Numerical Simulation of the Rayleigh Window at the Ocean Bottom

J.M. Carcione*, Istituto Nazionale di Oceanografia e di Geofisica Sperimentale (OGS), Trieste, Italy
H.B. Helle, Norsk Hydro ASA, E&P Research Center, Bergen, Norway
M.A.B. Botelho, Univesidade Federal da Bahia, Salvador, Brazil.

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Abstract

The Rayleigh window is a minimum in the P-wave reflection coefficient of fluid-solid interfaces, such as the ocean bottom. The phenomenon, which occurs beyond the P-wave critical angle, has been observed in the laboratory, and can be explained by using an anelastic model of the solid. For the purpose of simulating the Rayleigh window, we further develop and test the pseudospectral modeling algorithm for wave propagation at fluid-anelastic solid interfaces. The method is based on a domain-decomposition technique - one grid for the fluid part and another grid for the solid part - and the Fourier and Chebyshev differential operators. A wavenumber-frequency domain AVA method is used to compute the reflection coefficient and phase angle from the synthetic seismograms. This is the first numerical simulation of the Rayleigh window.

Introduction

The reflection coefficient of a water-stainless steel interface was measured experimentally by Becker and Richardson (1970). Their ultrasonic experiments were verified with an anelastic model in a later paper (Becker and Richardson, 1972), showing in particular that the Rayleigh window cannot be predicted by using reflection coefficients based on the elasticity theory (Brekhovskikh, 1960, p. 34; Carcione, 2001, p. 214). Borcherdt et al. (1986) present theoretical results for the ocean bottom, where the Rayleigh window also occurs. This viscoelastic effect implies that the energy incident on the boundary at angles within that window is substantially transmitted.

The problem of reflection, refraction and propagation at a plane boundary separating an acoustic medium (fluid) and a viscoelastic solid has practical application in seismic exploration, seismology, foundation engineering and non-destructive testing of materials. In seismic exploration, the relevant fluid-solid interface is the ocean bottom, whose properties are useful for data processing of multi-component seismic surveys acquired at the seafloor. Knowledge of S-wave velocities is required for static corrections and imaging of mode-converted PS-waves. Shear velocity is also important for multiple removal. Thus, the relevance of investigating the reflection and transmission properties of the ocean bottom.

The explicit modeling of the fluid-solid boundary condition is done by using domain decomposition and pseudospectral methods. The Fourier method is used along the interface direction and the Chebyshev method is used along the direction perpendicular to the interface. The approach for viscoelastic waves is illustrated in Carcione (1991, 1994). Modeling examples are given in Kessler and Kosloff (1991), Tessmer et al. (1992) and Carcione (1996) for elastic media. To our knowledge, the Rayleigh window has not been simulated with direct grid methods (see Carcione (2001) for a brief description of these methods).

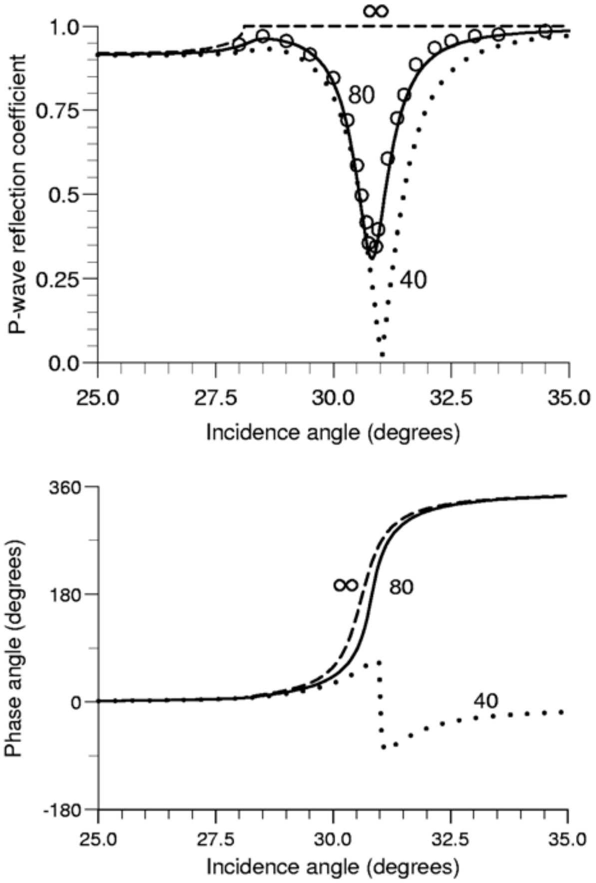


Figure 1 - Water-stainless steel interface. Absolute value of the P-wave reflection coefficient and phase versus incidence angle for different values of the shear-wave quality factor. The circles are the experimental data obtained by Becker and Richardson (1970).

