horizontal stand (railway sleeper). The observations were carried out as a rule on head waves of SSS and PPP types. The ratio of S and P frequencies was found approximately equal to the ratio of their velocities [2]. The frequencies of both P and S waves were ten times lower than the ratio between wave velocity to the source dimension. It was clear that both wave frequencies evolved but the rates differed. In any case, the approximately equal lengths of P and S waves were found in the various frequencies ranges. Study of wave absorption showed that in the ultrasonic frequency range, the absorption amplitude depended only on deformations of 10^{-6} -10^{-7} orders of magnitude. For the lower wave amplitude values the measurements also showed the enrichment of spectra with low frequencies that was interpreted as a case of absorption [3]. The frequency behavior of the compression, shear converted waves were considered to be caused mainly by inelastic action and the microstructure of real rocks. The microstructure may also give rise to highly localized stress concentration points invalidating the Hook's law although elastic behavior may be assumed on the average.

POROUS MEDIUM AS A DISCRETE SPACE

Porous and cracked media may be described by porosity, *f*, and specific surface, σ_0, with dimension of inverse length. Typical mean microstructure dimension, l_0, is related to σ_0 and to *f* by the relation

$$\sigma_0 l_0 = 4(1-f), \tag{1}$$

obtained from integral geometry [4]. The discrete nature of the structure reflected by the finite dimension l_0 results in nonequivalence of the difference and differential operators. Equivalence takes place only in the case of $l_0 \to 0.$ Derivatives of order higher than the second thus appear in the differential equations. The translation operator for the function $u(x \pm h)$ is known to be

$$p[u(x)] = u(x)\exp(\pm hD_x)\ , \tag{2}$$

where $D_x = \partial / \partial x.$ The difference operator is determined as

$$\Delta_1 = \frac{1}{h}\left[u(x+\frac{h}{2}) - u(x-\frac{h}{2})\right] = u(x)\frac{1}{h}\left[\exp(\frac{h}{2}D_x) - \exp(-\frac{h}{2}D_x)\right]. \tag{3}$$

The second difference corresponding to $\partial^2 / \partial x^2$ takes the form

$$\Delta_2 = u(x)\frac{1}{h^2}\left[\exp(\frac{h}{2}D_x) - \exp(-\frac{h}{2}D_x)\right]^2. \tag{4}$$

NONLOCAL EFFECTS

Classical continuum mechanics assumes that the cause and effect are localized. This, however, may not apply to microinhomogeneous media, where nonlocal effects may lead to differences of many orders. For porous media containing fluids and gases, the stress field varies strongly on the surface of each grain, from the point of contact between the grains to the point of contact with liquid or gas. Therefore, continuous mechanics formulation would require the concept of real field averaging, and relate the field to any structure point. For example, the center of gravity of grain, or the center of a cube with dimensions of l_0 determined by specific surface of σ_0 (for cracked media). One of the simplest versions of averaging is the combination of translation operators like

$$p[u(x)] = u(x)\exp(\pm hD_x) . \tag{5}$$

The arithmetic mean of the field is:

$$P[u(x)] = \frac{1}{6}u(x)\left[\exp(hD_x) + \exp(-hD_x) + \exp(hD_y) + \exp(-hD_y) + \exp(hD_z) + \exp(-hD_z)\right], \tag{6}$$

where P is the averaging operator. The operator may take a probabilistic meaning as

$$P[u(x)] = M_x u(x,1) = \sum p(1,x,y)u(y). \tag{7}$$

Note that all sums are positive and add up to 1. The operator $p(1,x,y)$ is the probability of transfer from the neighboring point *y* to point *x* in one step if

$$p(1, x, y + h_k) = \frac{1}{2l}, \tag{8}$$

where l has space dimension and k varies as $\pm 1, \pm 2, \pm ....l$, $h_{-k} = -h_k$. In equation (7) M_x is mathematical expectation. This averaging concept justifies the derivatives approximation by central differences. The linear operator $A = P - E$ (where E is the unit operator) has been long noticed to be a discrete analog of the Laplace operator $(h^2/2)\Delta$ [5]. For the elasticity theory the highest derivatives of the fourth order with small factor h^2 appear in the form

$$\frac{h^2}{2}\left[\mu(\Delta\Delta u_i) + (\lambda + \mu)\frac{\partial}{\partial x_i}\Delta div u_i\right]. \tag{9}$$

In one-dimensional case it is just a sum $(\lambda + 2\mu)(h^2/2)u_{xxx}$ that is familiar in one-dimensional chain theory [6].

