SBGf03099

A 2D SIMULATION OF THE PROTON RADIATION BELT WITH PELLPACK CODE

A. Gusev[1], [2] I. Martin [1], G. Pugacheva [1] , A. Christy [2], W. Spjeldvik [3]

1 IFGW/DRCC, UNICAMP, Campinas, SP, 13083-970, Brazil . 2 Department of Computer Science, Purdue University,1398 Computer Science Bilding, West Lafayette, Indiana 47907-1398, USA 3 Department of Physics, Weber State University, Ogden, Utah, USA

ABSTRACT

The numerical solution of diffusion equation for geomagnetically trapped protons taking into account deceleration of protons by Coulomb interactions with free and bounded electrons, the charge exchange process, the cosmic ray albedo neutron decay (CRAND) source and electric and magnetic radial diffusion was obtained using the PELLPACK code based on the finite element method. The advantage of the method in comparison with the traditional finite differences method is a several order greater speed of computation at the same precision. When boundary conditions at L=7 are given with the distribution function extracted from proton spectrum obtained on board of ATS 6 satellite, the PELLPACK code produces 2D unidirectional proton flux at the top of geomagnetic lines from L=1 up to L=7 that satisfactory agrees with the AP8 model proton flux for all proton energies more than ~ 300 - 500 keV. For less proton energies AP8 model predicts the trapped proton fluxes on several orders of magnitude greater than PELLPACK code at L < 4 that possibly could be explained by uncertainty of very low energy proton flux data at L=7. The detailed fitness of observational model proton fluxes by numerical theoretical solution of transport equation is still not attained.

INTRODUCTION

Energetic magnetospheric charged particle fluxes are mainly concentrated in the inner region of the Earth's magnetosphere where the drift magnetic shells are permanently closed (L = 1.15 to 6). It has been generally assumed that protons and electrons constitute the predominant part of that population.

The main sources and physical processes of formation of the charged particle population in the magnetosphere are known. We can infer for the sources nuclear interactions of cosmic rays with residual atmosphere and decay of the neutron albedo resulting these interactions, solar flares, and planetary magnetospheric origin. The principal physical processes are particle radial transport and pitch-angle diffusion due to perturbations in large scale electric and magnetic fields, injection, particle energy losses caused by interaction with the wave-particle interactions and Coulomb interactions. Diffusion theory intended to describe charged particle population in the magnetosphere accounts all these processes. The traditional steady radial diffusion version of the theory supposes that the quiet time structure of energetic particle population can be explained as equilibrium balance among adiabatic radial diffusive transport inward magnetosphere from a source located just within the first closed magnetic field lines in the outer region of the magnetosphere (L ~ 7) and the losses described above. Modern version of the theory also introduces inner source of particles presented by cosmic ray albedo neutron decay (*CRAND*). In the diffusion theory a description of the position and velocity of a trapped particle in the belt is equivalent to knowing its three adiabatic invariants: a mechanical moment M, second adiabatic invariant J and the third adiabatic invariant is a magnetic flux Φ through the drift L-shell :

$M = P^2/2mB;\ J = 2P\int\sqrt{(1 - B/B_{mir})}ds;\ \ \Phi = - 1.953/L\ \ Gauss\ R_E^2$

here P - is a particle moment; B, B_{mir} - a magnitude of geomagnetic field in a current point s of magnetic field line and in the mirroring points s', s''.

The diffusion equation may then be written in a form of a elliptic partial differential equation:

$$\frac{\partial f}{\partial t} = L^2\frac{\partial}{\partial L}\left[D_{LL}(L,M)\frac{1}{L^2}\frac{\partial f}{\partial L}\right] - \frac{\partial}{\partial M}\left[\left(\frac{\partial M}{\partial t}\right)_{fric} f\right] - \frac{\partial}{\partial J}\left[\left(\frac{\partial J}{\partial t}\right)_{fric} f\right] - \Lambda f + CRAND$$

Here D_{LL} represents the diffusion coefficient on the variable L; L is the Mac Ilwain parameter corresponding to a particle drift motion around the Earth; $(dM/dt)_{fric}$; dJ/dt_{fric} represent the loss terms, Λf is a term describing charge-exchange losses, i.e. neutralization of proton passing through residual atmosphere. The main feature of diffusion theory is a particle acceleration during cross-field transport: particles that flow inward to the Earth surface are accelerated due to conservation of the particle first M and a second J invariants.

