Signal-to-Noise Ratio Enhancement by Adaptive Filtering

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Abstract

An adaptive filtering technique was implemented in order to remove random noise from seismic data. The operator was calculated in the frequency-offset domain (FX), and it could enhance the lateral coherence of seismic events. For each frequency an optimum Wiener filter was designed following the classical approach using the complete seismic section. After that, an adaptive algorithm was implemented to actualize the filter coefficients in order to reduce the output error. This automatic actualization avoided us the use of predetermined design windows. The adaptive filter was based on a generalized expression of the least-mean square (LMS) filter dealing with complex frequency input signal. The performance was evaluated on synthetic seismic data contaminated with additive gaussian noise.

INTRODUCTION

One of the main characteristics of seismic reflections is their lateral continuity, and this continuity is used to distinguish events from random background noise. The main idea used is simple: signal is defined as that which is predictable from nearby traces, and noise is unpredictable from nearby traces. All kind of ”coherent seismic noise” (e.g. multiple energy) is treated as signal by the later definition, and consequently it cannot be eliminated.
Canales (1984) proposed a method for random noise attenuation, that considered linear events in t-x domain manifested as a superposition of harmonics in the f-x domain. This superposition can be modeled as an autorregresive process (AR), or if noise is taken into account as an autorregresive moving average process (ARMA). Hornbostel (1991) introduced a t-x prediction technique that allows changing data without requiring windowing. It consisted on an 2D adaptive least-mean-square (LMS) filter. After each filter application, the predicted value was kept and the prediction error was used to update the filter coefficients.
In our approach, the idea was the same, but the filter application was on the f-x complex domain. Such application required a complex adaptive filter that first predict the frequency values from nearby traces, and secondly use the prediction error to update the filter coefficients.
Furthermore a generalized expression of the LMS filter was used. It was based on an adaptive technique of predictive deconvolution implemented by Comínguez (1987) in order to remove multiple energy from shallow-water seismic data.
Following Mueller (1972) and Comínguez (1987), introducing a non-singular matrix into the recursive algorithm is useful for achieving more stability while the filter parameters change towards the optimum values. Our model was based on an AR model with adaptive parameters. If the statistical characteristics of any frequency have considerable changes along the seismic section, the adaptive property of the filter weights would become important.

THE MODEL

The seismic data was modeled as

$$y(t,x) = s(t,x) + n(t,x) \tag{1}$$

where, $s(t,x)$ was the signal and $n(t,x)$ was the random undesirable noise. Assuming events with linear moveout,

$$s(t,x) = w(t) * \delta(t+px) \tag{2}$$

where w(t) was the seismic wavelet that was considered constant for all the traces, and p was the slowness of the event. Transforming to the f-x domain,

$$S(f,x) = W(f)\exp(2\pi i f p x) \tag{3}$$

The above expression is a periodic function of x for each frequency. A superposition of p complex harmonics (p events with different slopes) may be modeled as a AR process of order p. If the random noise $n(t,x)$ is taken into account, the exact

model is an ARMA model of order (p,p) with identical parameters for the AR and MA parts.
The classical solution for calculating the AR parameters is the least-square approach for minimizing the error energy $J = E[\|\mathbf{S}\mathbf{a} - d\|^2]$ where $E[.]$ denotes the mean value of the statistical variable, $\mathbf{S} = (S(f,x), S(f,x-1), \ldots, S(f,x-p+1))$, $\mathbf{a}^H(f) = (a_1(f), a_2(f), \ldots, a_p(f))$ is the unknown filter, and in this case the desired output $d(f,x)$ is the next trace frequency value $S(f,x+1)$. In what follows, the dependence on f has been supressed for simplicity. Minimizing the error, the classical Wiener prediction solution is found:

$$\hat{\mathbf{a}} = (E[\mathbf{S}^{\mathrm{H}}\mathbf{S}])^{-\mathbf{1}}\mathbf{E}[\mathbf{S}^{\mathrm{H}}d] \tag{4}$$