DISSIPATION ENERGY

It was found in contact mechanics [7] that for elastic particles with finite area of contacts the normal forces to the contact surface do not cause infinite stress at any contact point. However, the translations are in the direction of the contact surface no matter how small is the disturbance. To exclude these infinite stresses, a model of partial creeping of particles was created in contact mechanics [7]. A similar situation takes place at the crack tips. For discrete solids, an asymmetry appears in the behavior of normal and shear of microstructure, which is not present in continuous media. A known solution of the problem may be used to obtain an expression for the dissipation energy, say ΔW. For shear between two grains, it has been found that [7]

$$\Delta W = \frac{1}{18\,a\,p}\frac{1}{P_0}\frac{2-\nu}{\mu}\tau^3 \tag{10}$$

In equation (10) ΔW is the energy lost in a cycle of loading and unloading; *a* is the contact radius; p is the friction coefficient; ν is the Poisson ratio; μ is the shear modulus of grain material; and τ is the tangent stress. Thus, the hysteresis loop may be approximated by ellipse with the major axis proportional to τ (or deformation γ) and the minor axis to τ^2 (or γ^2). The equation for an ellipse is given in the parametric form as:

$$\tau = a\cos u - b\sin u \quad \text{and} \quad \mu\gamma = a\cos u + b\sin u , \tag{11}$$

where $1 \le \cos u \ge -1$. On the other hand, in complex form the equation (11) can be rewritten as:

$$\tau = ae^{iu}[1 + i(b/a)] \quad \text{and} \quad \mu\gamma = ae^{iu}[1 - i(b/a)] . \tag{12}$$

We have the ratio $\dfrac{\tau}{\mu\gamma} = \dfrac{1 + i(b/a)}{1 - i(b/a)} \approx 1 + 2i(b/a)$ where b is proportional to γ^2, and *a* to γ. Hence, we can write, instead of classical Hook's law, a more general nonlinear relation in complex form

$$\tau = \mu\gamma(1 \pm is\gamma) , \tag{13}$$

where s is a real constant. The limit $b \to 0$ determines the single-valued relation between stresses and deformations. This corresponds to zero hysteresis loop area and dissipative energy. The generalization of equation (13) for the case of arbitrary placed contacts and arbitrary direction of action forces may be obtained by using a tangential stress of intensity $\sqrt{D_{\sigma 2}}$, where $D_{\sigma 2}$ is the second (square) invariant of stress tensor. In this case

$$D_{\sigma 2} = (\sigma_{ik} n_k)^2 - (\sigma_{ik} n_i n_k)^2 , \tag{14}$$

where the first term is the square of the absolute value of the load vector P_i , and the second term is the square of the load acting over the contact normal, i.e. P_n . Dissipative force is the Euler variational derivative of the dissipation energy with respect to the generalized coordinate σ_{ik} and then with respect to the coordinate x_k in accordance with the principle of least action, i.e.

$$f_i = i\frac{\partial}{\partial x_k}\left[\frac{\partial \Delta W}{\partial \sigma_{ik}}\right]. \quad (15)$$

Hook's law is thus modified to be

$$\sigma_{ik} = \lambda\theta\,\delta_{ik} + 2\mu\, e_{ik}\left(1+\sqrt{D_{\varepsilon 2}}\right), \quad (16)$$

where $D_{\varepsilon 2}$ is the second invariant of the deviator deformation tensor. The generalized form of equation (16) coincides with the known equation [7] for one-dimensional case. Moreover, the voluminal force created by inner stresses $\partial\sigma_{ik}/\partial x_k$ is variant with respect to the coordinate system rotation. Using the principle of least action, a purely imaginary dissipative force is obtained, namely:

$$f_i = i\, s\,\mu\sqrt{D_{\varepsilon 2}}\left(\partial\sigma_{ik}/\partial x_k\right), \quad (17)$$

where

$$s = 2\pi\frac{2-\nu}{3}\frac{r_o}{a}\frac{\mu}{pP_o}\eta(1-\eta)\eta \quad \text{with} \quad \eta = \frac{\sigma_o r_o}{3}. \quad (18)$$

