



# Multi-valued Two-Point Raytracing

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## Abstract

Imaging in complex geological areas some times require multi-valued traveltimes and amplitude information [1],[2]. Traditionally two-point raytracing is used to calculate the shortest traveltimes and raypaths between two points. However if there exists more than one ray solution, corresponding to multiple branches of a traveltimes curve, this method is computationally expensive and less successful. Here we present an efficient way to compute the minimum traveltimes based on a triangulated network by adaptive wavefront construction. We search for the minimum traveltimes for each arrival with maximum amplitude from multi-valued arrivals. The main contributions of this work compared to our previous algorithm [4] are two fold: (1) obtain more ray information from wavefront construction, which will improve the accuracy for two-point raytracing, and (2) decrease sufficiently the computational time of minimal traveltimes searching on each branch. Since we use the triangulated network for wavefront construction and implementing Fermat's principle, we are able to increase the efficiency, especially for multi-valued traveltimes. This, in turn, increases the efficiency for velocity model building, traveltimes table generation for 3-D prestack Kirchhoff migration, AVO analysis, tomographic analysis, and seismic data acquisition planning. The algorithms are tested and illustrated on a synthetic model. Finally the algorithms are tested by generating the traveltimes and amplitude maps on the Mahogany subsalt model.

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## INTRODUCTION

Wavefront construction via raytracing provides multi-valued traveltimes and amplitude information in complex geologic areas [1][2]. It is robust, accurate and efficient, especially for the large number of receivers in 3-D data. However, the drawback of these methods is the calculation of traveltimes information across complex regions. Velocity models are smoothed on those boundaries to avoid holes on the generated wavefronts. A more accurate velocity model without applying smoothing is necessary to compute the corrected traveltimes and raypaths. Wavefront construction with adaptive retiling has been introduced [2] to ensure that proper wavefronts are generated on a complex geologic model.

There are two traditional methods to compute the shortest raypath between two points: shooting and bending [8].

The shooting method uses different ray take-off parameters in order to find the raypaths closer to the given receivers. Snell's law is then solved at each interface until the ray emerges back at the observation surface. The advantage of this method is its speed for simple geologic models. However, for a complex geology, it becomes more complicated and less successful. Since raytracing is a non-linear process it is very difficult to deduce the way to shoot a ray arriving at the specific receiver location.

The bending method finds the minimum traveltimes between two points. It is an accurate and stable tool. However, if several thousand rays must be calculated to obtain the shortest raypath, this method is computationally expensive. In regions where there exists more than one solution, the bending method becomes a nightmare and is less successful.

Two-point raytracing by adaptive recursive subdivision [4] has been introduced to compute the minimum traveltimes by combining the above two methods. The first step is to implement Snell's law by shooting several rays with different take-off angles. Accurate and stable propagation through regions of velocity contrast in the subsurface model is handled by the local smooth, recursive subdivision method [8]. The intersected points between the rays and each layer are then triangulated. The second step is to implement Fermat's principle within a ray cell. We define the ray cell by connecting from the source point to three intersected points on each horizon until the rays hit the receivers. Recursive subdivision is used to calculate the minimum traveltimes within the ray cells until the computed traveltimes are stationary. However two problems exist. The first problem concerns rays that are missing before they arrive at the next horizon, where rays are diverging. The second problem concerns the computational expense, when a large number of layers are needed to compute the minimum traveltimes. In this algorithm, we implement: (1) wavefront construction rather than the shooting method, and (2) the sequential decomposition of the minimization procedure [5] using Fermat's principle. We test our algorithm by generating the raypaths, on a synthetic model after wavefront construction.

## METHOD

### 1. Implementation of Wavefront Construction

The first step of our algorithm is to implement wavefront construction. A set of rays from a point source, generated by a triangulated network, are shot to a target region. The relationship between rays is maintained by keeping track of the connectivity between, and on, each triangular mesh representing a wavefront. To construct successive wavefronts, rays passing through the vertices of the triangular mesh are continued by a constant time step[1]. This creates a region with two wavefronts in which the connectivity of the rays is well defined, enabling interpolation of traveltime, amplitude and raypath between the wavefronts. Rays propagating through the regions of velocity contrast, handled by recursive subdivision [8], are stable and accurate.

When there is divergence in the raypaths, the wavefront will be stretched. For insertion of a new ray, the midpoint is interpolated based on an adaptive surface curvature [6]. The wavefront may be very complicated. If holes are generated in the wavefront due to total internal reflection, local adaptive retiling is implemented for the missing hole to ensure the complete wavefront is generated[6]. The size of triangle is constrained by a density criterion[1]. The traveltime and raypaths are interpolated on the intersection between rays and the horizons. After the wavefield reflects on the specified horizon and arrives at the surface, a multi-valued wavefront with triangulated network is generated. The wavefront covers most of the receivers as shown in figure 1 and figure 2.