ADAPTIVE TECHNIQUES

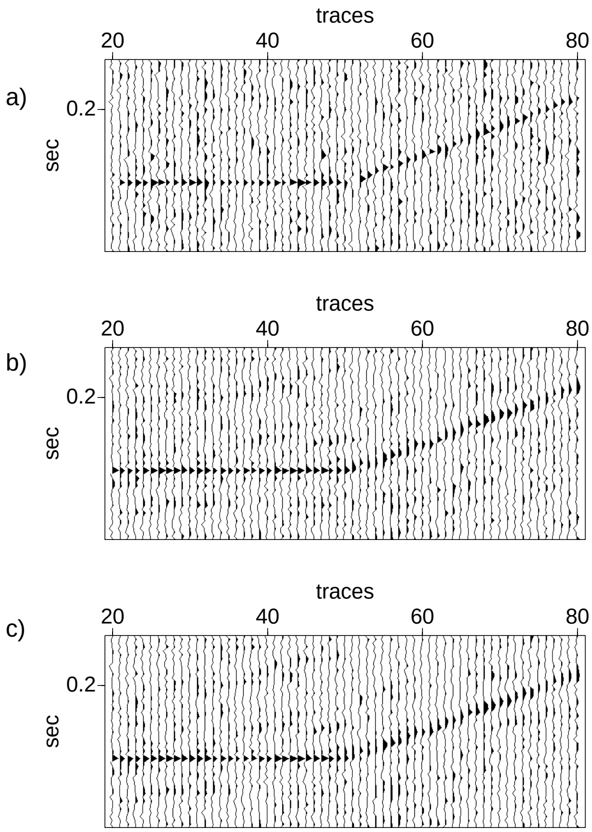

Figure 1: a) spatial window of the synthetic model. b) processed traces by an adaptive LMS filter $\alpha = 0.05$ c) processed traces by an adaptive GLMS filter $\alpha = 0.05$.

It is possible to find the minimum of J by the steepest descent method. Approximating the true autocorrelation matrix, and the croscorrelation vector by its 'instantaneous values' (Widrow and Hoff, 1960) a simple noisy gradient descent algorithm can be derived. It has been named LMS (least-mean square):

$$\mathbf{a}_{x+1} = \mathbf{a}_x - \mu\left(\mathbf{S}(x)\mathbf{a}_x - S(x+1)\right)\mathbf{S}^H(x) \tag{5}$$

The first term inside the brackets is the prediction $\hat{S}(x+1)$ calculated with the operator at offset x, and μ is a convergence real constant.
Following Mueller (1972), we have proposed a generalization of the algorithm introducing a nonsingular matrix in order to obtain more stability. It can be demonstrated (Comínguez, 1987) that a good choice for that matrix is the inverse of the autocorrelation matrix estimate.
For those cases where the input signal can be modeled as a stationary process, the best overall approach is undoubtedly the use of the normal equations 4. However, for those applications which occur too frequently in practice with time-varying statistical properties, the adaptive techniques offers potential advantage.
Consider a seismic stacked section composed of events with varying slopes, and strong geological features, such as folds and faults. The correlation measurements required for the normal equations necessitate averages over design windows and may involve averaging over significant changes in the data. So the operator obtained would not provide a satisfactory signal enhancement.

FILTER IMPLEMENTATION

The secuence for FX noise attenuation starts with Fourier transforming the data from t-x to f-x domain. For each frequency, a predictive complex filter is designed by means of LMS algorithm. The initial operators were chosen as the Wiener filters calculated by equation 4, replacing $E[.]$ by a simple average, in order to get more stability.
It can be demonstrated (Widrow et al, 1976) that the adaptive constant μ for the LMS algorithm of equation 5 must be chosen satisfaying the following inequalities $\mu = \frac{\alpha}{L{\sigma_x}^2} \quad and \quad 0 < \alpha < 2$, where ${\sigma_x}^2$ is the average power level of the input signal, and L is the operator lenght (autoregressive order). Naturally, lower μ values result in a slow and stable convergence, higher μ values are in accordance with rapid changes in order to track varying data but noisy performance.
From the before results, it was not difficult to deduce the generalized LMS (GLMS) algorithm parameters. The ${\sigma_x}^2$ parameter must not be included in the expression of μ because it is included in the diagonal terms of the autocorrelation matrix. On the other hand, as the gradient estimate is multiplied by this LxL matrix, the new μ value must be chosen proportional to $1/L^2$. Therefore, for the GLMS algorithm, the following expression will be adequate

$$\mathbf{a}_{x+1} = \mathbf{a}_x - \mu(\mathbf{S}^{\mathbf{H}}\mathbf{S})^{-1}\left(\hat{S}(x+1) - S(x+1)\right)\mathbf{S}^H(x) \tag{6}$$

where

$$\mu = \frac{\alpha}{L^2} \quad and \quad 0 < \alpha < 2 \tag{7}$$

The adaptive procedure was applied both forward and backward in space, that is from near to far offsets or CDP's, and in the reverse way. The final frequency-space value was obtained as a simple average. After the complex values for all traces and all frequencies were predicted, an inverse Fourier transform was performed to get the processed traces back in the time domain.

SYNTHETIC DATA PROCESSING RESULTS

In order to evaluate the adaptive operator ability to predict and enhance lateral coherency, a set of 100 synthetic traces were created with two linear events inmersed in gaussian noise (SNR=3). In figure 1 it is shown a spatial window of the input traces, where a flat reflector can be seen from traces 1 to 50, and a dipping reflector can be seen from traces 50 to 100. This environment is adequated for an FX adaptive prediction because the events, that must be enhanced, have linear moveout. An adaptive LMS operator of lenght 3 (equation 5) was used to process the input traces. Also, an adaptive GLMS operator of the same lenght was used. The results are shown in figures 1 and 2. The performance of both filters was good, but the LMS showed noisy processed traces due to the stochastic behaviour of the adaptive filter. While increasing the convergence constant α for tracking the dip change in time-offset domain (phase shift in FX domain)(figure 2), the operator become unstable giving no effective noise removal. When the same traces were processed by the GLMS adaptive operator of equation 6, the performance was increased. The algorithm allowed us to set the convergence constant α with a higher value in order to predict the dip change while leaving the output noise level low. In other words, the GLMS was more stable than the simple LMS algorithm.

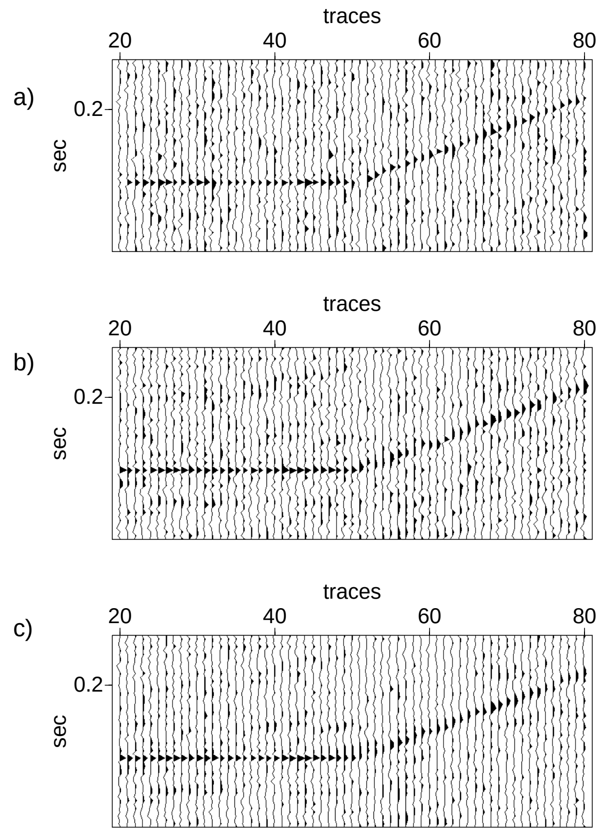

Figure 2: a) spatial window of the synthetic model. b) processed traces by an adaptive LMS filter $\alpha = 0.1$ c) processed traces by an adaptive GLMS filter $\alpha = 0.1$.

CONCLUSIONS

Adaptive LMS filters were implemented to enhance lateral coherency in synthetic seismic data. The application of the LMS filters (both the simple and the generalized) in the complex frequency domain has been found profitable. For each frequency, the adaptation was carried out while the filter was predicting the complex signal using the signal at previous offsets. The performances were quite similar, and evident improvement was obtained processing with the adaptive GLMS filter. Some evidences of instability have appeared when the convergence-constant α was increased. This fact has forced the use of small values of α, and therefore it has been difficult for the system to adaptate rapid enough to recognize statistical changes present in the signal. Such problem can be satisfactory solved with the generalized technique implemented in this paper. The good results obtained applying the adaptive techniques on synthetic data suggest that this method could be used with advantages on real noisy seismic data.

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