High Precision/Fast Adaptive Step Size Ray-Tracing by Curvature Criteria

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Abstract

Here I present one version of Hamilton based ray-tracing equations solution (Cerveny, 1987) (Popov, 1996) aiming at increasing precision as well as reducing and controling the computation's effort. The errors in numerical evaluating the differential ray-tracing equation can be classified in two kinds. First the error in interpolating the velocity field and its derivatives. Second the error in evaluating the next ray's coordinates and slowness vector by the Runge-Kutta method. The first one has well-known solutions like fourth order precision Bi-cubic spline interpolation (William et al., 1994). The second, the fourth order precision Runge-Kutta method, needs the time step. If we apply a constant time step, it requires the use of a high-density coordinate's sampling when used for a non-homogeneous medium. The reason is that the precision of ray-tracing is directly related to the curvature radio of the ray and the last with the gradient of the square slowness. Then to obtain a high precision ray-tracing we need to decrease the size of the time step in regions where there are large variations of the square slowness field or on the other hand increase where there are no such variations. The result is an adaptive algorithm, which automatically adapts the size of the ray coordinates interval to the necessity of the medium by geophysical control. Despite the existence of the "adaptive control for Runge-Kutta method" (William et al., 1994) it's based in numerical considerations. Here I developed an alternative based on solid physical arguments most properly useful for wave propagation purposes which can easily control the accuracy of ray's coordinates and in consequence the ray's amplitudes (Lambaré, G. and Lucio, P. S. and Hanyga A., 1996) and time of processing by just changing a parameter.

THEORY

One option to ray-tracing equations solution (Cerveny, 1987) is a numerical evaluation of the initial value contour problem:

$$\frac{d}{d\tau}\begin{bmatrix} \vec{X}(\tau) \\ \vec{P}(\tau) \end{bmatrix} = \begin{bmatrix} \vec{P}(\tau) \\ \frac{1}{2}\nabla\left[\frac{1}{c^2(x(\tau),y(\tau),z(\tau))}\right] \end{bmatrix}, \qquad \frac{d\tau}{dt} = c^2(\tau) = c(\tau)\frac{d\tau}{ds}, \tag{1}$$

This eq. 1, in 3D case, is a system of six differential equations, three for derivatives of ray coordinates $\vec{X}(\tau)$ and three for derivatives of the slowness vector $\vec{P} = \frac{\vec{t}}{c}$, where $\vec{t}$ is the unitary vector tangent to the ray path, t is the time and τ a parameter related with time by the second equation of eq. 1 and with purpose to simplify the system. By a change of variables:

$$\vec{Y}(\tau) = \begin{bmatrix} \vec{X}(\tau) \\ \vec{P}(\tau) \end{bmatrix}, \qquad \vec{F}(\tau) = \begin{bmatrix} \vec{P}(\tau) \\ \frac{1}{2}\nabla\left[\frac{1}{c^2(x(\tau),y(\tau),z(\tau))}\right] \end{bmatrix}, \tag{2}$$

we have:

$$\frac{d}{d\tau}\left[\vec{Y}(\tau)\right] = \left[\vec{F}(\tau)\right], \qquad \vec{Y}(\tau_0) = \vec{Y}_0, \qquad \vec{F}(\tau_0) = \vec{F}_0, \tag{3}$$

where $\vec{Y}_0$ and $\vec{F}_0$ are the initial values. The numerical solution of this differential equation eq. 3 by the fourth order Runge-Kutta method is equivalent to the algebraic system:

$$\vec{Y}(\tau+\Delta\tau) = \vec{Y}(\tau) + \frac{\Delta\tau}{6}\left[\vec{F}(\tau) + 4\vec{F}\left(\tau+\frac{\Delta\tau}{2}\right) + \vec{F}(\tau+\Delta\tau)\right], \qquad \frac{\Delta\tau}{\Delta t} = c^2(\tau) = c(\tau)\frac{\Delta\tau}{\Delta s}, \tag{4}$$