Physics of wave propagation

The geophysical problem has been investigated by Borcherdt et al. (1986), who found that the Rayleigh window should be observable in appropriate sets of wide-angle reflection data and that can be useful in estimating attenuation for various ocean-bottom reflectors. The reduction in amplitude occurs for angles of incidence near the so-called Rayleigh critical angle, when the apparent velocity for the incident wave is near to that of a Rayleigh surface wave.

Let us consider the water-stainless steel interface. The compressional and shear velocities of steel are 5740 m/s and 3142 m/s, respectively, and the density is 7932 kg/m^3. The P- and S-wave quality factors at 10 MHz are 140 and 80, respectively. Figure 1 represents the P-wave reflection coefficient and phase. The amplitude reaches zero for a quality factor of 44, and below this value there is a phase reversal. Figure 2 shows the window for the oceanic crust, which has P- and S-wave velocities of 4850 m/s and 2800 m/s, respectively, and a density of 2600 kg/m^3. The P-wave quality factor is 1000 (although this value has practically no influence). Different values of the S-wave quality factor are indicated. The value of least reflection is slightly higher than 10. For this value, there is a phase reversal. Simulations, corresponding to this case, are presented below.

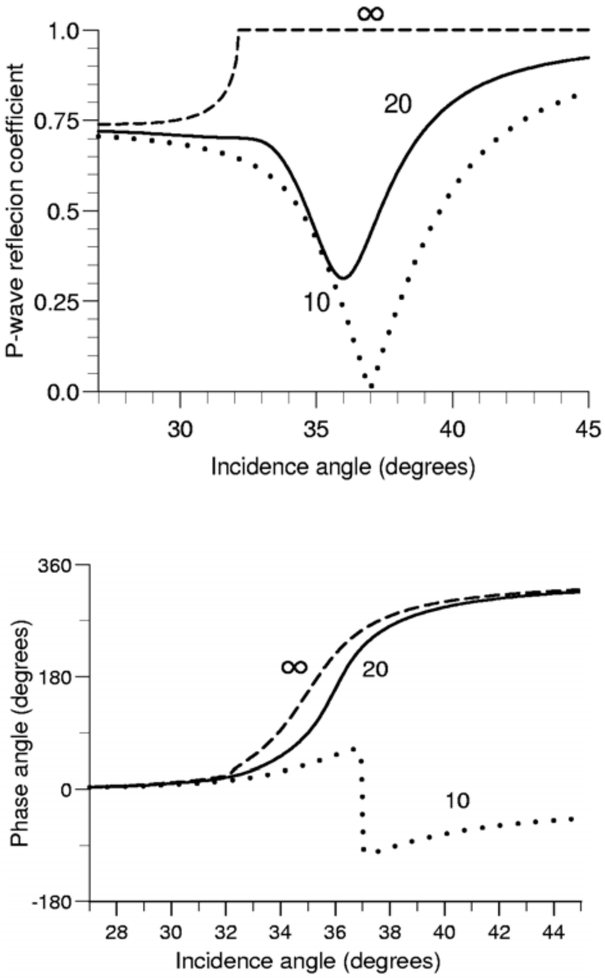


Figure 2 - Ocean-crust interface. Absolute value of the P-wave reflection coefficient and phase versus incidence angle for different values of the shear-wave quality factor.

The main factors affecting the window are the S-wave attenuation and the shear velocity, which controls its angular location. This is shown in Figure 3, which illustrates the elastic (a) and anelastic (b) reflection coefficients for S-wave velocities of 2800, 2300, 1900 and 1500 m/s (from left to right) and an S-wave quality factor equal to 10. The medium is a Poisson solid and the density equals $312\,V_P^{0.25}$, where V_P is the P-wave velocity in m/s. Note how the window moves to the right when going from stiff to soft ocean bottoms (see Figure 3b). When the S-wave velocity equals the sound velocity of water, the window disappears.