The adiabatic diffusion theory seems to be qualitatively adequate for description of practically all important (including not stationary) phenomena. But in spite of that a comprehensive **quantitative description of the trapped population dynamics is still not created**. The main reason of it is a problem of quantitative description of such complicated and multiparameter system as the geomagnetosphere. Analytical solutions of the diffusion equation only exists for a few simplified stationary cases and rather have a character of estimation and qualitative illustrations and generally are restricted to the equatorial plane with a vanishing the second adiabatic invariant i.e., with an equatorial particle pitch angle of 90° (i.e. two dimensional or 2D task; 3D task concerns fluxes with pitch-angle not equal to 90°. In the same time many numerical solutions were obtained on the basis of the diffusion theory (Nakada and Mead, 1965; Cornwall, 1972; Lyons and Thorn, 1973; Spjeldvik, 1977; Riley and Wolf, 1992). Numerical methods give possibility to obtain more realistic and detailed description.

The recent example of exact numerical solution of 3D task for stationary case of protons was published by Beutier et al., (1995). Here we see also general agreement with the experimental values (Figure 1, (Beutier et al., 1995) but the difference of 1 order of magnitude between computed fluxes and observational fluxes of several MeV protons still remains. Thus, we may state that the parameters of diffusion theory still need to be adjusted to correspond observational data concerning even steady state of radiation belt particle fluxes. There exist stationary empirical models based on averages of satellite data from sixties, particularly those of NASA: AE8 - electron flux model and AP8 - proton flux model (Vette, 1991). The models are based on hundreds satellite experiments, made in the periods of minimum and maximum Solar activity. In this paper we will try to get numerical solution of diffusion equation for main population of radiation belt, - for proton fluxes, accounting Coulomb energy losses in the residual atmosphere, albedo cosmic ray source, using modern values of diffusion coefficients, charge exchange process for neutralization-reionization of protons etc. We will vary the parameters of diffusion equation and will compare the results of numerical solution with the observational model AP-8 proton fluxes.

THE PROTON CONTINUITY EQUATION

In the In phase space the stably state diffusion equation describing the time evolution of the phase space distribution function *f* at the top of geomagnetic field line (2D task, J=0), including Coulomb, charge-exchange losses and CRAND source is given as

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left[D_{LL}(L,M) L^{-2} \frac{\partial f}{\partial L} \right] + G(L) M^{-1/2} \frac{\partial f}{\partial M} - \Lambda f + CRAND = 0$$

Here the term $G(L)M^{-1/2}df/dM$ describes Coulomb energy losses on bound and free electrons. For transport equation with active-variable M, L a space phase distribution function is determined *as* $f = dN/dE/P^2$ here *P* is a particle moment and *dN/dE* is a particle differential spectrum.

Diffusion parameters. Particle transport mechanism across the geomagnetic field lines is driven by fluctuations in geomagnetic field $D_{LL}{}^{m}$ and in large scale convection electric field $D_{LL}{}^{e}$. Generally, it is assumed that $D_{LL}{}^{m} = \mathbf{D_0}^{\mathbf{m}} L^{10}$ and $D_{LL}{}^{e} = K_e L^{10}/(L^4 + M^2)$ where $\mathbf{D_0}^{\mathbf{m}}$ and K_e are parameters which depend on magnetic and electrostatic field fluctuations. In the publications made in 60thies - 70thies (Tverskoy, 1968; Cornwall 1972; Spjeldvik, 1977 and others) $\mathbf{D_0}^{\mathbf{m}}$ and K_e were accepted as $(2 - 5)10^{-15}$ 1/s; $(2 - 5)10^{-10}$ 1/s correspondingly. In the recent publication by Beutier et al., (1995) $\mathbf{D_0}^{\mathbf{m}} = 1.1\ 10^{-13}$ 1/ , i.e. is the 2 orders of magnitude greater than in the previous works. Below we show how particle space L-distribution changes with change of diffusion coefficients.

