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Electromagnetic waves generation by the hybrid modulational instability

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Abstract

An improved model of the fundamental plasma emission of type III solar radio bursts is presented. It is shown that nonlinear conversion of a traveling Langmuir pump wave into an electromagnetic wave can occur via either convective or absolute hybrid modulational instabilities, resulting from the coupling of two wave triplets, with the Langmuir pump wave and ion-acoustic daughter wave common to both wave triplets. The properties of these four-wave hybrid modulational instabilities are studied using the observed interplanetary parameters.

INTRODUCTION

There is increasing observational evidence in support of nonlinear wave-wave interactions involving Langmuir waves (L), electromagnetic waves (T) and ion-acoustic waves (S) in association with type III solar radio bursts (Lin et al. 1986; Gurnett et al. 1993; Hospodarsky and Gurnett 1995). Traditionally, type III events are interpreted in terms of three-wave processes L $\rightarrow$ T $\pm$ S and L $\rightarrow$ L $\pm$ S (Ginzburg and Zheleznyakov 1958; Chian and Alves 1988; Melrose 1991; Robinson, Cairns and Willes 1994; Chian and Abalde 1995). In this paper, we study the nonlinear generation of type III solar radio bursts by a four-wave *hybrid* (coupled electromagnetic and electrostatic) modulational instability, L $\rightarrow$ T + L + S, driven by a traveling Langmuir pump wave. Three-wave electromagnetic or electrostatic three-wave decay/fusion instabilities turn out to be oversimplified descriptions of nonlinear interaction. Studies indicated that hybrid parametric instabilities, involving nonlinear coupling of two or more wave triplets, are easily produced by a Langmuir pump wave (Akimoto 1988; Rizzato and Chian 1992; Chian and Abalde 1997).

THEORY

The nonlinear coupling of Langmuir waves, electromagnetic waves and ion-acoustic waves is governed by the generalized Zakharov equations (Akimoto 1988, Rizzato and Chian 1992, Chian and Abalde 1997):

$$\left[\partial_t^2 + \nu_e \partial_t + c^2 \nabla\times\nabla\times - \gamma_e v_{th}^2 \nabla(\nabla\cdot) + \omega_p^2\right] E = -\frac{\omega_P^2}{n_0} nE \quad , \tag{1}$$

$$\left(\partial_t^2 + \nu_i \partial_t - v_s^2 \nabla^2\right) n = \frac{\varepsilon_0}{2 m_i} \nabla^2 < E^2 > \tag{2}$$

where $\mathbf{E}$ is the high-frequency electric field, $\mathbf{n}$ is the ion density fluctuation, $\boldsymbol{\omega_p = (n_0 e^2/m_e \varepsilon_0)^{1/2}}$ is the electron plasma frequency, $\mathbf{v_{th}} = \mathbf{(KT_e/m_e)}^{1/2}$ the electron thermal velocity, $\mathbf{v_s = [K(\gamma_\varepsilon T_e + \gamma_i T_i)/m_i]^{1/2}}$ is the ion-acoustic velocity, $\boldsymbol{\nu_\varepsilon}$ is the damping frequency for electrons, $\boldsymbol{\nu_\iota}$ is the damping frequency for ions, γ_ε is the ratio of the specific heats for electrons, γ_ι is the ratio of the specific heats for ions, and the angular brackets denote the fast time average. The *hybrid* nature of coupled high-frequency *electromagnetic* and *electrostatic* waves is evident in equation (1).
A traveling Langmuir pump wave $\mathbf{E_0}$ (ω_0, $\mathbf{k_0}$) with dispersion relation $\omega_0^2 = \omega_p^2 p + \gamma_\varepsilon v_{th}^2 k_0^2$ can excite two types of four-wave hybrid modulational instabilities (Akimoto 1988; Chian and Abalde 1997); $L_0 \rightarrow T^+ + L^- + S$ and $L_0 \rightarrow T^- + L^+ + S$, respectively, provided the following frequency and wave-vector matching conditions are fulfilled

$$\omega_\alpha^- \approx \omega_0 - \omega^*, \qquad \omega_\alpha^+ \approx \omega_0 + \omega, \qquad k_\alpha^\pm = k_0 \pm k \quad , \tag{3}$$

