Developments in a model to describe low-frequency electrical polarization of rocks - (I): Theory

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Abstract

The author reworks his *total current conductivity* function introduced in a previous paper, related to electrical polarization of rocks in the frequency range 1 MHz to 10^{-3} Hz. Five new parameters are introduced in that function, now having clear petrophysical and electrochemical meanings and better defined ranges of variation. Some classical models are derived as particular cases of this model alongside its function analysis.

INTRODUCTION

Electrical polarization in rocks containing disseminated metallic and/or ion-exchange capable minerals in contact with ionic solutions filling their pore spaces has been studied by several researchers in exploration geophysics since middle 1940s. This phenomenon has been observed in frequency domain, in the range 1 MHz to 10^{-3} Hz, under an applied harmonic electric field (time dependence $e^{i\omega t}$), and correspondingly in the time domain. This type of electrical polarization is understood to be primarily due to the flux of "free" ions in the neighborhood of the interface between the ionic solution and the conductive (metallic) or ion-exchange competent (clay, zeolite and some organic) particles. The flow of ions and their possible chemical reactions, in the liquid phase, result in an ohmic (induced) and a diffusion-produced conduction, eventually combined with chemically generated displacement currents. And in parallel, capacitive effects due to electrical oscillations of a "fixed layer" of positive ions, adsorbed on the particle surface, and the outer limit of the excess electrical charge concentration in the ionic solution (Madden and Marshall, 1959; Ward and Fraser, 1967; Dias, 1968, 1972; Angoran and Madden, 1977; Klein et al., 1984; Olhoeft, 1985).

The work by Dias (1968), partly published in Dias (1972), clarified basic points on this matter which make up the fundamentals of our original model. For the sake of clarity of our assumptions, their limitations and possible consequences for our later model development, these points are summarized in the following: 1) recognition of simultaneous dispersion of both conductivity and dielectric permittivity, each exhibiting in-phase and quadrature components; 2) representation of this rock polarization using a single constitutive electrical parameter, defined as *total current conductivity*, 3) description of most of the reliable experimental data available at the time, using the same type of (complex) conductivity function. These data consisted of spectra of conductivity amplitude and independent measurements of σ and K, both of these parameters taken *a priori* as real quantities; 4) description of both kinds of rock polarization, associated either to disseminated metallics (electrode polarization) or clay particles (membrane polarization), using the same type of conductivity function. 5) demonstration that an equivalent circuit analog, defined as *the fundamental circuit* can generate the desired function representation for the conductivity. From a physical standpoint, this approach is equivalent to saying that the fundamental circuit represents an average unit cell of the electrical behavior of the polarizable medium; and that the medium is therefore composed by many of those units in such a way that it leaves the conductivity unchanged in its function representation; 6) selection of the fundamental circuit reproduced in Figure 1 (Dias, 1968, 1972), comprising the following two parallel combined basic components, taken within the realm of linearity in the frequency domain: (a) a simple resistor which describes the pure ohmic conduction associated to a free ionic path. Its regular presence in the unit cell dominates the asymptotic behavior for low values of frequency. It also relaxes the need to deal with some second order and more complex non-linear processes verified at low frequencies for just a single interface; and (b) the circuit model which describes best the medium to high frequencies asymptotic behavior of the polarization associated to the metal (or clay)-electrolyte interface. For this purpose three major contributions were analysed (Grahame, 1952; Chang and Jaffé, 1952; Madden and Marshall, 1959). It is our understanding that the cases discussed by these authors, involving the possibility of transportation of electrical charge through induction and/or diffusion, chemical reactions (with or without ionic discharge, formation and accumulation of a product), chain reactions and ionic recombination can all be reduced for medium to high values of frequency to the lower branch in the circuit of Figure 1.

This paper has as objective to present the constitutive function σ proposed by Dias (1968, 1972) in a new form which is more convenient to use in electromagnetic and petrophysical inversion problems involving polarizable rocks.

THE MODEL (the new conductivity function)

The original *total current conductivity* σ given by Dias (1968, 1972), associated with the circuit analog of Figure 1, is reworked, in terms of the new parameters defined in this section. This function was originally written in terms of five independent parameters, which have undefined limits of variation and unclear physical meanings. These reasons led us

to redefine the parameters, however keeping the same function. In its new form, derived from Figure 1 after some

$$\frac{\sigma}{\sigma_o} = 1 + \frac{\dfrac{R\,\lambda}{Z_W}}{1 + \dfrac{(r + R_s)\lambda'}{Z_W}} \,,\tag{1}$$

algebra, it is given as:
where

$$\lambda = 1 + \mu; \; \lambda' = 1 + \left(1 - \frac{r}{r + R_s}\right)\mu \;\; and \;\; \mu = \frac{r + Z_W}{Z_{C_{dl}}}.\tag{2}$$