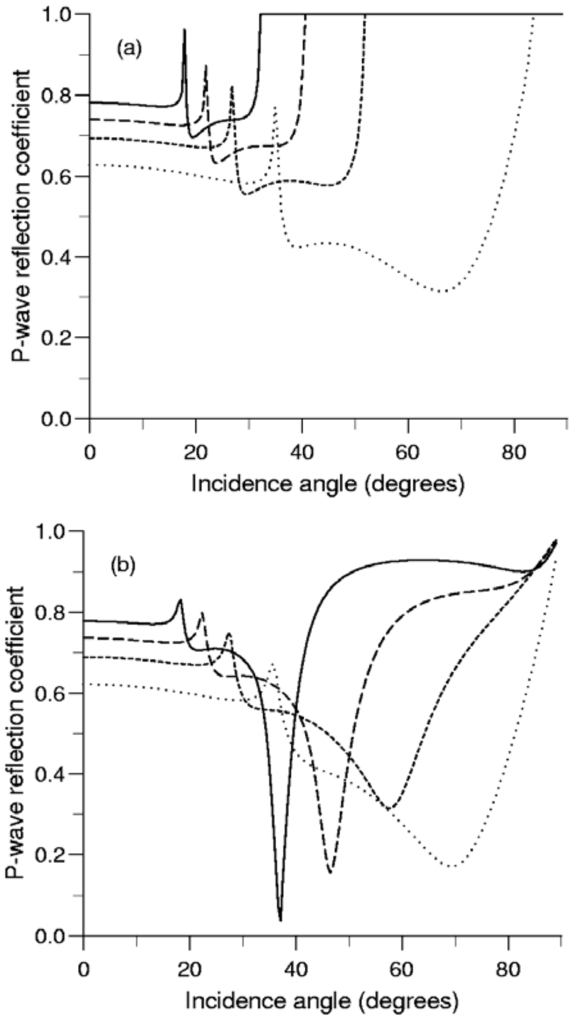


Figure 3 - Absolute value of the P-wave reflection coefficient and phase versus incidence angle for different values of the shear-wave velocity in elastic (a) and anelastic (b) ocean bottoms.

Modeling algorithm

The time-domain equations for propagation in a heterogeneous viscoelastic medium can be found in Carcione (2001, p. 110). The anelasticity is described by the standard linear solid, also called Zener's model, that gives relaxation and creep functions in agreement with experimental results. We solve the two-dimensional velocity-stress equations for anelastic propagation, by using memory-variable differential equations.

Two grids model the fluid and solid subdomains. The solution on each grid is obtained by using the Runge-Kutta method as time stepping algorithm and the Fourier and Chebyshev differential operators to compute the spatial derivatives in the horizontal and vertical directions, respectively (Carcione, 2001). In order to combine the two grids, the wave field is decomposed into incoming and outgoing wave modes at the interface between the solid and the fluid. The inward propagating waves depend on the solution exterior to the subdomains and therefore are computed from the boundary conditions, while the behavior of the outward propagating waves is determined by the solution inside the subdomain.

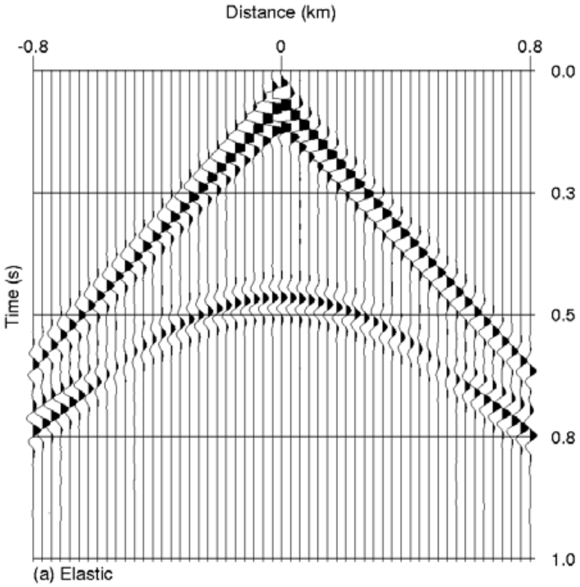
AVA algorithm

In order to obtain the reflection coefficient from the synthetic seismograms, we use an AVA (amplitude variation with angle) method developed by Kindelan et al. (1989) for elastic media. It consists of the following:

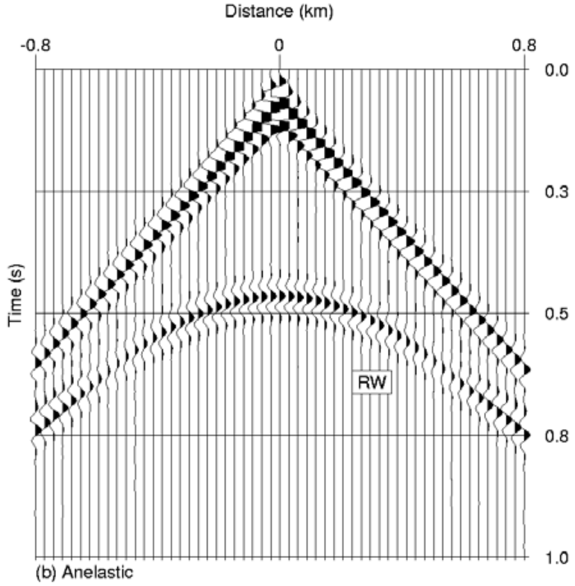
1. Generate a synthetic seismogram of the pressure field by using a dilatational point source in water. Place a line of receivers at each grid point above the interface. This record contains the incident and reflected fields.
2. Compute the synthetic seismogram without interface (without ocean bottom) at the same location. This seismogram contains the incident field only.
3. Perform the difference between the first and second seismograms. The difference contains the reflected field only.
4. Perform frequency-wavenumber transforms of the incident and reflected fields and their ratio to obtain the reflection coefficients and phase angle.