Coulomb energy losses are caused by proton collisions with free electrons of plasmasphere and with neutral atoms with their following ionization. The Coulomb losses are given by expression (Schulz and Lanzerotti, 1974):

$$\frac{dE}{dt} = -\frac{4\pi e^4}{m_e v}\left[N_i Z_i \left(\beta^2 - Log A_i\right) + N_e\left(\beta^2 - Log C_e\right)\right]$$

where $A_i = 2m_e c^2(\gamma^2-1)/I_i$; $C_e = 4\pi^2 \Lambda_D m_e \beta c/h$; Λ_D- Debye length; h is Plank constant; N_e is the averaged densities of free electrons in plasmasphere, and N_i is the number density of the exospheric gas molecules, each one containing Z_i bound electrons, and $I_i=13Z_i$ eV is the mean excitation energy for the bound electrons. For free electron densities, the function is used (Cornwall, 1972):

$N_e(L) = 250(L/4.1)^{-4.64}$ 1/cm^3 for L < 4.1
$N_e(L) = 13(L/4.1)^{-4.64}$ 1/cm^3 for L > 4.1

In the expression dE/dt there is used plasmasphere temperature model. According to Cornwall (1972) we put T = 5000K for determination of Debay radius in ionization term describing proton interactions with free electrons.

Charge Exchange process. Energetic protons when collide with neutral geocoronal atomic hydrogen strip of its bound electron and become fast non-thermal neutral hydrogen atoms, that are not affected or confined by the geomagnetic field and escape from radiation belt region. The change in phase space distribution function caused by the neutralization process is given by term df/dt = - Λf where $\Lambda = \sigma v N_h$; here v - is a proton velocity; σ is a charge exchange cross section for energetic protons in an atomic hydrogen (Alisson, 1958; Orsini et al,1994); N_h is the neutral hydrogen concentration. This last value depends on L and exospheric temperature and the model of N_h was been taken from Spjeldvik (1977) at the temperature 950K. In the future we will renovate exospheric hydrogen model on the base of modern atmospheric constituent model MSIS1986 (Hedin, 1986) and will study how it influences on solution results.

Boundary conditions. Observational model proton fluxes are determined at L = 1. 0 - 7.0 and in E_{kin} range of 0.1 - 1000 MeV. We accepted L-shell L=7 as a boundary and proton differential spectrum dN/dE at L = 7 in the energy range 0.1 - 1000 MeV derived from AP- 8 is taken for boundary spectrum. We used also for boundary spectrum the spectrum suggested by Spjeldvik (1977), that was modeled from quiet time ion observations on board the geostationary satellite ATS 6 (Fritz et al, 1977). Then, we extend a boundary spectrum down to low energy of 0.05 keV. The solution intervals of L and E are the followings: 1 < L < 7 ; 0.05 keV < E_{kin} < 1000 MeV. The low energy edge used is very low and has no physical sense. It was made to avoid at low L-shells the influence of the edge conditions on the distribution function of the physically important low energy (of about 50 keV) protons because particles are accelerated due to cross transport from L = 7 to L = 1 with an energy increase of about L^3, i.e. 350 times, and the 50 keV proton at L ~ 1 before penetration to this L-shell has energy of 0.15 keV at L = 7. A boundary condition at L = 1 is f(L,M)=0.

Numerical technique used is based on the method of finite elements. In the traditional method of finite differences, a rectangular grid is placed over the domain in order to determine approximations to the solution at each grid point. An algebraic equation is written for each such point that approximates the differential equation locally. The finite element method uses a set of basis functions (Bj(x, y), j=1,..N), and then determines coefficients Cj. The domain is approximated by elements (rectangular, triangular, etc in 2d; tetrahedral, etc in 3d) to define the basis functions. A grid point resolution in L space is 0.05 R_E, and 15 grid points per decade were assigned in M space and consequently in the energy spectra. In the whole L, M space there were used 6252 grid points to get estimations and 138100 grid points to get main results. To get estimations it is necessary 1-2 minutes.

RESULTS AND DISCUSSIONS

In Figure 1 we compare the unidirectional differential proton fluxes for 8 energies (0.1; 0.5; 1; 2.5; 5; 10; 50; 100 MeV) resulting of PELLPACK code and of the AP-8 model in the geomagnetic equatorial plane with the boundary proton spectrum from Spjeldvik(1977). Computed L-distributions with Spjeldvik (1977) boundary spectrum at L = 7 agree very well at all energies greater than 500 keV with the observational AP-8 proton flux model. At the energies less 500 keV computed and observational proton fluxes do not agree and the reason of it possibly could be found in a boundary spectrum at the very low energies. We will study it soon. In the Figure 2 we present results of computed and AP-8 model proton L-distributions (with the Spjeldvik 1977 boundary spectrum) for 10 times greater magnetic radial diffusion coefficient $D_0^m = 10^{-14}$ 1/s and with the same electric diffusion coefficient as in the Figure 1. One can see that computed fluxes with low energies, less than 5 MeV, come to strong disagreement with observational AP-8 fluxes when 10 times great diffusion coefficient is used. In Figure 3 the AP-8 model proton fluxes and the fluxes resulting from PELLPACK code but with the boundary proton flux extracted from AP-8 model at L = 7 are presented. One can see that at the energies of several MeV AP-8 model is not self-consistent from the point of view of theory: computed L-distributions of proton flux with boundary conditions of AP-8 at L=7 satisfactory agree with observational proton L-distributions from AP-8 model only at energies of about 1 MeV. The difference in the flux values reachs several order of the flux magnitude at several MeV energies (see Figure 3).

