A "Multi Source" version of the Reverse Time Migration (RTM)

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Abstract

The conventional 2D/3D Reverse Time Migration (RTM), currently applied in the oil industry, is done in the common shot or receptor domains as established by the Green's function reciprocity theorem. The main reason for using the RTM method is that it applies the full differential wave equation to propagate the stress field. In this way, it is implicit the presence of the exact point Green's functions (despite the well solved problem of numerical dispersion) centered in the shot / receptor stations, according to the current application. The experience with real data shows that such results often have higher quality than those obtained by other methods. Nevertheless, its main limitation is the expensive computational costs, which are proportional to the number of shot / receptor stations. The parallel processing in clusters of LINUX turns viable the use of the RTM method, although the time of processing is still high nowadays. The use of the coarse transverse grid generates problems like "numerical anisotropy" (Alford et al., 1974) (Cunha, 1997). The main objectives of this work is to apply the "Multi Source" version of the RTM by using the special "Multi Source" Green's functions for groups of shots / receptors, and to describe its theoretical principles. This method has a potential to reduce by hundreds / thousands orders of magnitude the processing time, without a great quality loss, even in the presence of noise. A suggestion to this type of strategy can be found in (Faria, 1986). *"These ideas can be applied to any pre-stack depth or time migration method (PSDM, PSTM), but are particularly useful for the RTM"*.

Introduction

The conventional elastic, acoustical, onshore and offshore pre-stack depth and time seismic migrations, in any one of their modalities of implementation (Kirchhoff, RTM, etc), intends to recover the wave field as it was just after the scattering in the geological structures during seismic acquisition. Any migration process can be split in two stages:

1. Back propagation of the wave field recorded on surface receptors in the inverse time sense;
2. Recovering of the field just as it was in the exact moment of scattering (image condition).

Since the seismic acquisition is performed by means of point source / receptors (Airgun, dynamite, etc) it turns necessary to build the Green's function with basis in the velocity field. This allows us to obtain the amplitudes and times of transit among the source / receptor stations and the subsurface scattering points. The PSDM is the main stage of the seismic processing, and is still extremely, expensive even with the recent advances in hardware technology. The goal of this work is to describe a PSDM / PSTM method for decreasing the processing time, by performing simultaneous migration of shot / receptor groups, which are dependent on their respective "Multi Source" Green's functions. The results show that the following sequence:

1. Seismic acquisition by illuminating the earth interior by point sources;
2. Migration of each source by PSDM / PSTM;
3. Staking of these images;

is approximately equal (equal without noise) to:

1. Appropriate common source / receptor stacking of seismograms of adjacent sources / receptors;
2. Migration of these groups of sources / receptors with "Multi Source" Green's functions;
3. Stacking the images of these groups.

"Within the domain of the elastic deformations, the field superposition's principle guaranties the validity of this procedure"

Conventional Reverse Time Migration

The time reversibility principle (invariance of the wave propagation operator by the transformation $t \rightarrow -t$) and the Huygen's principle, allow us to use the wave equation for both the forward modeling and migration. During migration the field recovered in the receptor stations is re-injected at the same place in the inverse time sense. These stations will assume the function of point sources as in the right side of equation-1,

$$\nabla^2 U_{s_j}(\vec{x},t) - \frac{1}{c^2(\vec{x})}\frac{\partial^2 U_{s_j}(\vec{x},t)}{\partial t^2} = \sum_{i=1}^{N_g} S_{s_j}(\vec{x}_{g_i},t)\,\delta(\vec{x}-\vec{x}_{g_i}), \tag{1}$$

where $S_{s_j}(\vec{x}_{g_i},t)$ is the seismogram for a point source; $\vec{x}_{s_j}$ the j^{th} source station; $\vec{x}_{g_i}$ the i^{th} receptor station; N_g the total number of receptor stations and $U_{s_j}(\vec{x},t)$ the back-propagation field corresponding to the j^{th} source station.

Punctual Maximum Field Image Condition (PMFIC).

