Radon transform: beyond aliasing with irregular sampling

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Abstract

Aliasing imposes a strict limitation on the maximum frequency allowed in a data set, or the maximum possible sampling interval that can be used in the sampling of a time series. In the Fourier transform this is completely justified, as regular sampling is almost always used and FFT algorithms based on regular sampling are much more faster than any other technique to estimate the transformed data. However, in other fields, the advantages of regular sampling may be less clear. That could be the case in seismic processing where sometimes irregular sampling is imposed by practical considerations. Even when further processing usually requires regular sampling in offset, modeling in the Radon domain allows the re-sampling to a regular grid. Furthermore, High Resolution RT allows us to interpolate and extrapolate the final grid, with an increase in aperture. Here we show some simple examples of information beyond Nyquist in the context of the Radon transform.

INTRODUCTION

The Radon transform (RT) is a very useful tool for filtering events according to their velocity. As explained in the bibliography (Turner, 1990, Marfurt et al., 1996) the RT shares the same aliasing problems than the Fourier transform (FT). In this work we want to make some comments about the effect of irregular sampling on aliasing. Similar considerations on NMO and stacking can be found in Wisecup (1998), but we want to show this phenomenon in the context of the Radon transform. The linear RT (LRT) is best suited to study the effect as there is an analytical solution for the aliasing condition. In the Parabolic RT (PRT) the analysis is more complicated as regular x sampling implies irregular x^2 sampling (Hugonnet and Canadas, 1995). A first look at the simple 1D frequency domain will clarify the problem. As is well known the FT of a digitized continuous signal is equal to the convolution of the Fourier transform (FT) of the sampling function with the Fourier transform of the continuous function. Because the FT of a regular sampling function is another sampling function with a different spacing, given by $\Delta F = 1/dt$, (Figure 1(a-b)), the final effect in the Fourier domain is the repetition of the spectrum at spacing given by ΔF. Interesting enough, the FT of an irregular sampling function is a spike with a background level (Figure 1(c-d)). When a continuous signal is digitized with an irregular sampling function, the resulting spectrum does not contain repeated copies of the same spectrum, but a distortion due to a non-orthogonality that must be removed with a regularized deconvolution as follows (Marfurt et al., 1996)

$$\mathbf{V}(\omega) = (\mathbf{F^H F} + \epsilon \mathbf{I})^{-1} \mathbf{F u(x)} \tag{1}$$

Note that when the dt spacing is constant, $(\mathbf{F^H F}) = \mathbf{I}$ and no deconvolution is needed. As a consequence, aliasing does not exist, but the deconvolution introduces some loss of resolution because of the regularization.

Now, let us see what happens in a two dimensional transform, in particular the RT. In this case there is another complication because the spacing in the Radon domain for the transform of the offset sampling function is frequency dependent. In other words, the "$p_{Nyquist}$" is different at every frequency. Hence, the aliased event will not be localized at a particular frequency, but will be dispersed throughout the whole Radon domain. To avoid aliasing we need to keep the maximum slope in our data below the value given by (Turner, 1990)

$$p_{max} = \frac{1}{f_{max} dx}. \tag{2}$$

Similarly to what happens with the FT, p_N does not impose a strict limit on the highest frequency represented by a given sampling rate. However, the consequences are more important. In the FT,

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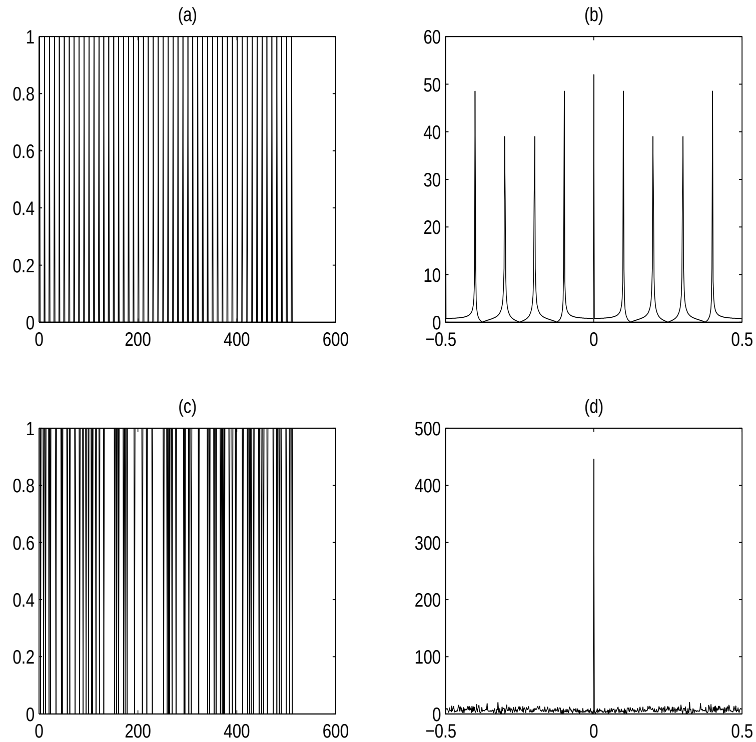

Figure 1: (a) Regular sampling function, (b) Fourier transform of sampling function. (c) Irregular sampling function. (d) Fourier Transform of Sampling Function

regular sampling has many advantages since the FFT can be used, and the transform is orthogonal ($\mathbf{F^H F = I}$), making deconvolution unnecessary. In the RT, on the other hand, non-orthogonality is always present and deconvolution is required whether the sampling in offset is regular or not. In the RT, therefore, advantages of regular sampling are not nearly as important as in the 1D case.

