

# Multiple Attenuation in the Plane Wave Domain by Match Filtering

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#### Abstract

In this paper we present a method of multiple attenuation in the plane wave domain that uses a filter estimation procedure to match the angle dependent primaries to predict the observed multiples. The filter is obtained through the solution of an optimization problem. Instead of using a source function estimate we use the primaries from the shallow part of the data (reflectivity) to estimate the multiples associated with these events that are correct in traveltime, but not necessarily in amplitude. Using this filter the amplitude and phase of the multiples are estimated and the predicted multiples are subtracted from the data to obtain multiple free data. The application of this method is illustrated on synthetic and real data and the results demonstrate its effectiveness in attenuating multiples. The proposed method is computationally very efficient.

#### Introduction

The elimination of free-surface multiples remains a very important problem in seismic processing. Multiples are normally high-amplitude reflection events which obscure the underlying less energetic reflections of geologic interest. Multiple suppression techniques are based on identifying different characteristics that discriminate multiples from the primary reflections. For example, the differential normal moveout (NMO) between multiples and primaries and the periodic nature of multiples are some identifying characteristics used by several multiple attenuation techniques. Shallow water short-period multiples are approximately periodic and may be attenuated using predictive deconvolution filters (Backus, 1959). Predictive deconvolution is commonly used for multiple attenuation in marine data, but it performs best on small offset data where the multiple arrivals appear at approximately periodic time intervals in the intercept time and ray-parameter  $(\tau - p)$  domain. When the water bottom is flat, the multiples are periodic for all p values and therefore, predictive deconvolution often works well in this domain.

The CDP stack and velocity filtering work when multiples can be distinguished from primaries based on differences in moveout. When the seismic data contain primary and multiple events with significantly different NMO,velocity filtering, such as f-k filtering, can remove most of the multiple events from the data. However, when these differences are small (i.e., near-offset data) the velocity filter will be ineffective and it might remove or distort the primary signal.

Several new approaches to the suppression of multiples are based upon wave-equation methods. Recently, Verschuur et al. (1992), Fokkema and Van den Berg (1994), Carvalho and Weglein (1994), and Lokshtanov (1993), proposed multiple elimination methods that are based on the wave equation. Although these methods are based on the same wave equation, they differ in their details of the derivation and in their implementation (Sen et al., 1998). Verschuur et al. (1992), Fokkema and Van den Berg (1994) and Carvalho and Weglein (1994) require an estimation of the source function. These methods are commonly called source based methods. While in the reflectivity method, proposed by Lokshtanov (1993) an estimate of the sea floor reflectivity is required in order to yield the multiple-free data.

In this paper, we apply the 1-D multiple elimination operator (Sen et al., (1998) in the  $\tau - p$  domain, like Lokshtanow (1993). The required estimate of the complete reflectivity is approximated by estimating the primaries from the shallow part of the data only. With this estimate of the primaries, a first estimate of the multiples can be obtained by convolving the primaries with the data. The estimated multiples will likely match the traveltimes of the multiples present in the data. However, the amplitudes may be poorly estimated. This can be overcome by estimating a source related filter. The filter is allowed to vary for each ray parameter trace, i.e., for each ray parameter trace, a unique filter is determined using a number of adjacent traces on either side of the trace of interest and the best filter is obtained using the criterion that the multiple-free data have minimum energy.

#### **Multiple Attenuation by Match Filter Estimation**

For the 1-D case, the reflectivity method (Sen et al. (1998)), can be used for estimating the data without surface multiples. Following Liu et al. (1998), we will only consider the multiples associated with the first k layers and in this case we have:

$$d^{o_k}(\omega, p) = d(\omega, p) + R_k(\omega, p) \ d(\omega, p)$$
(1)

and we need to have an estimate of the reflectivity of the k layers  $(R_k)$  in order to suppress the multiples related with these k layers.

We can also rewrite equation (1) as

$$d^{o_k}(\omega, p) = d(\omega, p) \left[ 1 + A(\omega, p) P_k(\omega, p) \right]$$
(2)

where  $P_k(\omega, p)$  represents the primaries of the first k layers and  $A(\omega, p) = S^{-1}(\omega, p)$  is the inverse of the source. Then, the estimation of the reflectivity (equation 1) is replaced by an estimate of the primaries for just the shallow layers.

In our approach, an initial estimate of the multiples is obtained by convolving the data with the primaries, or simple multiplication in the  $\omega - p$  domain, as follows

$$M(\omega, p) = P_k(\omega, p) \ d(\omega, p).$$
(3)

This initial estimate will likely closely match the predicted traveltimes with those of the multiples in the data, but not

(5)

necessarily their amplitudes. So, the predicted data will need to be corrected by estimating the source related term  $A(\omega, p)$ . Then, an optimal filter needs to be found such that:

$$E(A) = \left\| d + A\hat{M} \right\|^{2} = minimum$$
(4)

where M represent the multiples present in the data (Note in addition to adjustments the amplitude adjustments to the phase will also be estimated subject to the least-square error criterion) to estimate the filter  $A(\omega)$  for each frequency in the data we have:

 $A(\omega, p) = -F^{-1} b$ 

 $b = \sum d(\omega, p_i) \overline{\hat{M}}(\omega, p_i)$ 

where,

and

$$F^{-1} = \sum_{i} \hat{M}(\omega, p_i) \overline{\hat{M}}(\omega, p_i)$$

where  $\overline{M}$  is the complex conjugate of the multiples M. Here, for each p (ray parameter) trace the filter  $A(\omega, p)$  is determined using a number of adjacent p-traces. An equal number of traces on either side of the trace of interest is used to determine the filter. When the filter is calculated using only one trace, the solution above is exactly the conventional least-square solution with *b* being the crosscorrelation between the data and the predict multiples and *F*, the autocorrelation of the predicted multiples.

## **Numerical Examples**

Here we will show a synthetic example generated in the  $\tau - p$  domain for a 1-D acoustic earth model. The model contains eight interfaces separated by layers of constant velocity. The Figure 1(a) shows the ideal plane wave seismogram for this model with surface multiples and intrabed multiples included, after processing to remove the direct waves and ghosts (both source and receiver side). This data will be used as input to illustrate the surface multiple elimination method described in the previous section. The multiple-elimination result of the Figure 1(a) using the least-squares approach (equation 5) is shown in Figure 1(b) with the primaries up to 0.28 s being provided. Using these primaries, the multiples were estimated and subtracted from the data. The difference between the data (Figure 1(a)) and the data after multiples elimination (Figure 1(b)) is shown in Figure 1(d).

## Application to a Gulf of Mexico Data Set

The Gulf of Mexico line shown Figure 2 has 1000 shots with 180 traces per shot. The shot and group intervals were both 26.7 m. These 2-D data were acquired in a deep offshore area over a shallow salt pillow, which appears in the center of the section approximately between 2.2 and 2.8 s. It is a good example showing multiples from the water bottom that appear near 3.4 s. Also present are two upward reflections from the top of the salt layer which appear near 4.0 and 4.5 s. Also, two more multiples from the bottom of the salt layer are visible in the deepest part of the section. All these multiples have a downward reflection at the water surface and belong to the surface related class of multiples.

Figure 2 shows the stacked section obtained by applying DM0 but without any attempt to remove multiples. Nearly

all of the structure below the salt is contaminated by multiple energy.

Before application of the multiple elimination method described here, we transformed all the shots to the  $\tau - p$  domain. To explain our approach in detail we use shot number 500, located in the middle of the section (Figure 2) after  $\tau - p$  transform (Figure 3a). First we need to estimate the primaries from the shallow part of the data and thus we will only attempt to remove the surface multiples for layers less 3.2 s. After isolating the primaries from the data in the  $\tau - p$  domain we estimated the multiples. The multiples were obtained by convolving the primaries with the data followed by the least-square filter (equation 5) in the  $\omega - p$  domain) to better match the multiples present in the data. The estimated multiples are shown in Figure 3(b).

Next the estimated multiples were subtracted from the data. The multiple free data are shown in Figure 3(c). After inverse  $\tau - p$  transform, the shot gather of Figure 3(a), the estimated multiples (Figure 3(b)) and data after multiple attenuation (Figure 3(c)) are shown in Figure 4(a), Figure 4(b) and Figure 4(c), respectively. We notice that most of the multiple energy has been attenuated. The multiples near 3.4 s (from the water bottom) and also the two multiples from the top of the salt layer (4.0 and 4.5 s) were very well attenuated, particularly at the near offsets.

The same procedure was applied to all the shots and the final stack section is shown in Figure 5. The method proposed here has removed a significant amount of the multiple energy associated with the salt dome. Before the application of the method (Figure 2) all the primaries below 3.5 s appear were masked by the multiples. We also notice some residual multiples in the section between 4 and 4.5 s. But, comparison to the data before (Figure 2) and after multiple attenuation (Figure 5) indicates that much of the multiple energy has been removed by the method described in this paper. For example, many events below 3.5 s now appear clearer after the demultiple procedure. The residual multiples shown in the stack section after multiple elimination using the match filter were further diminished by applying a f-k filter after our demutiple filter. The final stack section obtained by applying our multiple elimination method,f-k filter and DMO is shown in Figure 6.

# Conclusion

In this paper we have developed a fast procedure for multiple attenuation in the plane wave domain using match filtering. The predicted multiples are obtained by convolving the data with the shallow primaries in the section. To improve the match of the amplitude and phase of the predicted multiples with those of the multiples observed in the data we estimate a filter based on the criteria that the multiple free data have minimum energy. This filter was allowed to vary for each ray parameter trace in shot gather for the entire line. The method proposed here was applied to synthetic and real data. The synthetic results show that the demultiple procedure removes a significant amount of multiple energy from the data. For the real data example we used data from the Gulf of Mexico where strong surface multiples associated with the free surface and top and bottom of a salt layer are present. The results obtained with the proposed multiple attenuation method were compared with the results from processing using conventional multiple attenuation methods.

A combination of our match filter approach with conventional f-k filter gave the best results.

# Acknowledgements

The first author gratefully acknowledge the Conselho Nacional de Desenvolvimento Científico e Tecnlogico (CNPq) (process no. 200.196/89-8 (NV)) and the Institute for Geophysics (UT-Austin) for financial support.

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Fig. 1. (a) Synthetic shot gather in the plane wave domain. (b) Shot gather after multiple removal for the shallow primaries. (c) Shot gather after removal of multiples using the source method. (d) The multiples removed from the data and difference between (a) and (b).



Fig. 2. Gulf of Mexico stack section before multiple attenuation. The multiple from the water botton appeares at 3.5 s in the central part of the section and most of the structure below the salt is covered by strong multiples form the base and top of sal.



Fig. 3. (a) Shot gather from the central part of the Gulf of Mexico line transformed to the  $\tau - p$  domain. (b) The estimated multiples using the primaries for the shallow part of the data. (c) The result after the demultiple method.



Fig. 4. (a) The same shot gather as Fig. 3a, but in the offset-time domain. (b) The estimated multiples. (c) The shot gather after multiple attenuation, i.e., difference between Figure 4a and 4b.



Fig. 5. Stack section obtained by applying the plane wave match filter of multiple attenuation.



Fig. 6. Stack section obtained by applying the plane wave match filter of multiple attenuation and f-k multiple elimination.