The image condition $\tau_{s_j}(\vec{x},\vec{x}_{s_j})$ is the time necessary by the maximum field to move from the point source $\vec{x}_{s_j}$ to the grid point $\vec{x}$. The FFD method is particularly suitable to determine the MFIC, see fig-1-A. With only a few code lines it is possible to determine τ_{s_j} and the amplitude correction $A_{s_j}(\vec{x},\vec{x}_{s_j}) = u_{max}(\vec{x})$ to each grid point.However, the seismogram injection must be made at the opposite time sense, beginning with the last sample T_{max} (register time), until the first sample. Consequently the PMFIC for the s_j

station source necessary to freeze and to catch the field must be $\tau_{g_i}(\vec{r}) = T_{max} - \tau_{s_j}(\vec{r})$ which corresponds exactly to the time necessary for the field to move from the receptor station to the grid point (see fig-2-A). As is well known, *migraton* is the association of the back-propagation field in the inverse time sense followed by the image condition. The MFIC is specially useful for complex media, improving the imaging quality, because it is capable to catch the field by using multiple arrivals.

A "Multi Source" RTM

As in the case of the conventional RTM the "Multi Source" version uses the same differential equation-1 where we substitute the punctual seismogram by its "Multi Source" version $\hat{S}(\vec{x}_{g_i}, t)$ (see equation 2):

$$\hat{S}(\vec{x}_{g_i}, t) = \sum_{j}^{N_j} D^t_{1/2}\left[S_{s_j}(\vec{x}_{g_i}, t - T(s_j))\right] , \qquad (2)$$

where the delay $T(s_j)$ is determined by the direction of the slowness vector $\vec{p} = \frac{\vec{t}}{c}$ during the back-propagation (see fig-1-C and equations 3) and $D^t_{1/2}$ is half of the time derivative of the punctual seismogram $\sqrt{iw}$ in frequency domain.

$$\frac{\Delta t}{\Delta x} = \frac{\cos\beta}{c} = \vec{p}\cdot\hat{e}_x = \frac{\vec{K}}{\omega}\cdot\hat{e}_x = \frac{\vec{K_x}}{\omega} . \qquad (3)$$

"Multi Source" Maximum Field Image Condition (MMFIC)

One of the advantages of applying the "Multi Source" *RTM* is due to the fact that the MMIC can be determined without changes in the code of the fig-1-A by using the acoustical wave equation 4. In this equation: $f(t)$ is the source modeling signature; $T(x_{s_j})$ the time delay for the J^{th} source station of the gather $\vec{X}_s = \vec{x}_{s_1}, ..., \vec{x}_{s_{N_s}}$; $\hat{G}(\vec{x}, \vec{X}_s, t)$ the wave field during MMIC determination; N_s the total number of source stations of this source gather.

$$\left(\nabla^2 - \frac{1}{c^2(\vec{x})}\frac{\partial^2}{\partial t^2}\right)\hat{G}(\vec{x}, \vec{X}_s, t) = \sum_{j=1}^{N_s} f(t - T(\vec{x}_{s_j}))\delta(\vec{x} - \vec{x}_{s_j}), \qquad (4)$$

The solution $\hat{G}(\vec{x}, \vec{X}_s, t)$ of the equation-4, for complex geological structures, achieves multiple arrivals image condition. But, by using the code lines of fig-1-A we are catching only the *MMFIC* $\tau^m_{s_j}(\vec{x}, \vec{x}_1, ..., \vec{x}_{N_J})$ and "Multi Source" maximum amplitude $A^m_{s_j}(\vec{x}, \vec{x}_1, ..., \vec{x}_{N_J})$, building a "Multi Source" maximum field *Band limited Green's* function analogous for a group of station sources $\vec{x}_{s_1}, ..., \vec{x}_{s_{N_J}}$ (see Eq-5).

$$\hat{G}^m(\vec{x}, \vec{X}_s, t) = A^m(\vec{x}, \vec{X}_s) \ \ \delta(t - \tau(\vec{x}, \vec{X}_s)) \qquad (5)$$

By doing this we are catching both the "Multi Source" maximum field image condition and the amplitude correction, for a gather of sources (fig-2-B). The "Multi Source" *RTM* will employ the same acoustical equation-1 but, in this case using *MMFIC*, the "Multi Source" seismogram of Eq-2, and catching the migrated field by the code lines of fig-1-B.