where $\mathbf{w}$ and $\mathbf{k}$ are the frequency and wave vector of the low-frequency ion mode, respectively, α = T or L, with $|\mathbf{k}_T^\pm| < (|\mathbf{k_0}|, |\mathbf{k}_L^\pm|)$ and $|\mathbf{k}| \approx |\mathbf{k_0}|$, the asterisk denotes the complex conjugate. The wave-vector kinematics for $L_0 \rightarrow T^+ + L^- + S$ is illustrated in Figure 1. In this paper, we shall focus on the process $L_0 \rightarrow T^+ + L^- + S$ since it generates an upconverted anti-Stokes electromagnetic wave ($\omega_T^+ = \omega_0 + \omega$) which can readily leave the source region. In contrast, the process $L_0 \rightarrow T^+ + L^- + S$ discussed by Akimoto (1988) generates a downconverted Stokes electromagnetic wave ($\omega_T^- = \omega_0 - \omega^*$) which can easily be absorbed in the source region. The existing theories of type III events treat the processes, $L_0 \rightarrow T^+ \pm S$ and $L_0 \rightarrow L^- \pm S$, as two uncoupled processes (Ginzburg and Zheleznyakov 1958; Robinson, Cairns and Willes 1994; Chian and Abalde 1995). The purpose of this paper is to show that in the presence of a large-amplitude traveling Langmuir wave, the two wave triplets $L_0 + S \rightarrow T^+$ and $L_0 \rightarrow L^- + S$ (or $L_0 \rightarrow T^- + S$ and $L_0 + S \rightarrow L^+$, see Figure 2 of Chian and Abalde 1997) are actually wave triplets coupled to each other, resulting in a hybrid modulational instability, as depicted in

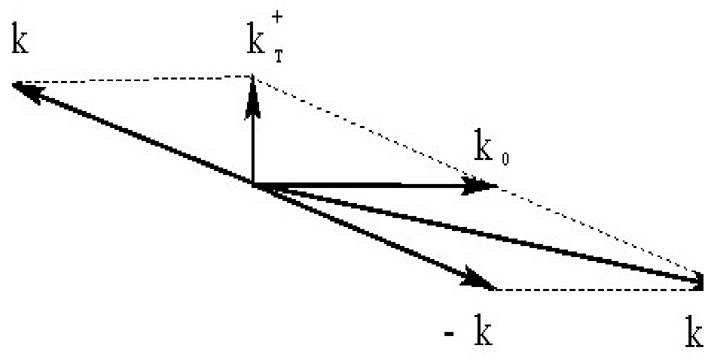

Figure 1 - Geometry of wave-vector matching conditions for the hybrid modulational instability with upconverted anti-Stokes electromagnetic wave.

Figure 1.

DISCUSSION

The nonlinear dispersion relation for the hybrid modulational instability $L_0 \rightarrow T^+ + L^- + S$ can be derived from a Fourier analysis of equations (1) and (2), making use of the phase-matching conditions (3), which yields

$$D_S(\omega, k) = \Lambda\left[1/D_T^+(\omega^+, k_T^+) + 1/D_L^{-*}(\omega^-, k_L^-)\right] \quad , \quad (4)$$

where $\Lambda = e^2 k_s^2 |\mathbf{E}_0|^2/(m_e m_i)$, $D_s(\omega,\mathbf{k})=\omega^2+i\nu_s\,\omega-v_s^2 k^2$, $D_T^+(\omega^+, \mathbf{k}^+)=(\omega_0+\omega)^2+i\nu_T(\omega_0+\omega)-c^2(\mathbf{k}_0+\mathbf{k})^2-w_p^2$, and $D_L^-(\omega^-,\mathbf{k}^{--})=(\omega_0-\omega)^2+i\nu_T(\omega_0-\omega)-\gamma_e v_{th}^2(\mathbf{k}_0-\mathbf{k})^2-w_p^2$. We assume $\mathbf{k}_T$ perpendicular to $\mathbf{k}_0$. Making the resonant approximation for the high frequency electromagnetic and Langmuir waves, equation (4) becomes

$$\omega^2+i\,2\nu_s\omega-\mu\tau k_0^2=\frac{\mu\tau k_0^2 W_0}{4}\left[\left\{\omega+3/2k_0^2-1/2\left(c/v_{th}\right)^2 k_T^2+i\nu_T\right\}^{-1}-\left\{\omega-9/2k_0^2+i\nu_L\right\}^{-1}\right] \quad (5)$$

where $\mu = m_e/m_i$, $\tau=(\gamma_\varepsilon T_e+\gamma_i T_i)/T_e$, $W_0=\varepsilon_0|E_0|^2/(2n_0KT_e)$ is a dimensionless parameter that measures the energy density of the Langmuir pump wave, $\mathbf{E}_0=1/2E_0 \exp i(\mathbf{k}_0.\mathbf{r}-\omega_0 t)$+c.c., $\lambda_D=\varepsilon_0 KT_e(n_0e^2)^{1/2}$ is the Debye length and we have introduced the normalizations $\omega/\omega_\bullet \rightarrow \omega$ and $\mathbf{k}\,\lambda_D \rightarrow \mathbf{k}$. The three-wave electromagnetic fusion instability $L_0 + S \rightarrow T^+$ is obtained from equation (4) by treating the daughter Langmuir wave off-resonant ($D_L^- \neq 0$), giving $D_sD_T^+ = \Lambda$. Under the assumption of resonant ion-acoustic wave ($\omega=v_s\,\mathbf{k}+i\Gamma$, $\Gamma<<v_s\mathbf{k}\equiv w_s$), the threshold is $W_{th}\geq 8\nu_T\nu_S v_S/(\mu\tau\mathbf{k}_0)$ and growth rate $\Gamma=[\mu\tau\mathbf{k}_0W_0/(8v_S)]^{1/2}$. This instability operates when

$$k_0 < \tfrac{2}{3}\left(\mu\tau\right)^{1/2} \quad . \quad (6)$$

The full dispersion relation (5) contains both convective and absolute hybrid modulational instabilities. Under the assumption of purely growing low-frequency ion mode ($\omega=i\Gamma$), the minimum threshold is $W_{th}=4(\nu_T\nu_L)^{1/2}$ which is independent of ν_S, and the growth rate is

$$\Gamma=\left(\frac{k_0}{2^{1/2}}\right)\left\{\left[9\mu\tau W_0+(81/4)\left(k_0^2-\mu\tau\right)^2\right]^{1/2}-\left[(81/4)k_0^2+\mu\tau\right]\right\}^{1/2}, \quad (7)$$

where the relation $\mathbf{k}_T=12^{1/2}v_{th}\mathbf{k}_0/c$ was imposed. This instability operates when

$$k_0 < \tfrac{1}{3}W_0^{1/2} \quad (8)$$

Equation (5) is solved numerically using the typical parameters of interplanetary type III events (Lin et al. 1986) according to Table 1. For a given $\mathbf{k}_0$, we vary $\mathbf{k}_T$ to find the point where the growth rate is maximum. Figures 2 and 3 display this maximum growth rate (Γ) and the corresponding real part of frequency (Re ω) as a function of $\mathbf{k}_0$, for small and large pump wave numbers, respectively. For small $\mathbf{k}_0$ ($\mathbf{k}_0 <(1/3)W_0^{1/2}$), two unstable modes of hybrid modulational instability coexist, as shown in Figure 2. One mode is purely growing (Re ω=0), whereas the other mode is nearly purely growing (Re $\omega\approx 0$). Hence, we call this region the absolute regime. In this regime, both Stokes and anti-Stokes modes are resonant. For large $\mathbf{k}_0$ ($\mathbf{k}_0 > (2/3)(\mu\tau)^{1/2}$). the instability is convective (Re $\omega\neq 0$), as shown in Figure 3. In this regime, the anti-Stokes electromagnetic mode dominates over the Stokes Langmuir mode hence the hybrid modulational instability reduces essentially to the three-wave electromagnetic fusion instability. Note that in contrast to the case of strong Langmuir pump (W_0 = 0.1) considered by Akimoto (1988) whereby the absolute and convective regimes merge into a single wideband unstable region, for typical amplitudes of interplanetary Langmuir waves such as reported by Lin et al. (1986) the absolute and convective regimes decouple from each other, each operates in its own wave number range, as seen in Figures 2 and 3. An analysis of Figures 2 and 3 shows that the ion-acoustic wave is a resonant mode (Re $\omega\approx\omega_S$) only in a restricted region when $\mathbf{k}_0>(2/3)(\mu\tau)^{1/2}$, where the Stokes mode is off-resonant. On the other hand, in the regime where both Stokes and anti-Stokes waves are resonant ($\mathbf{k}_0<(1/3)W_0^{1/2}$) the ion-acoustic wave is a nonresonant mode (Re $\omega\neq\omega_S$). Thus, the hybrid modulational instability under study is difficult to be excited in the stimulated modulational regime discussed in Section 3 of Chian and Abalde (1997). Note, however, that nonlinear four-wave coupling $L \Leftrightarrow T+L+S$ such as discussed in Section 4 of Chian and Abalde (1997) can operate through coupling to resonant ion-acoustic waves evolved nonlinearly from other parametric instabilities or beam-driven ion-acoustic instability.

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Table 1 - List of some typical parameters of two interplanetary type III events provided by Lin et al. (1986). In our paper the frequencies are normalized to the electron plasma frequency (ω_p) and de wave vectors are normalized to the Debye length (λ_D).

Plasma, beam, and wave parameters		
	Values	
Parameters	1979 March 11	1979 February 8
Solar wind plasma:		
Electron temperature, T_e	2 X 10^5 K	1.7 X 10^5 K
Ion temperature, T_i	2 X 10^4 K	6 X 10^4 K
Debye length, $\lambda_\Delta=[(\varepsilon_0 KT_e)/(ne^2)]^{1/2}$ . .	2.2 X 10^3 cm	1.1 X 10^3 cm
Electron plasma frequency f_p	13 kHz	24 kHz
Fast electrons:		
Beam velocity, v_b	~ 3.5 X 10^9 cm s^{-1}	~ 3.5 X 10^9 cm s^{-1}
Langmuir pump waves:		
Beam resonant wave number, k . .	2.3 X 10^{-5} cm^{-1}	4.3 X 10^{-5} cm^{-1}
Maximum normalized energy density, $W_0=\varepsilon_0|E_0|^2/(2n_0KT_e)$	8 X 10^{-7}	6 X 10^{-9}

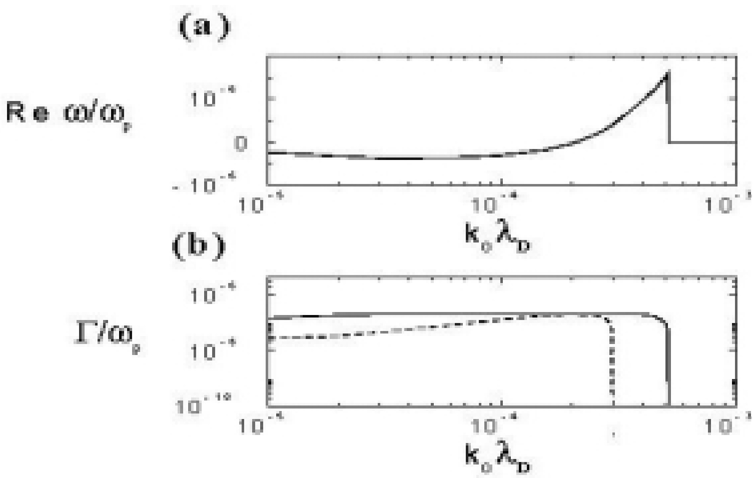

Figure 2 - The real wave frequency (a) and maximum growth rate (b) of the convective (solid line) and absolute (dashed line) hybrid modulational instabilities for *small* $\mathbf{k}_0$.; $W_0=8X10^{-7}$,$T_e=5T_i$ and $v_{th}=1.8X10^6$m/sec.

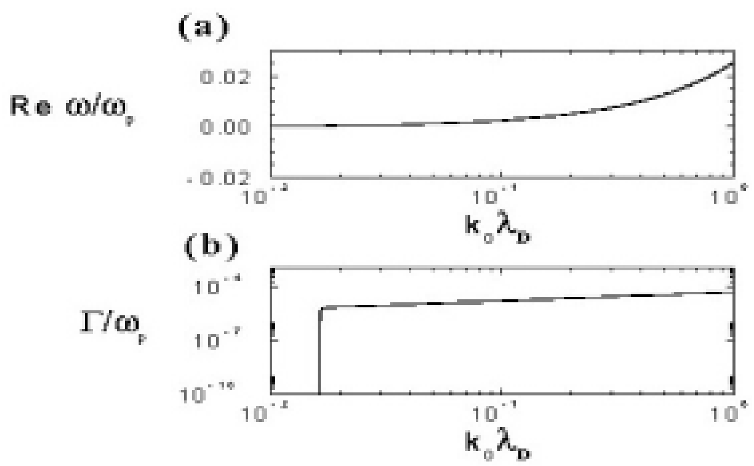

Figure 3 - The real wave frequency (a) and maximum growth rate (b) of the convective (solid line) and absolute (dashed line) hybrid modulational instabilities for *large* $\mathbf{k}_0$.; $W_0=8X10^{-7}$,$T_e=5T_i$ and $v_{th}=1.8X10^6$m/sec.