EXAMPLES

The first example is a very simple time series with two harmonics, $f_1 = 0.25$ and $f_2 = 0.7Hz$. We digitized the time series both, at regular intervals (Figure 2(a)) and at intervals drawn from a normal distribution (Figure 2(b)). We can see that the computed spectrum from the regular sampling is aliased (Figure 2(c)), whereas, the spectrum of the irregularly sampled data is not (Figure 2(d)).
The second example shows the same phenomenon in the RT. We have generated a synthetic data set with two linear events. The first event at 0.6 s simulates what would be the air wave with a velocity of 330 m/s. The second event at 0.8 s has a velocity of 2500 m/s. To avoid aliasing in p the maximum dp is given by (Turner, 1990)

$$\Delta p = \frac{1}{f_{max}(x_{max} - x_{min})}. \tag{3}$$

Hence, considering a maximum frequency of 40 Hz and and the limits $x_{max} = 945m$, $x_{min} = 0m$, the maximum dp (dp_c) is $2.64E - 05s/m$. We take $dp = 0.8 * dp_c = 2.1E - 5$. To avoid spatial aliasing p_{max} is given by Eq. (2). In this example $f_{max} = 40Hz$ and $dx = 15m$, so that $p_{max} = 0.00166s/m$ which is less than $p_{max} = 0.003s/m$ given by the air wave. For this example we used 200 p traces, obtaining a $p_{max} = 0.0042s/m$.
As a consequence the low velocity event will be mapped to the corresponding p value (0.003 s/m) but will also be aliased. Unfortunately, the aliased location is frequency dependent and we can not localize the event anymore. If we apply a mute to remove the air wave, only the lower frequency components will be removed and the event will still appear after reconstruction with the inverse Radon transform. An example can be seen in Figure 3. Figure 3(a) shows the original regularly spaced data set. Figure 3(b) shows the recovered data after muting low velocity events in the Radon domain (p larger than 0.001 s/m).

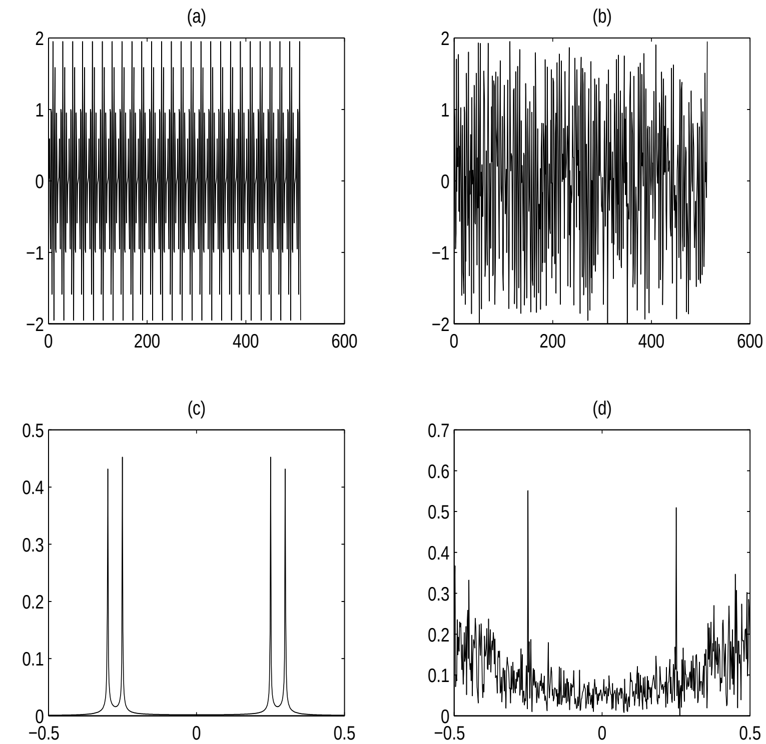

Figure 2: (a) Regularly sampled time series, (b) Irregularly sampled time series. (c) Fourier transform of time series in (a). (d) Fourier transform of time series in (b)