Summary and Conclusions

1. Figure 4-A shows the *SEG/EAGE* test model (Aminzadeh et al., 1997).
2. Figure 3-A shows the seismogram resulted from simultaneous shooting of all point sources without time delay followed by *Gaussian* noise addition.
3. Figure 3-B shows the seismogram resulted from the stacking of the 616 point source seismograms (to each one it was previously added the same *Gaussian* noise), with zero time delay. We can observe the high similarity as expected by the principle of field superposition. This is the "Multi Source" seismogram with $p_x\Delta x = 0.0$
4. Figure 3-C shows the "Multi Source" stacking seismogram $p_x\Delta x = 0.00250$
5. Figure 3-D shows the "Multi Source", stacking seismogram $p_x\Delta x = -.00250$
6. Figures 4-B,C and D show the results of three "Multi Source" migrations, which achieve different imaging angles keeping however unchanged the reflectors positions, which is expected by the theory.
7. Figure-4-E shows the image condition for PT300.
8. Figure-4-F shows the "Multi Source" image condition.
9. Figure-4-G is the result of individual *RTM* migration of a 616 punctual source seismograms (to each one was made rigorous *Gaussian* noise addition) followed by the stacking of the respective migrated sections. Time of processing of 4.27 hours.
10. The figure-4-H is the result of the stacking of three "Multi Source" *RTM* migrated sections 4-B,C and D. Time of processing of 75 seconds.
11. We can observe that, in spite of the processing time it is hundreds of times lower, the "Multi Source" image doesn't give inferior quality in the same proportion, showing even an improvement in the shallow part of the model.

References

Alford, R. M., Kelly, K. R., and Boore, D. M., 1974, Accuracy of finite-difference modeling of the acoustic wave equation: Geophysics, **6**, 843–852.

Aminzadeh, F., Brac, J., and Kuns, T., 1997, 3-d salt and overthrust models: SEG/EAGE.

Cunha, P. E. M., 1997, Estratégias eficientes para migração reversa no tempo pré empilhamento 3-D em profundidade pelo método das diferenas finitas: Master's thesis, Universidade Federal da Bahia, Salvador.

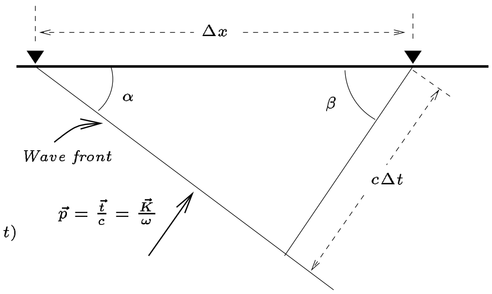
Faria, E. L. d., 1986, Migração antes do empilhamento utilizando propagação reversa no tempo: Master's thesis, Universidade Federal da Bahia, Salvador.

For All: $(\vec{x}, t)$
Propagate: $U(\vec{x}, t)$
if
$U(\vec{x}, t) > U_{max}(\vec{x}, t)$
then
$U_{max}(\vec{x}, t) = U(\vec{x}, t)$
$\tau(\vec{x}) = t$
end if
End For: $(\vec{x}, t)$ e t
$A(\vec{x}) = U_{max}(\vec{x}, t)$

A: *RTM* image condition.

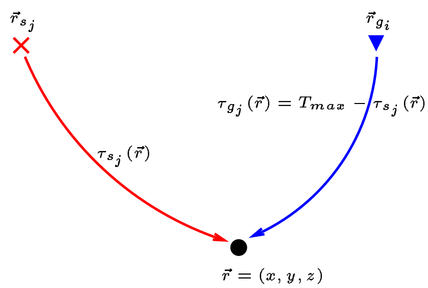
For All: $(\vec{x}, t)$
Propagate: $U(\vec{x}, t)$
if
$t = \tau(\vec{x})$
then
$U_{mig}(\vec{x}, t) = U(\vec{x}, t)/A(\vec{x}, t)$
end if
End For: $(\vec{x}$ e t

B: *RTM* migration.

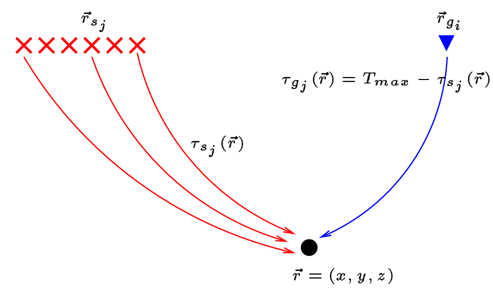


C: "Multi Source" *RTM* image condition.