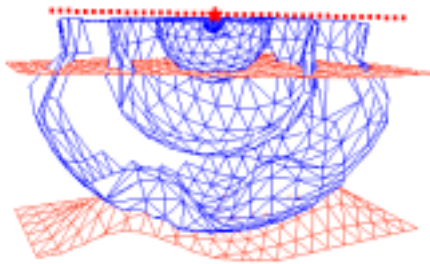


Figure [1] A series of wavefields propagated downward.

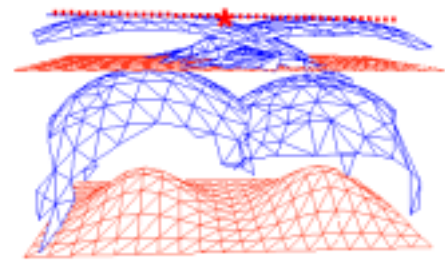


Figure [2] A series of wavefields propagated upward with three branches of multi-valued traveltime on the surface.

## 2. IMPLEMENTATION OF FERMAT'S PRINCIPLE

The second step is to define the ray cells by connecting three rays from the source point to three intersected points on each layer until the rays hit the receiver[4]. We implement Fermat's principle by the sequential decomposition of the minimization procedure within the ray cells, shown in Figure [3], rather than search for the minimum traveltime in a ray cell [4]. We place the nodes on the center of the cell for each loop so that Fermat's principle can be implemented in a network. For each loop the point or node is connected to its adjacent triangle. We implement recursive subdivision[6] to find the minimum traveltime on each loop by dividing each triangle into four triangles until it is stationary, It is therefore possible to compute the minimum traveltime from the first to the last loop via connections. We are able to find the index of each triangle for all the loops, which gives the minimum traveltime and corresponding shortest raypath through the network, as shown in Figure [3].

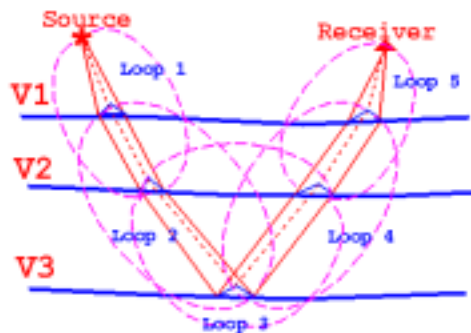


Figure [3] Several loops within the ray cells for computing the minimum traveltime.

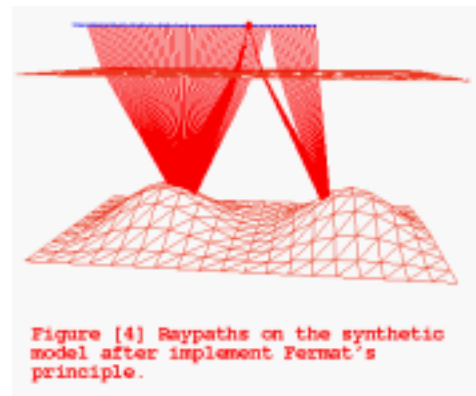


Figure [4] Raypaths on the synthetic model after implement Fermat's principle.

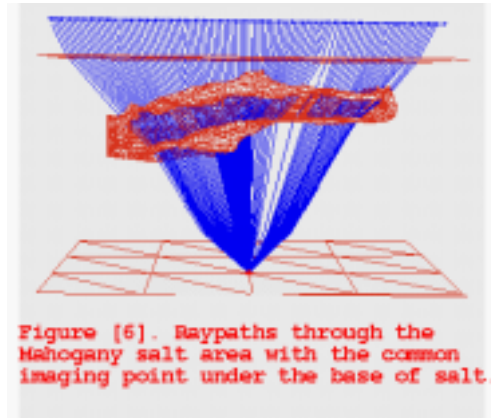
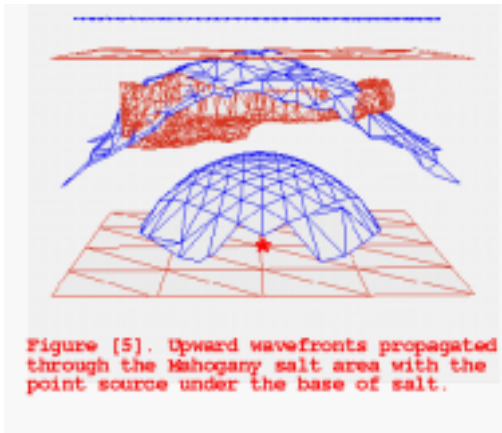
Figure[4] shows the raypaths on this synthetic model, after the implementation of Fermat's principle on each ray cell for all the receivers. The raypaths are uniform on the surface, but are not uniform in the subsurface, due to the result of searching for a maximum amplitude from multi-valued arrivals.