Above, σ_o is the specific surface; r_o is the average radius of grain; *n* the mean average number of contacts; *a* the radius of contact. The dissipation force containing quadratic terms in the deformations cannot be ignored because of the large factors μ/pP_o and r_o/a. For the cracked media, an analogous problem was solved in reference [8] for the rigid-plastic case. The dissipative was found in the form:

$$\Delta W = iM\frac{\tau_s}{pP_o}\mu D_{\varepsilon 2}^{3/2}\sigma_o z\frac{1}{\gamma_e}\frac{1}{4\gamma^4}, \quad (19)$$

where *M* is the constant depending on the creep line configuration; *z* the average length of crack rectilinear segment; γ_e the elasticity limit of deformation; γ the ratio V_s/V_p and $\tau_s = \mu\gamma_e$. In this case the factor $1/\gamma_e$ accounts for the nonlinear term. An infinite friction coefficient $p\to\infty$ results in absence of creep and deformation of structure.

EQUATION OF MOTION

The equation of motion becomes:

$$\mu\left(\Delta u_i + \frac{h^2}{2}\Delta\Delta u_i\right) + (\lambda+\mu)\frac{\partial}{\partial x_i}\left(div\, u + \frac{h^2}{2}\Delta div\, u\right) + 2is\mu\frac{\partial e_{ik}}{\partial x_k}\sqrt{D_{\varepsilon 2}} + 2is\mu\frac{1}{2\sqrt{D_{\varepsilon 2}}}\frac{\partial D_{\varepsilon 2}}{\partial x_k} = \rho\ddot{u}_i. \quad (20)$$

For one-dimensional case, equation (20) has the form:

$$\frac{1}{c_o^2}u_{tt} = u_{xx} + l_o^2 u_{xxxx} \pm \varepsilon u_x u_{xx}. \quad (21)$$

Note that

$$\varepsilon = \varepsilon_o\frac{1-\nu/2}{pP_o}\frac{\mu n\gamma_o}{a}, \quad c_o^2 = \frac{\lambda+2\mu}{\rho}\ or\ \frac{\mu}{\rho}. \quad (22)$$

ε_o is the amplitude value of deformation, and the + and – signs correspond to the loading an unloading processes. In one-dimensional coordinates: $x^* = x/\lambda_o$, $l^* = c_o l_o/\lambda_o$, where λ_o is the mean wavelength, the equation of motion becomes

$$u_{tt} = u_{xx} \pm 2i\varepsilon u_x u_{xx} + \beta u_{xxxx}, \qquad \text{where } \beta = l_o^2/\lambda_o^2. \quad (23)$$

PLANE WAVES

In discrete medium, *P* and *S* waves are taken as the average such that the mean intensity of the tangential stress of the *S* wave is $\left(\sqrt{3/2}\right)1/\gamma$ times higher than the compressional wave. The shear wave velocity is $1/\gamma$ times lower than that of the compressional wave, hence

$$\varepsilon_p = \left(\sqrt{2/3}\right)\gamma\varepsilon_s \quad \text{and} \quad \beta_s = (1/\gamma^2)\beta_p. \quad (24)$$