CONCLUSIONS

In conclusion we would like to underline that undertaken attempt to describe stable magnetospheric proton fluxes with numerical theory is important for 2 reasons: at the present time we need to know more exactly stable conditions of Space Weather in the nearest Earth's space when the era of human colonization of space is coming with the Third Millennium; and for the future predictions of particle dynamics, necessary for human security in space, we need to take possession and to develop the perfect theoretical methods of descriptions both stable and dynamical magnetospheric phenomena.

REFERENCES

Allison, S. K., Experimental results on Charge-Changing Collisions of hydrogen and helium atoms and ions at kinetic energies above 0.2 keV, Rev. Mod. Phys., ***30,*** *1137, 1958.*

Beutier, T., D. Boscher, and M. France, SALAMMBO: A three-dimensional simulation *of* the proton radiation belt, JGR*, 100, 17181-17188, 1995.*

Beutier, T., and D. M. Boscher, A three-dimensional analysis of the electron radiation belt by Salammbo code, J. Geophys. Res.,100*, 14853-14861, 1995.*

*Cornwall, J.M., Radial Diffusion o*f Ionized Helium and Protons: A *Probe for Magnetospheric Dynam*ics, J. Geophys. Res., 77, *A10, 1756, 1972.*

Fritz, T.A., et al, Significant initial results from ATS-6, NASA, Tech Rep., *1977.*

Hedin, A. E., MSIS-86 Thermospheric Model, J. Geophys. Res., 92, 4649-4662, 1987.

Lyons, L.R., and R.M. Thorne, Equilibrium structure of radiation belt electrons, J. Geophys. Res., 78, 2142-2149, 1973.

Nakada, M.P., and G.D. Mead, Diffusion of protons in the outer radiation belt, J. Geophys. Res., 70, 4777 - 4791, 1965.

Orsini, S., I. A. Daglis, M. Candidi, K. C. Hsieh, S. Livi, and B. Wilken, Model calculation of energetic neutral atom precipitation at low altitudes J. Geophys. Res., 99, 13489, 1994.

Riley, P., and R.A. Wolf, Comparison of diffusion and particle drift descriptions of radial transport in the Earth's inner magnetosphere, J. Geophys. Res., 97, 16865 - 16876, 1992.

Schulz , M., and L.J. Lanzerotti, Particle Diffusion in the Radiation Belts, Springer-Verlag, pp.215, New York 1974.

Tverskoy B. A. The Earth's radiation belts , Science, Moscow, (in Russian), 223, 1968.

Vette, J.I., The AE-8 trapped electron model environment, Rep. NSSDC 91-24 , NASA Goddard Space Flight

Center, Greenbelt, Md., November, 1991.

ACKNOWLEDGMENTS.

The work was supported by CNPq (52211/96-3), Fapesp-sp (93/4978-0), Faep of UNICAMP.

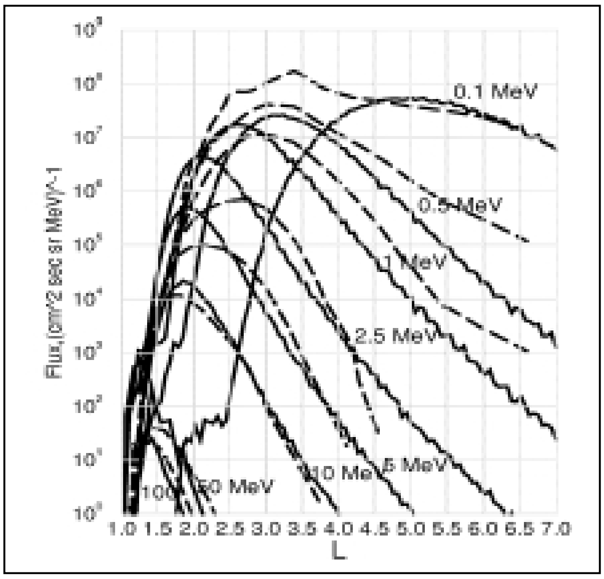

Figure 1. L-distributions of computed (solid) and AP-8 model (dashed) proton fluxes with boundary proton flux from Spjeldvik 1977.