## EXAMPLE

We test our algorithm by putting a single shot point under the base of the Mahogany salt model and examine the

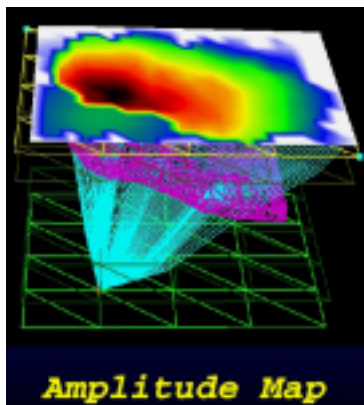
amplitude and the time map on the surface. The main reason for doing this test is to know how our algorithm can be used for AVO analysis to better understand the subsalt geology.

Figure [5] shows the shot point under the Mahogany salt sill area and a series of wavefronts propagated upward. The wavefronts are uniform under the salt. However the wavefronts are distorted above the salt due to the large velocity contrast.

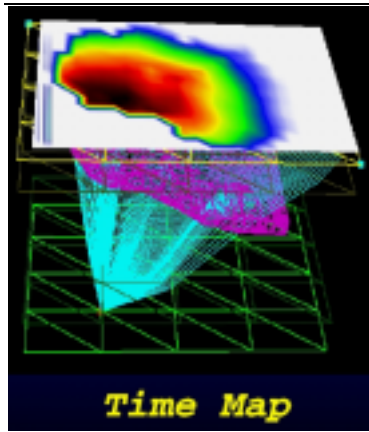


After the wavefronts arrive at the surface, we compute the minimum travelttime by implementing Fermat's principle on each ray cell for each receiver. For the multi-valued arrivals, we search for the minimum travelttime with maximum amplitude. Figure [6] shows the raypaths on all the receivers with a common imaging point beneath the salt. The raypaths are not uniform under the salt, but they are uniform on the surface, due to the result of searching for the maximum amplitude from the multi-valued arrivals.

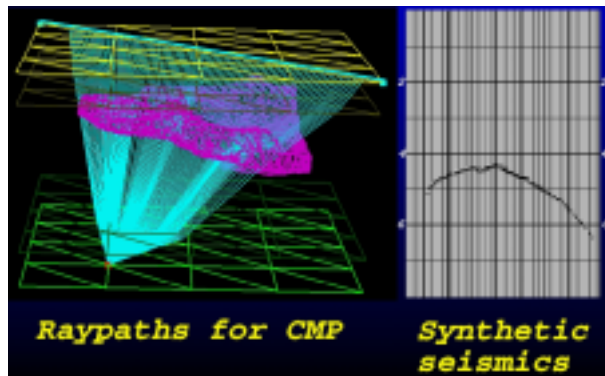
Figure[7] shows the amplitude map on the surface, after we search for the maximum energy for all the receivers. Figure[8] shows the corresponding travelttime map, which has the maximum amplitude on each receiver. The two figures show the values are uniform at the top salt, but change abruptly at the edge of the salt. Figure[9] shows the synthetic seismics on the target line of base map. The discrepancy between the synthetic and the real seismics can be used for updating the velocity model.



Figure[7]. Amplitude map on the surface with the corresponding raypaths on the Mahogany model.



Figure[8] Time map on the surface with the corresponding raypaths on the Mahogany model.



Figure[9] Synthetic seismics on the target line with the corresponding raypaths on the Mahogany model.

## CONCLUSION

Multi-valued two-point raytracing is a flexible and reliable tool to calculate the minimum traveltimes with the shortest raypath between a shot point and the receivers. However, if the algorithms are only used for computing traveltimes between a single shot and a few receivers, the computational time is expensive. Since most geophysical applications require a large number of receivers, this is a powerful tool for seismic exploration. Implementing wavefront construction while the rays propagate through the medium stabilizes the raypath. The algorithms, without applying any smoothing on the velocity model, is accurate and efficient. Implementing Fermat's principle by the sequential decomposition of the minimization procedure decreases the computation time for two-point raytracing. It is a robust tool for complex areas, and is an important tool for the imaging of subsalt geology.

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