The forth derivative is the result of discretization of the medium. For sedimentary rocks, ε and β are of the order of 10^{-2}-10^{-4}. With accuracy of ε and β squared, equation (23) may take a form:

$$\frac{\partial w}{\partial\eta} \pm i\varepsilon w\frac{\partial w}{\partial\xi} - \beta\frac{\partial^3 w}{\partial\xi^3} = 0, \quad (25)$$

where $w = u_x$ and $\xi = t - x$, $\eta = t + x$ are the characteristic arguments. Such a reduction has been carried out before in [6], and it is related to the fact that differentiation with respect to η coordinate leads to small values of ε or β. An ideal wave process would not depend on the η coordinate or the values of the second order and may be neglected. equation (25) is a Korteweg-de-Vries (KdV) equation with a purely imaginary nonlinear term. In the case of $\beta = 0$, it has an exact solution:

$$w = F(\xi \, \mu \, i\varepsilon\eta w) \, . \tag{26}$$

In general, it has an approximate solution with the accuracy of ε and β squared:

$$w(\xi,\eta) = F[\xi \, \mu \, i\varepsilon\eta w - \kappa^2(\xi)\eta] = F(\zeta) \, , \tag{27}$$

where F is an arbitrary function, while $\kappa^2(\xi)$ satisfies the equation

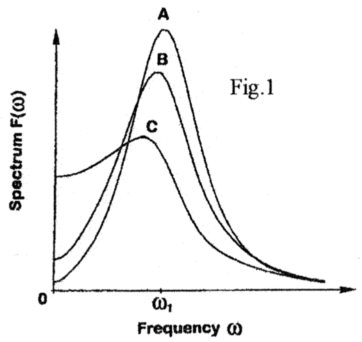
$$F'''(0,t) + \kappa^2(t)F'(0,t) - 3F''(0,t)\frac{\partial^2\zeta}{\partial\xi^2}(0,t) = 0 \, . \tag{28}$$

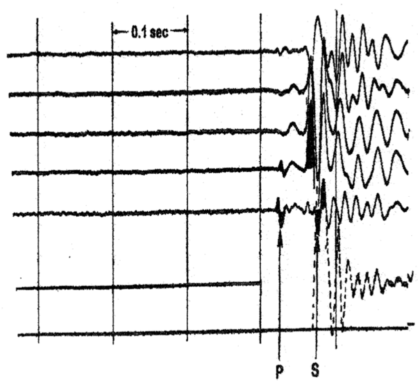
In equation (28) the factor of the second derivative is of the first order of triviality relative to ε and β. Hence, for the zero approximation, equation (28) may be simplified to

$$F'''(0,t) + \kappa^2(t)F'(0,t) = 0 \, . \tag{29}$$

CONCLUSIONS

The solution of equation (25) was obtained in order to describe the evolution with a Berlage pulse, at a distance which is set by the equation $P(0,t) = t^2 \exp(-\alpha t)\sin(\omega_1 t)$ with x=0. Unlike the classical KdV equation, the imaginary nonlinear term causes absorption of the wave only but not the increase of the front steeples of the waves. The dispersion term expands the impulse as in the classical KdV equation. The nonlinear term causes both absorption and expansion of the impulse; it is $\left(\sqrt{3/2}\right)1/\gamma$ times higher for S waves. The dispersion term leaves the absorption unaffected and casts the low frequencies in the forefront leaving the higher frequencies behind. That may mistakenly be considered as absorption. Large values of both dispersion and absorption for shear waves are observed such that S wave frequencies become lower than that of P waves. Both waves should have the same lengths in the limit [2]. In Figure 1, the impulse changes of plane P and S waves are shown for different distances from the source. Curve A corresponds to the same P and S wave spectra while Curves B and C correspond to P wave and S wave spectrum, respectively. As an example, the Berlage impulse was set with the index n=2. It is evident that S waves have higher absorption and expansion than P waves. Equation (25) shows that in the different frequency ranges, there are actually different equations of motion, since for long and short waves the influence of dispersion and absorption differs significantly. Observations of direct waves using different orientation of borehole devices shows a clear difference between P and S waves frequencies in sand and clay sediments in West Kazakhstan. In Figure 2 the record of direct waves excited by a small amount of detonator near the borehole orifice is shown. The contents of horizontal receivers orientated in different azimuths holds one vertical device registering compression and then shear waves. The depth of the device was 14m. The horizontal devices hardly register compression waves. It is clear from Figure 2 that visible frequencies of S waves are half as high as the ones of P waves that were emitted with the same source. The ratio V_s / V_p is about 0.5.





... Figure 1. Amplitude Spectra. Figure 2. Seismograms.

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