with the same initial value conditions. Then if we know $\vec{Y}(\tau)$ we can evaluate the next point $\vec{Y}(\tau+\Delta\tau)$ by using eq. 4. The error in determing the next ray coordinate is $O\left[(\Delta\tau)^4\right]$ or fourth order in each one of the parameters Δt, Δs or $\Delta\tau$. It's a common sense to fix one of the parameters: Δt , $\Delta\tau$ or Δs. If for example we choice a constant time algoritm fixing Δt we have a problem to determine what value is good enogh for all model. A large value decrease the time of processing but can generate errors in regions of large variations of velocity field. If we decrease Δt for this critical regions, than it will ocur oversampling in smooth regions and the time of processing will increase. Here we seek an alternative which don't fix no one of these parameters, but adapts them to the heterogeneities of the medium keeping constant the errors. This alternative is based in the assumption that the errors in evaluating the next ray coordinate by the Runge-Kutta method is directly related with the ray curvature. In this manner, if we get the second differential equation of the system eq. 1, aproximate $\frac{d\vec{P}}{d\tau} \simeq \frac{\Delta\vec{P}}{\Delta\tau}$ and get only the second coordinate of $\vec{Y}$, we have a second order precision approximate relation between de variation of the tangent slowness vector $\Delta\vec{P}(\tau) \simeq \frac{\Delta\tau}{2}\nabla\left[\frac{1}{c^2}\right]$ and the velocity field. Then by creating a parameter $\Delta P = \left|\Delta\vec{P}\right| / \left|\vec{P}\right|$ and isolating $\Delta\tau$ we have:

$$\Delta\tau \;=\; \frac{2\Delta P}{c\left(x(\tau),y(\tau),z(\tau)\right)\left|\nabla\left[\frac{1}{c^2(x(\tau),y(\tau),z(\tau))}\right]\right|}. \tag{5}$$

The interval $\Delta\tau$ between two ray's coordinates will be automatically determined by fixing this parameter ΔP. In regions where the square slowness gradient $\nabla\frac{1}{c^2}$ is large the $\Delta\tau$ is small and in consequence the dt and ds will be both small. For the other hand in regions where the $\nabla\frac{1}{c^2}$ is small than dt and ds will be both large. Then we have a problem in points where $\nabla\frac{1}{c^2}\to\infty$ or $\nabla\frac{1}{c^2}\to 0$. In first case $\Delta\tau\to 0$ and we have sequence of points each time closer and the algoritm doesn't finish. In second $\Delta\tau\to\infty$ and the next point will be outside the model boundaries. To solve these problems we need to keep $\Delta\tau$ inside certain bounds $\Delta\tau_{min}\le\Delta\tau\le\Delta\tau_{max}$.

EXAMPLES

The examples compares the method of constante interval time *CIT* dt_{const} with the adaptive interval time *AIT* using the equation 5. The figures 1 and 2 shows a simple model for constante gradiente square slowness field with equation: $\frac{1}{c(x,z)^2} = A + B(z-z_0)$ with $A = 1\times10^{-6}$ and $B = -2\times10^{-9}$. This model has well known theoretical solution (Cerveny, 1987) for coordinates: $z = \frac{B}{4}(\tau-\tau_0)^2 + P_{z_0}(\tau-\tau_0) + z_0$. The trajectory is parabolic with apex $(\tau_{z_{max}}-\tau_0) = \frac{-2P_{z_0}^2}{B} = -\frac{sin^2\theta}{Bc_0^2}$, where P_{z_0} is the z component of the slowness vector at initial coordinates $(x(\tau_0),y(\tau_0),z(\tau_0))$. For vertical launch ($\theta = 0$) the analytical solution gives $z_{max} = 510m$. The apex for the *CIT* was $z_{max} = 502.755$ and for *AIT* $z_{max} = 510.000$. The time of processing was almost equal and the precision of the *AIT* thousand times larger. The fig. 4 is the surpeposition of the two methods ray's. The *CIT* use $dt_{const} = 12.5ms$ and the *AIT* use bounds $\Delta\tau_{min} = dt_{const}10^{-6}\le\Delta\tau\le\Delta\tau_{max} = dt_{const}$ showing that in regions of large field gradient the two ray's separates in different coordinates. However the *AIT* has always higher precision since $\Delta\tau_{max}\le c^2 dt_{const}$. The fig. 3 is 3D example.

CONCLUSIONS

The adaptive time interval method achieves it's objetive to increase the precision without large increase in time processing. Really is possible high decreasing in time processing by exploiting the potencial of the adaptive interval time method to walk large paths in homogeneous regions keeping the error under control.