Figure 1:

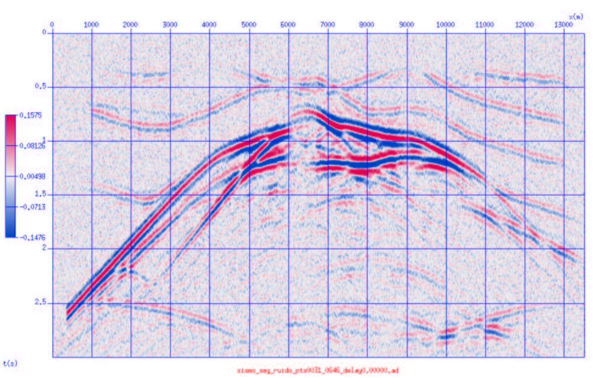


A: Punctual image condition.

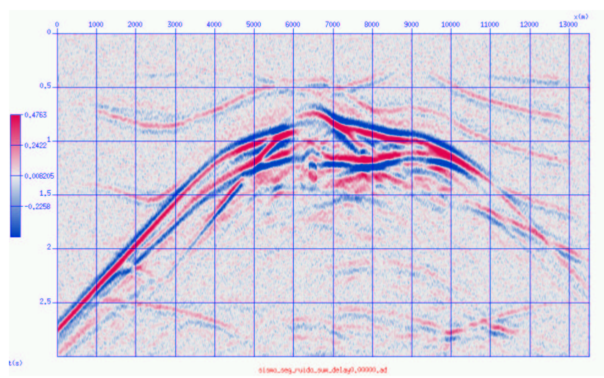


B: "Multi Source" image condition.

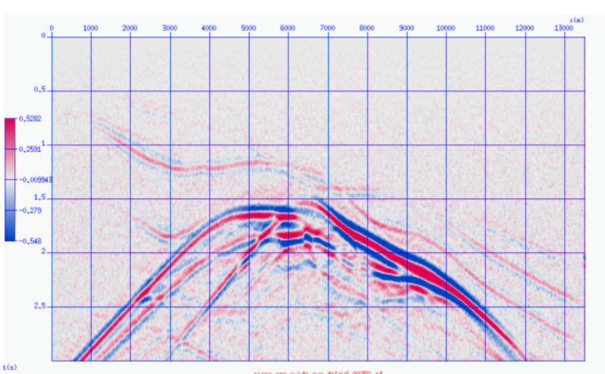
Figure 2:



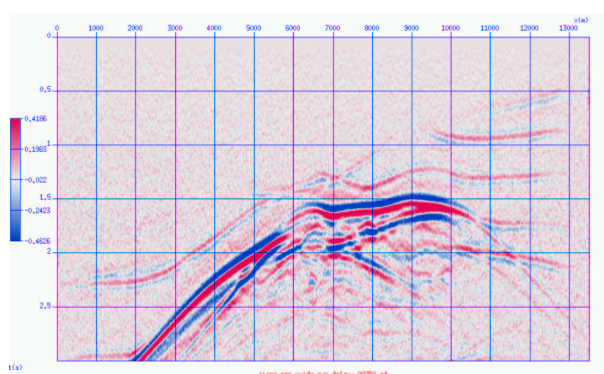
A: One shot synthetic seismogram plus noise