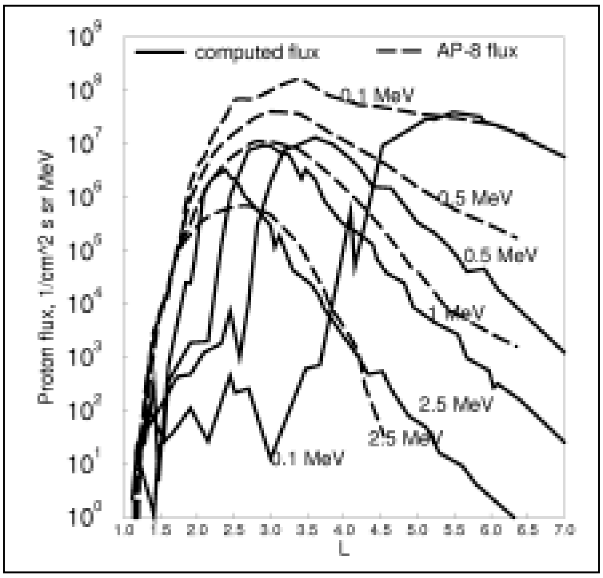

Figure 2. L-distributions of computed proton fluxes with magnetic radial diffusion coefficient $D_0^m = 10^{-14}$ 1/s

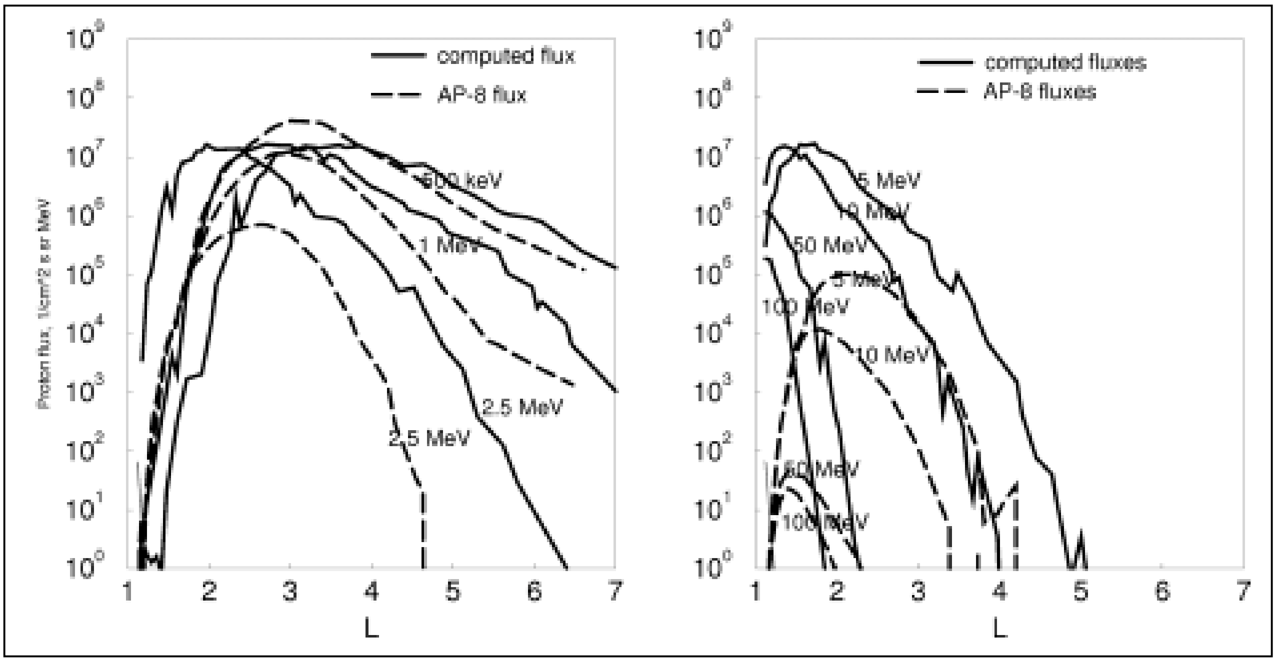

Figure 3. L-distributions of AP-8 model proton fluxes and computed fluxes with the boundary proton flux from AP-8.