Simulation

We consider a stiff ocean bottom, whose reflection coefficient is the dotted line in Figure 2. Receivers (hydrophones) are located 1.3 m above the ocean bottom. Figure 4 shows synthetic seismograms in the space-time domain. It is difficult to observe the Rayleigh-window phenomenon, since the reflected pulse is probably masked by the head wave, because the window is located beyond the critical angle. The visualization requires an accurate AVA analysis, such as that described in the previous section. Figure 5 shows the numerical evaluation of the P-wave reflection coefficient (a) and phase angle (b) for the oceanic crust using the AVA algorithm. As can be seen, the modeling algorithm correctly simulates the Rayleigh window, i.e., the magnitude of the reflection coefficient and phase-change slope. The mismatch between theory and numerical experiments is due to the fact that the receivers are located at 1.3 m above the interface. Then, there is a phase shift between the incident wave and the reflected wave. The results, i.e., the perfect agreement between analytical and numerical results, constitute a further confirmation of the correctness of the modeling method. To our knowledge, this is the first simulation of this phenomenon. Inspection of data reported by Stoffa et al. (1992) may suggest amplitude variation associated with



(a) Elastic



(b) Anelastic

Figure 4 - Pressure seismograms for the elastic (a) and anelastic (b) ocean bottoms (the source and receivers are located at 300 m above the bottom). The two events are the direct and reflected wavefields, and the symbol RW indicates the location of the window. The reduction in amplitude cannot be seen in (b) when compared with the elastic seismogram. Therefore, the analysis requires an accurate AVO analysis of the reflection event.

the Rayleigh window. However, gain adjustment and lateral velocity variations may complicate the analysis. Better true-amplitude data is therefore required for the observation of the window in geophysical surveys.

Conclusions

We have simulated the Rayleigh-window phenomenon with a direct grid method. This effect can be used to obtain information about the shear-wave velocity and quality factor of the ocean bottom from real data. The AVA analysis used to obtain the reflection amplitudes can also be used to process ocean-bottom cable data. The analysis has required the improvement of the pseudospectral method to model the interface boundary condition between the ocean and the sea-bottom sediments (or ocean crust). The new features of the modeling method involve the inclusion of viscoelastic dissipation and the use of a domain decomposition technique. The modeling allows for the presence of the sea surface and general material variability along the vertical and horizontal directions. It can be used to investigate the propagation of Scholte and leaky Rayleigh waves, and generate realistic seismograms for various applications (Carcione and Helle, 2002).

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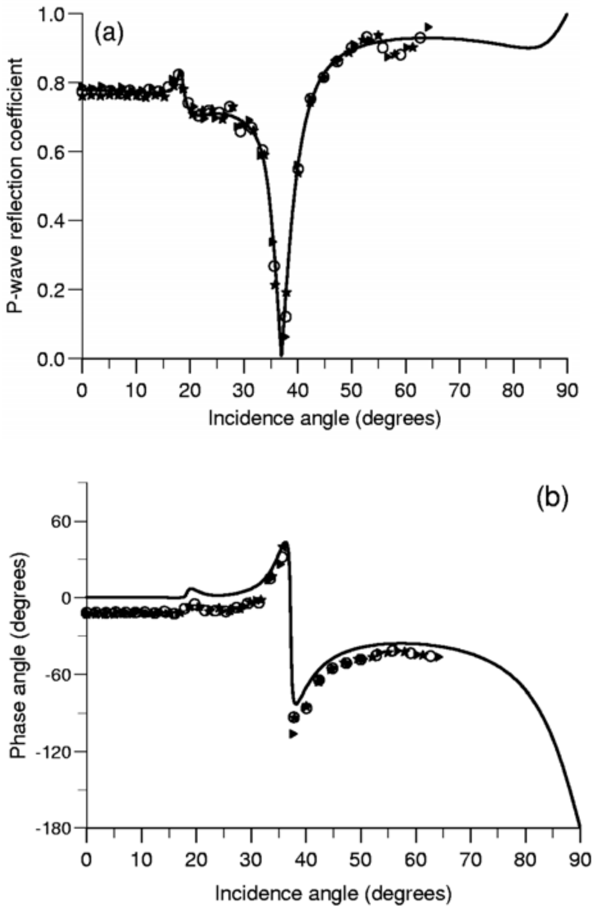

Figure 5 - The Rayleigh window at the ocean-crust interface ($Q_S = 10$ (dotted line in Figure 2)). P-wave reflection coefficient (a) and phase angle (b) versus incidence angle. The symbols correspond to the numerical evaluation of the AVA response at 18, 19 and 20 Hz.