Now let us represent function (1) using five new parameters. For this, let us introduce the following definitions:
a) Relaxation time, given by

$$\tau = rC_{dl}.\tag{3}$$

This parameter is positive and corresponds to the relaxation time of the electrical double layer zone in the unit cell, when diffusion is overwhelmed by the other mecanisms. It seems related to the average particle size causing the polarization (Olhoeft, 1985, Fig. 10).
b) Electrochemical parameter, given by

$$\eta = \frac{a}{r}.\tag{4}$$

This parameter has physical dimension $s^{-1/2}$ and is related to the relative significance between the ohmic (induced) electrical current component and the diffusion-produced component inside the electrical double layer at 1 rad/s frequency in the unit cell. It is related to the characteristics of the electrochemical environment producing polarization. The corresponding parameter, for a single metal-electrolyte interface, is equal to twice the ratio of the square root of the average value of the ionic diffusivities to the thickness of the electrical double layer (Chang and Jaffé, 1952). The value of η varies with ionic concentration and the plate separation L in a solution (about 1 to 10 for L varying from 4 to 1 mm, according to Jaffé and Rider, 1952). Extending this result to the rocks, the value of η could range from say 1 to about 150 or so. It is expected to increase with concentration, both of the disseminated "sources of IP" and of the ions in solution, although less critically with the last one.
c) Pore length fraction affected by the source of polarization, given by

$$\delta = \frac{r}{r + R_s}.\tag{5}$$

This parameter is dimensionless and defined on the interval $0 \leq \delta < 1$. It refers solely to the pore path that represents the locus where polarization is produced and it involves only the components of the ohmic (induced) conduction. The parameter r is proportional to the length of the average zone affected by the presence of the electrical double layer and R_s to the extension not influenced by it in the same pore. So δ is roughly the ratio of the size of the "affected zone" to the size of this zone added of the size of the "homogeneous solution zone" next to it, in the pore path where polarization is set, in the average unit cell.
d) Chargeability, according to Seigel (1959) given by

$$m = \frac{\sigma_{\omega \to \infty} - \sigma_{\omega \to o}}{\sigma_{\omega \to \infty}}.\tag{6}$$

This parameter, as δ, is dimensionless and defined in the interval $0 \leq m < 1$. It corresponds to the maximum dispersion in the amplitude of σ, as a varying function of frequency, normalized to its maximum value (the value reached when this type of polarization is over) at the high frequencies. The saturation frequencies are of the order of MHz at the highest and 10^{-3} Hz at the lowest (Dias, 1968, 1972). When using the limit values, this parameter becomes

$$m = \frac{R}{R + R_s}.\tag{7}$$

Chargeability is the best defined and most reliable parameter used until now to identify rock polarization in field measurements. Its value is associated to the intensity of the polarization effect.
Introducing the new parameters and performing some algebra, we rewrite (1), as

$$\sigma = \sigma_o\left[1 + \alpha\frac{\lambda\beta(i\omega)^{1/2}}{1 + \lambda'\beta(i\omega)^{1/2}}\right],\tag{8}$$

where

$$\lambda = 1 + \mu; \; \lambda' = 1 + (1 - \delta)\mu; \; \mu = i\omega\tau\left[1 + \eta(i\omega)^{-1/2}\right]\alpha = \frac{m(1 - \delta)}{(1 - m)} \;\; and \;\; \beta = (\eta\delta)^{-1}.\tag{9}$$

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The function σ is now expressed in terms of one variable (ω) and five independent parameters (σ_o, τ, η, δ and m). It is important to note that, in traditional exploration campaigns using induced polarization, geophysicists have exploited only three parameters, σ_o, τ and m (PFE is just a simplification of m). An expression for the resistivity, as the inverse of σ, after some algebra is given by

$$\rho = \rho_o \left[1 - m \left(1 - \frac{1}{1 + i\omega\tau' \left(1 + \frac{1}{\mu} \right)} \right) \right], \tag{10}$$

where $\mu = i\omega\tau + (i\omega\tau'')^{1/2}$, $\tau = rC_{dl}$, $\tau' = (R+R_s)C_{dl}$, $\tau'' = (aC_{dl})^2$, $m = (\rho_o - \rho_\infty)/\rho_o$ (as before), and ρ_o is the real dc value of ρ. Note that $\tau'/\tau = (1/\delta)(1 - \delta)/(1 - m)$ and $\tau''/\tau^2 = \eta^2$. It is also interesting to note that the relaxation times (τ, τ' and τ''), appearing in equation (10), seem related to the different relaxation modes of the fundamental circuit.

Particular cases deduced from the model

For this purpose, consider equation (10). If $\mu = i\omega\tau + (i\omega\tau'')^{1/2}$ has modulus on the order of unity, then the complete function given by equation (10) must be maintained. If that is not so, two cases are to be considered:

1)   If $|\mu| >> 1$, we can write straightforward

$$\rho = \rho_o \left[1 - m \left(1 - \frac{1}{1 + i\omega\tau'} \right) \right], \tag{11}$$

where $\tau' = (R + R_s)C_{dl}$

Expression (11) corresponds to the classical Debye model. In terms of the fundamental circuit elements, this is equivalent to saying that $|(r + Z_W)/Z_{Cdl}| >> 1$. In terms of the conduction mechanisms, this indicates that the double layer capacitive effect conducts current more efficiently than the series combination of induction and diffusion mechanisms inside the double layer.