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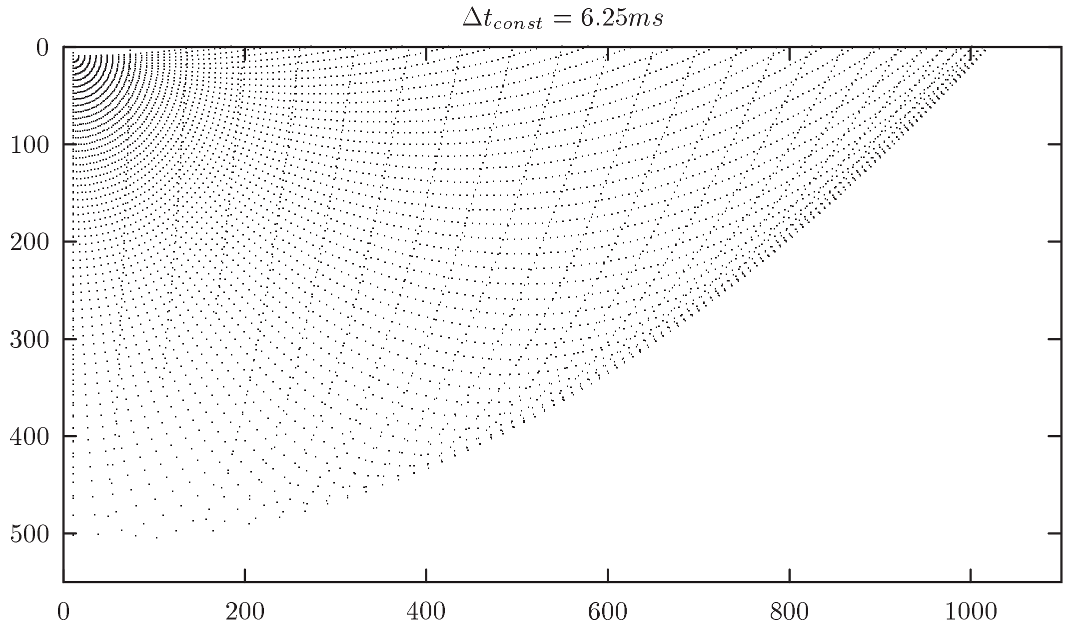

Figure 1: Model-01: $\frac{1}{c(x,z)^2} = 1 \times 10^{-6} - 2 \times 10^{-9}(z - z_0)$

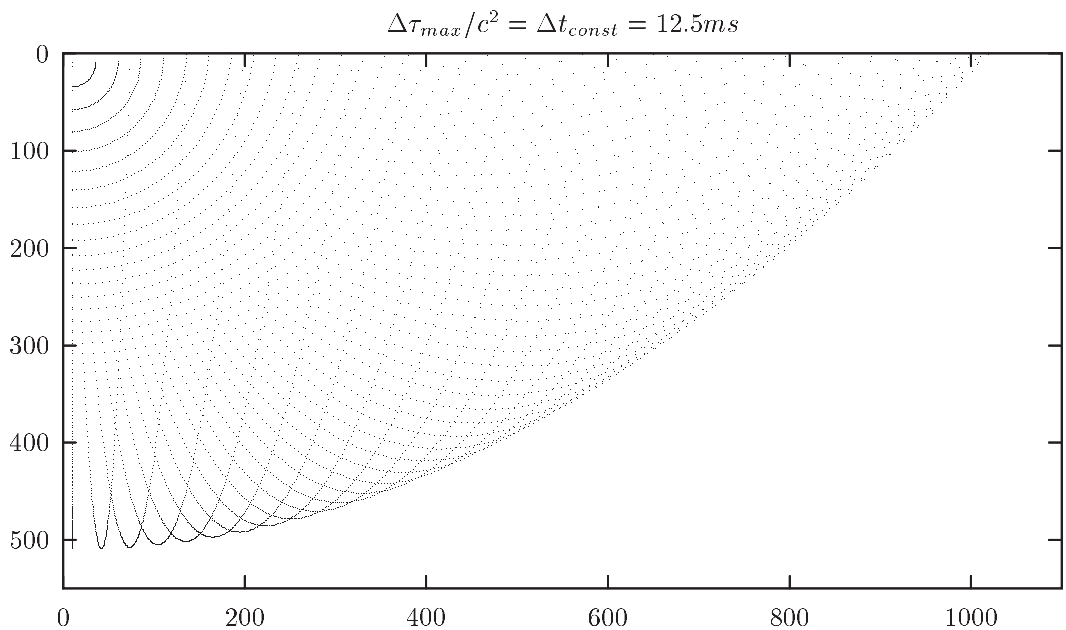

Figure 2: Model-01: $\frac{1}{c(x,z)^2} = 1 \times 10^{-6} - 2 \times 10^{-9}(z - z_0)$

$dt_{const} = 12.5ms,\ \Delta\tau_{max}/c^2 = dt_{const},\ \phi = .25\pi$