Beam-driven Langmuir waves have $\mathbf{k}_0\approx\mathbf{k}_b=\omega_p/\mathbf{v}_b$, where $\mathbf{v}_b$ is the electron beam velocity. For the two interplanetary type III events reported by Lin et al. (1986), $\mathbf{k}_b=2.333X10^{-3}$ m^{-1} for 1979 March 11 event and $\mathbf{k}_b=4.308X10^{-3}$ m^{-1} for 1979 February 8 event, respectively. It follows that for both events the condition (6) is satisfied. Hence, the beam-driven Langmuir waves operate in the convective regime. Figure 3 gives Re ω=103,73 rad/sec and Γ=0.923 rad/sec for $\mathbf{k}_0=2.333X10^{-3}$ m^{-1}. The beam-driven Langmuir waves may subsequently cascade to lower wave numbers through successive forward and backward scatterings if $\mathbf{k}_b>>(m_e/m_i)^{1/2}=0.0233$. Alternatively, direct scattering of $\mathbf{k}_b$ to lower wave numbers can occur if $\mathbf{k}_b\geq(m_e/m_i)^{1/2}$ (Chian, Lopes and Alves 1994). As the result of the above scattering processes, Langmuir wave energy is built up in the region $\mathbf{k}_0\approx0$ leading to the formation of Langmuir wave condensate. According to Figure 2, Langmuir waves in the condensate state can emit radio waves via the absolute hybrid modulational instabilities. In the condensate state, nucleated collapse of Langmuir waves can take place; evidence of such phenomenon in the solar wind was seen by Kellogg et al. (1992). In order to compare the four-wave hybrid modulational instability with the three-wave electromagnetic fusion instability, we plot in Figure 4 the growth rate as a function of $\mathbf{k}_T$ for a fixed value of $\mathbf{k}_0$. The parameters were taken from the interplanetary type III event of 1979 March 11 reported by Lin et al. (1986) (see Table 1). Figure 4 shows that the hybrid modulational instability gives a higher growth rate and wider bandwidth than the three-wave electromagnetic fusion instability. It is worth mentioning that despite the difference in the wavevector kinematics and the corresponding nonlinear dispersion relation, the other hybrid modulational instability $L_0 \rightarrow T^- + L^+ + S$ (Akimoto 1988; Chian and Abalde 1997) yields the same results as the process considered in this paper. For simplicity, we discuss only the case where the wave vector $\mathbf{k}_T$ of the transverse electromagnetic wave is orthogonal to the wave vector $\mathbf{k}_0$ of the pump Langmuir wave. The generalization to oblique

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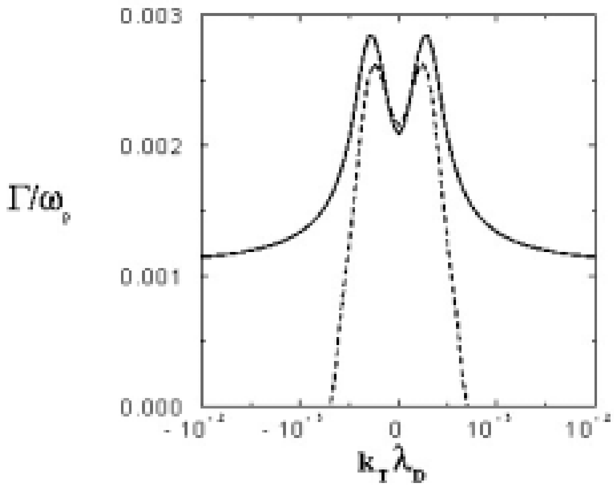

Figure 4 - The real wave frequency (a0 and maximum growth rate (b) of the convective hybrid modulational instability for large k_0; W_0=8X10^{-7}, T_e=5T_i and v_{th}=1.8X10^{6} m/sec and $\mathbf{k}_0\lambda_D$=0.0506.

wavevector geometry can be readily achieved by adding angular factors to the nonlinear dispersion relation (4) (Chian and Abalde 1997). However, the assumption of orthogonal geometry is a reasonable one since it gives the most efficient coupling between the Langmuir pump wave and electromagnetic wave, with $\mathbf{E}_0$ being parallel to $\mathbf{E}_T$.

CONCLUSION

In conclusion, we have shown that the four-wave hybrid modulational instability provides a more accurate description of the fundamental plasma emission of type III solar radio bursts. The nonlinear conversion of Langmuir waves into radio waves occurs via a convective instability for large Langmuir pump wave numbers, and via absolute instabilities for small Langmuir pump wave numbers. For a given Langmuir pump wave energy level, the growth rate of the convective instability is higher than the growth rate of the absolute instabilities. However, in the convective regime the radio waves can propagate out of the source region before reaching sufficiently large amplitudes, whereas in the absolute regime the radio waves can grow to higher amplitudes in the localized source region until nonlinear saturation sets in. Hence, both convective and absolute hybrid modulational instabilities are likely to contribute to the generation of type III solar radio bursts.

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