2) If $|\mu| << 1$, we can approximate equation (10) by

$$\rho = \rho_o \left[1 - m \left(1 - \frac{1}{1 + \frac{i\omega(\tau + \tau') + (i\omega\tau)^{1/2}}{i\omega\tau + (i\omega\tau'')^{1/2}}} \right) \right]. \tag{12}$$

In terms of the equivalent fundamental circuit, this corresponds to $|(r + Z_W)/ Z_{Cdl}| << 1$, and in terms of conduction mechanisms, it means that the capacitive effect is insignificant compared to the induction-diffusion series combined effect. Two subcases can then be distinguished:

(2.a) When $(\omega\tau'')^{1/2} >> \omega\tau$, the resulting resistivity is

$$\rho = \rho_o \left[1 - m \left(1 - \frac{1}{1 + (i\omega\tau_W)^{1/2}} \right) \right]. \tag{13}$$

Expression (13) corresponds to the so called Warburg model (Pelton et al., 1978). In terms of the equivalent circuit, this corresponds to the situation where $|Z_W/r| >> 1$. In terms of physical mechanisms, it means that the electrical current through the double layer is determined by a diffusion process.

(2.b) When $(\omega\tau'')^{1/2} << \omega\tau$, the resulting resistivity is

$$\rho = \rho_o \left[1 - m \left(1 - \frac{1}{1 + \frac{\tau'}{\tau} \left(1 - \frac{1}{(i\omega\tau^*)^{1/2}} \right)} \right) \right], \tag{14}$$

where $\tau'/\tau = (R + R_s)/r$ and $\tau^* = \tau^2/\tau'' = (r/a)^2$.

In terms of the equivalent circuit, this corresponds to $|Z_W/r| << 1$. In this case, induction dominates the current flowing through the double layer zone. It corresponds to a peculiar situation, in which the amplitude of ρ passes through an intermediate plateau distinct from the extremal asymptotic values while the phase passes through a depression. Such a peculiar frequency interval, when it exists, is given by

$$\eta^2 << \omega << \frac{1}{\tau}. \tag{15}$$

It is important to note that this model has, as particular cases, three functions corresponding to Debye, Warburg and real constant. Coincidentally the same situation occurs with the Cole-Cole model (Pelton et al., 1978), by assigning the values 1, 1/2 and 0 to the parameter c in the exponent of $(i\omega\tau)^c$ in the Cole-Cole function, where $0 \leq c \leq 1$. Intuitively, it seems reasonable to assume that all curves generated from Cole-Cole type function could be generated by the Dias model. Yet, since the Dias model is a five parameter function and the Cole-Cole a four parameter function, the opposite can not be true.

CONCLUSION

The model presented here has as its main virtue to stay on phenomenological grounds and have five parameters with well defined petrophysical meanings from origin. The function itself, in its new form, has more appeal to be used in inverse problems, both for electromagnetic and petrophysical purposes.

Some classical models are derived as particular cases of our model.

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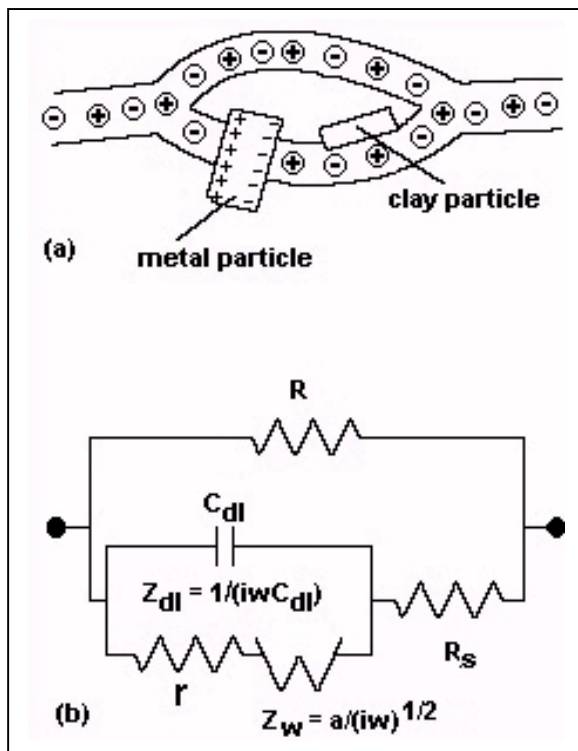
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FIG.1. (a) Schematic picture of low-frequency rock electrical polarization. (b) The equivalent *fundamental circuit* analog as the unit cell for electrical polarization of rocks (Dias, 1968, 1972).