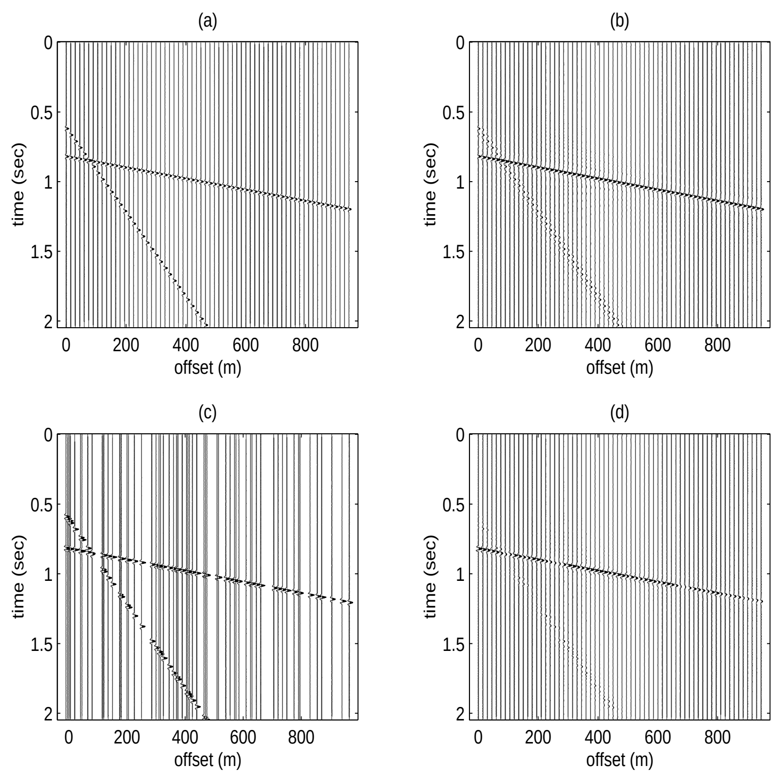

Figure 3: (a) Regularly sampled data set ,(b) Recovered after muting in the RT. (c) Irregularly sampled data set. (d) Recovered and re-sampled after muting in the RT

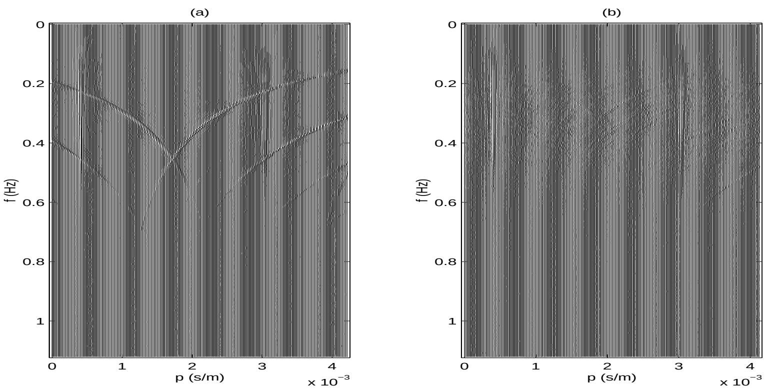

Figure 4: (a) (f-p) domain for regularly sampled data,(b) (f-p) domain for irregularly sampled data,

Now let us take the same model but sampled at non regularly spaced offsets. We use an offset axis with a mean $dx = 15m$ as before, but now with a perturbation normally distributed (Figure 3(c)). The irregular sampling produces less aliasing than before and the same muting in the Radon domain after re-sampling and inverse RT gives the results in Figure 3(d). In Figure 4(a) the Radon domain is displayed in $f - p$. For lower frequencies, there is no aliasing but at higher frequencies the frequency dependent p_{max} values follows a $1/f$ curve, yielding the overlapping of high and low velocity events. Figure 4(b) shows the same domain for irregular offset sampled data. It is clear from these plots that the mute in the Radon domain for the irregular sampled data will produce a better separation between high and low velocity events.

CONCLUSIONS

The Nyquist frequency has always represented the limit beyond which frequencies are mixed and information is ambiguous. As most limits tend to do, however, the Nyquist limit appears to dissolve under certain conditions. For example the Nyquist frequency is, indeed, a limit when regular sampling is enforced. The same frequency has not such interpretation when irregular sampling is in effect. In Radon transform, where advantages of regular sampling that allow use of the FFT are no longer so important, irregular sampling can be advantageous and computationally efficient.

REFERENCES

Hugonnet, P. and Canadas, G., 1995, Aliasing in the parabolic Radon transform: 65th Ann. Internat. Mtg., Eur. Assoc. Explo. Geophys., Expanded Abstracts, 1366–1369.

Marfurt, K. J., Schneider, R. J., and Mueller, M. C., 1996, Pitfalls of using conventional and discrete Radon transforms on poorly sampled data: Geophysics, 61, *5, 1467–1482.*

Turner, G., 1990, Aliasing in the τp transform and the removal of spatial aliased coherent noise: Geophysics, 55, *1496-1503.*

Wisecup, D., 1998, Unambiguous signal recovery above the Nyquist using random-sample-interval imaging: Geophysics, 63, *No 2, 763-771.*