B: "Multi Source" seismogram, $p_x \Delta x = 0.0$

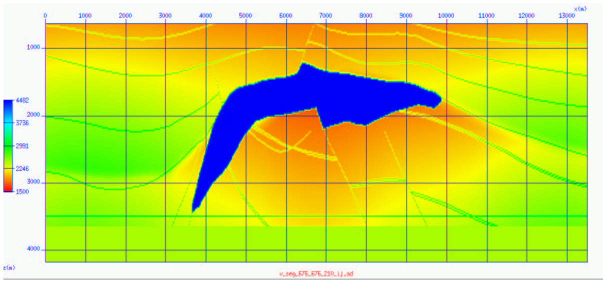


C: "Multi Source" stacking seismogram, $p_x \Delta x = 0.00250$

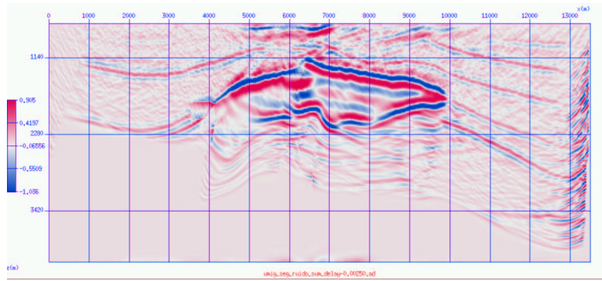


D: "Multi Source" stacking seismogram, $p_x \Delta x = -0.00250$

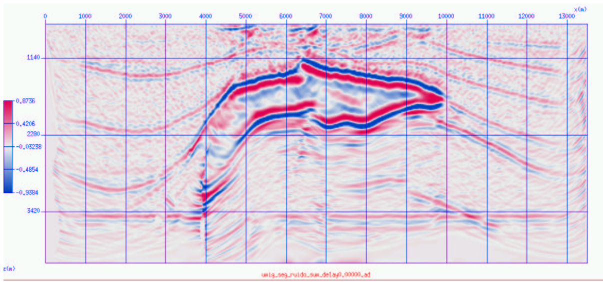
Figure 3:



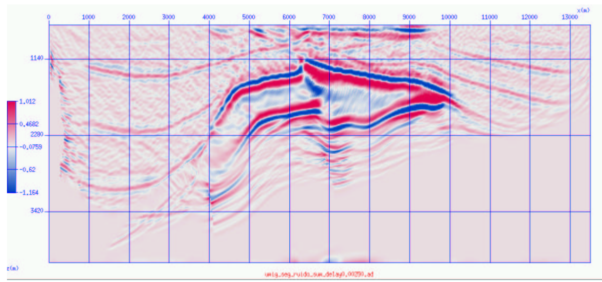
A: *SEG/EAGE* model



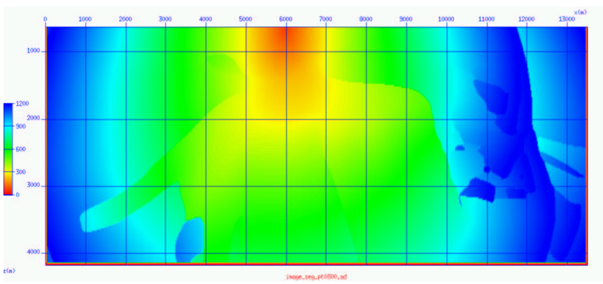
B: “Multi Source”, $p_x \Delta x = -.00250$



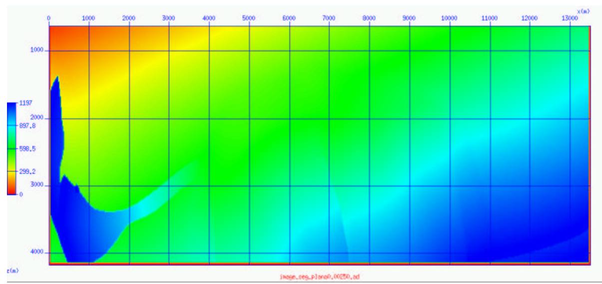
C: “Multi Source”, $p_x \Delta x = 0.0$



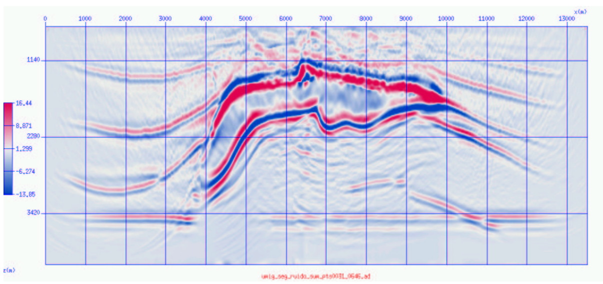
D: “Multi Source”, $p_x \Delta x = 0.00250$



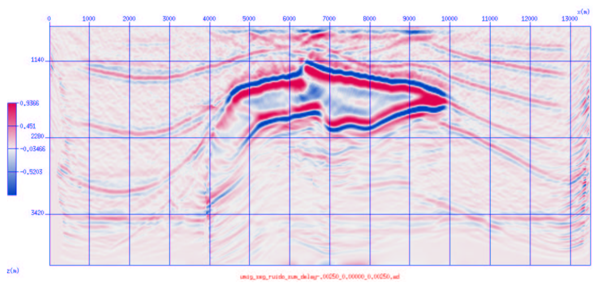
E: Punctual source image condition PT300



F: “Multi Source” image condition



G: Conventional RTM, t=4.27 hours.



H: “Multi Source” RTM, t=75 seconds

Figure 4: