Common Opening Angle Migration/Inversion by Summation Over the Acquisition Surface

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Abstract

We show that the technique of migration/inversion (m/i) as an integral over dip angles at an image point at depth can be derived by the standard Kirchhoff m/i technique. In this method, the opening angle between rays from source and receiver is constrained to make a fixed angle at the image point. Thus, this is a common opening angle migration/inversion, in contrast to the more standard common-offset or common-shot methods. This method has the advantage of dealing more routinely with multipathing of rays than the latter two methods do. However, integration over dip angle can have some computational disadvantages. We overcome these by recasting the formula as an integral over all sources and receivers, and migrating in parallel to generate a suite of common-opening angle gathers. We show that the transformation of coordinates from image point to surface points leads to an additional amplitude weight involving only geometrical factors and ray-generated factors already required for true amplitude processing. Previous derivations of this method use the theory of generalized Radon theory and pseudo-differential operators/Fourier integral operators, or they use least squares methodology. Thus, our results are not new—although some of the formulas are. Our derivation here arises from more classical Kirchhoff modeling and inversion theory, hopefully more accessible to the geophysical community. Further, it demonstrates that this m/i method is an alternative Kirchhoff method for a sorting of the data that is different from common-shot or common-offset. Integrating over dip angle at depth requires easy access to all traces, since the required traces are defined by ray tracing from the image point. Integrating over all sources and receivers eliminates that difficulty. The new expressions for the necessary scale factors introduced by this transformation make that process more feasible.

Introduction

Migration/inversion (m/i) over dip angles at an image point for a fixed opening angle between source and receiver rays can be derived by the classical technique for Kirchhoff inversion. "Migration-dip angle" is the direction of the gradient of the total travel time function from source and receiver points to the image point in the background wave speed medium. The method treats multipathing of rays between surface and depth points in a more natural manner than do standard Kirchhoff common-shot or common-offset methods. This inversion in fixed opening angle is initially stated as an integral over migration-dip angles at an image point, in contrast to the integration over the surface source and receiver coordinates in the aforementioned methods. Xu et al. [2001] demonstrate, alternatively, that one can also carry out the integration over source and receiver surface coordinates with a windowing criterion that fixes the opening angle of the rays at the output point. That approach introduces an additional Jacobian of transformation connecting coordinates at the image point with the coordinates at the source and receiver point. We present new results here about that Jacobian in 2D and 3D. We claim that these new results lead to computational advantages in carrying out the processing in this manner.

Integrating over dip angle requires that the data be sorted by common opening angle between the associated rays to source and receiver locations. Figure 1 shows the relevant variables in 2D. In this figure, the unit vector $\hat{\nu}$ points in the direction of the gradient of the sum of travel times from source and receiver; ϕ denotes the angle that $\hat{\nu}$ makes with the vertical and defines the dip direction. Thus, initially, we are proposing an integration over dip angles ϕ in our Kirchhoff inversion. The angle between $\hat{\alpha}_r$ or $\hat{\alpha}_s$ and the dip vector is denoted by θ , in which case 2θ is the opening angle between the rays. This angle remains fixed in our m/i scheme.

Since opening angle at depth does not readily translate into offset at the surface, the method requires access to essentially all of common-shot or common-offset panels for each output summation. Furthermore, there are issues of adequate sampling; uniform steps in a sum over subsurface dip angle do not translate into uniform steps in source/receiver coordinates. The proposed transformation to a sum over source and receivers addresses both of these problems.

On the other hand, m/i, as an integral over dip angles, offers the following advantages over common-shot or common-offset m/i:

1. The output is a reflectivity map at known (constant) specular angle 2θ, since that angle is an input variable for each computation. For the output, this method replaces more familiar common-offset output panels, such as in Figure 2, with common opening angle output panels, such as in Figure 3.

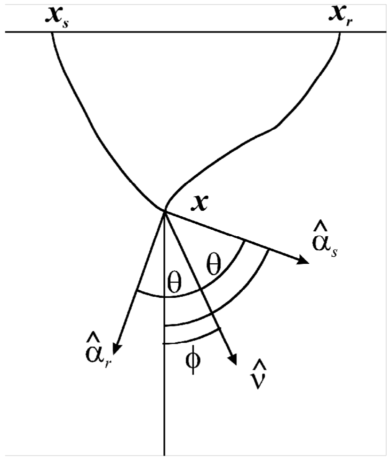

Figure 1: Angle definitions for common opening angle Kirchhoff migration-inversion carried out as an integration over dip angles. $\hat{v}$, the unit vector in the direction of the sum of the gradients of the travel time; this vector defines the migration dip direction. ϕ is the dip, the angle of $\hat{v}$ with respect to the vertical. $\hat{\alpha}_s$ is a unit vector in the direction of the ray from the source x_s to the output point x, and $\hat{\alpha}_r$ is a unit vector in the direction of the ray from the receiver x_r to the output point x. θ is the common opening angle between $\hat{v}$ and $\hat{\alpha}_s$ and also the angle between $\hat{v}$ and $\hat{\alpha}_r$. Finally, $\alpha_s = \phi + \theta$, $\alpha_r = \phi - \theta$ are the angles that $\hat{\alpha}_r$ and $\hat{\alpha}_s$ make with the vertical.

2. The peak amplitude at an image location is proportional to the reflection coefficient at that opening angle, with a known constant scale factor. Thus, an amplitude-versus-angle (AVA) plot is easily computed from the peak amplitudes.
3. If, in addition, travel times and other ray quantities are computed from points at depth rather than from points at the upper surface, the Beylkin determinant—an important factor in the inversion process—is constant. In particular, the intense computation of the 3D common-offset Beylkin determinant is avoided in this approach. On the other hand, we show how integration over dip can be transformed back into a sum over sources and receivers at the upper surface. This transformation, then, relates our results back to those of Xu et al. [2001].

De Hoop et al. [1994] originally proposed m/i in dip angle, with follow-on discussions by de Hoop [1998] and de Hoop and Brandsberg-Dahl [2000]. The presentation of de Hoop and Brandsberg-Dahl [2000] requires knowledge of generalized Radon transforms and an understanding of pseudo-differential/Fourier integral operators. Further, they present formulas for only the most general anisotropic elastic cases, from which it is not a trivial matter to produce simpler results.

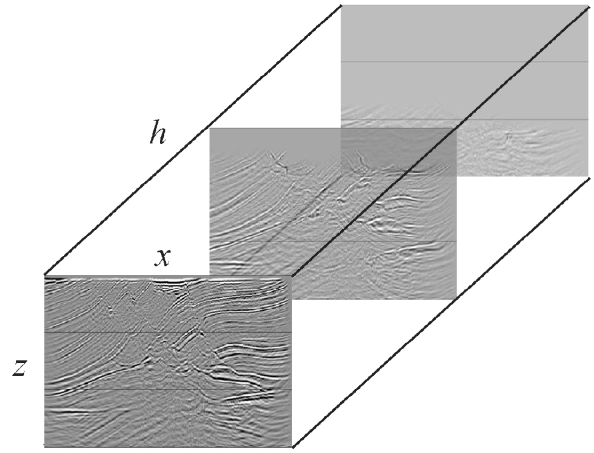

Figure 2: A 2D depiction of common-offset panels. Offset increases from front to back.

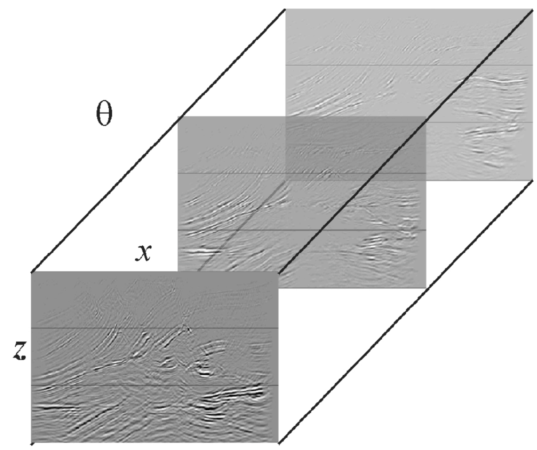

Figure 3: A 2D depiction of common-opening angle panels. Opening angle increases from front to back.

Xu et al. [2001] derived an acoustic m/i formula using a least-squares approach; the type of processing we derive here is their first iterate. Their results are presented in 2D and they state the formula for dip angle inversion as a sum over sources and receivers. The corresponding formula for summation over dip angle requires just a one-line derivation to deduce this formula from their equations (33-35). Their equation (33) is a formula for model-perturbation update as an integration over dip angle; (34) is a formula for model perturbation (percentage difference of, say, velocity) as an integration over all sources and receivers; (35) is a formula for reflectivity (reflection coefficient times a delta function as defined in our Kirchhoff inversion formulation) as an integral over sources and receivers; our result is a formula for reflectivity as an integral over dip angle. In the integrals over sources and receivers, Xu et al. [2001] use a window function to restrict the source-receiver pairs to those having approximately the right opening angle at the output point. (We use this trick, as well, to derive the formulas for summation over sources and receivers from the formulas for summation over dip angle.) Xu et al. [2001] also provide numerical examples in 2D that demonstrate the value of using all arrivals in the presence of multipath ray trajectories. Their images were generated from the formula for model perturbation as an integral over sources and receivers. We expect similar quality of output for the reflectivity function.

There are two new results here. The first is that common-opening angle inversion as an integral in dip angle can be derived via the more classical Kirchhoff inversion technique as described in Bleistein et al. [2001]. We believe that this derivation is more accessible to a geophysical readership, and further demonstrates that this is just a natural extension of the more familiar Kirchhoff method. This approach places m/i as an integral over dip angles in the class of Kirchhoff migration formulas. Furthermore, the explicit formulas for the 3D and 2.5D acoustic cases is also new. Detailed derivations will be provided in a paper now in preparation.

The second new result is the simplifications of the 2D and 3D Jacobians that arise in connection with the mapping between parameters at the image point and their values at source and receiver points. We believe that these simplifications overcome many of the drawbacks of integration over dip angle at the image point. Furthermore, in 3D, where computation of the Beylkin determinant for common-offset data is computer intensive, the computations here are much simpler. Thus, we believe that, without the extra raytracing overhead, migrating in parallel to obtain a suite of common-opening angle gathers can actually be faster than migrating in parallel to obtain a suite of common-offset gathers.

Other examples of computer output for examples of common-opening angle inversion can be found in Brandsberg-Dahl [2001], Koren and Kosloff [2001], Koren et al. [2002] as well as Xu et al. [2001]. Note, however, that only in Xu et al. [2001] is the computation carried out as a sum over all sources and receivers. It is this type of processing (common opening angle inversion by summing over sources and receivers) that is made simpler by the new formulas for Jacobians connecting image point coordinates and source/receiver coordinates.

When multiple arrivals from a single subsurface location occur, common-shot or common-offset m/i typically picks up only one of these to accumulate into the sums that produce the output---perhaps the first arrival or the most energetic arrival. Picking up the contributions from multiple specular returns requires extra complexity in the migration program. This results in a migration which, at a given image location, is a sum over two or more specular terms wherever such multiple events occur, making amplitude studies problematic.

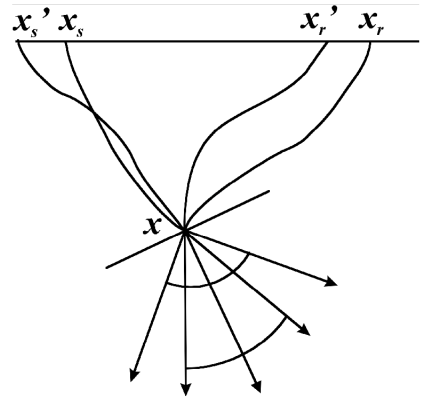

Figure 4: A dipping reflector with two sets of specular rays from the source-receiver pairs, x_s, x_r and x_s', x_r', respectively. They have the same offset, but different opening angles at depth.

Figure 4 shows two specular ray pairs for a point x on a dipping reflector and for source-receiver pairs, x_s, x_r and x_s', x_r', respectively. Furthermore, these two ray pairs ~~of rays~~ have the same offset between source and receiver, but different opening angles at x. The opening angles are indicated by the two arcs connecting the directions of the source and receiver rays for each pair at the specular point. A common-offset migration or inversion that uses multiple passes to pick up all returns will add the two outputs at the point x.

Even when a single specular return produces an image at a point, the opening angle of the specular rays producing the output is typically unknown *a priori*. Thus, amplitude behavior must be resolved by further processing and analysis. In one approach [Bleistein, et al. 2001], two inversion formulas are processed; the quotient of the outputs at peak amplitude estimates the opening angle between the specular rays and, hence, the angle of the angularly-dependent reflection coefficient. Although the different formulas for these inversions are minimal, it is necessary to retain two migrations in memory, pick peak amplitudes and take a quotient to estimate the opening angle.

Common-opening angle m/i, carried out as a sum over all possible dip angles, provides a natural approach for dealing with these shortcomings. For the example of Figure 4, the migration will pick up both of these specular returns, but on different common-opening angle panels. Hence, no summing of contributions from the two different specular arrivals occurs. This improved ap-proach comes with an extra cost, as we describe below.

In our implementation, we shoot rays from the image locations at depth, opposite to the directions of $\hat{\alpha}_s$ and $\hat{\alpha}_r$ of Figure 1. Their arrival points at the upper surface are the source and receiver locations, respectively. In 2D, these locations define the midpoint and offset of a recorded trace. For each image point and half-opening angle θ, we must have access to traces in the range of offsets and midpoints defined by these arrivals. The same type of calculation is necessary for the Jacobians that arise in computing the Green's function. The very act of shooting rays from depth makes the computation of ray trajectories, travel times and amplitudes less efficient than procedures that shoot rays down from the upper surface. This is a significant shortcoming of common-opening angle m/i.

3D migration is further complicated by the orientation or azimuth of the basic triad of directions, $\hat{\nu}$, $\hat{\alpha}_s$, and $\hat{\alpha}_r$, as shown in Figure 5.

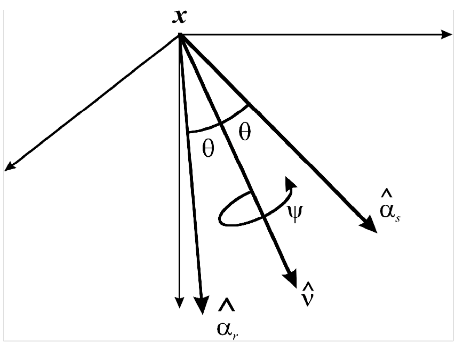

Figure 5: Coordinates for migration in dip angle characterized by dip direction $\hat{\nu}$, in azimuthal angle ψ, and at fixed opening angle θ between rays to source and receiver. All are referenced to the output point $\boldsymbol{x}$. $\hat{\alpha}_s$, and $\hat{\alpha}_r$ are unit vectors in the directions of the rays from source and receiver, respectively. $\hat{\nu}$ $\hat{\alpha}_s$, and $\hat{\alpha}_r$ are in the same plane, so that $\hat{\alpha}_s$, and $\hat{\alpha}_r$, spin around $\hat{\nu}$ as ψ varies. Note that two angles are required to define the direction of $\hat{\nu}$.

If we think of shooting the rays back to the acquisition surface, their arrivals there are governed by the properties of the wave speed in the overburden. As in 2D, their locations define a source/receiver pair, that is, an input trace location. (If this is not the case, we must modify the theory to accommodate this deficiency in acquisition, for example by performing a least-squares m/i.) The vector $\hat{\nu}$ ranges over a unit hemisphere centered at the image point. Recall that inversion requires computation of the Beylkin determinant. The formula for the Beylkin determinant is

$$\left|h(\boldsymbol{x},\hat{\nu},\theta)\right| = \left[\frac{2\cos\theta}{c(\boldsymbol{x})}\right]^3 \left|\frac{\partial\hat{\nu}}{\partial\alpha_1} \times \frac{\partial\hat{\nu}}{\partial\alpha_2}\right| . \qquad (1)$$

Here, α_1 and α_2 are the parameters that describe the source/receiver pairs. These could be the surface coordinates of a midpoint in a common-midpoint survey, the coordinates of the receiver in a common-shot survey, or the parameters that define the location of $\hat{\nu}$ on the hemisphere at the image point. The second factor in this determinant characterizes the ratio of differential area elements in the acquisition variables and the corresponding patch on this unit hemisphere. That is,

$$\left|h(\boldsymbol{x},\hat{\nu},\theta)\right| d\alpha_1 d\alpha_2 = \left[\frac{2\cos\theta}{c(\boldsymbol{x})}\right]^3 dS. \qquad (2)$$

When the parameters α_1 and α_2 are acquisition surface parameters, access to the traces is systematic, but the size and shape of the image patch on the hemisphere is irregular, as in Figure 6. On the other hand, if the parameters in (1) are polar or cylindrical coordinates, on

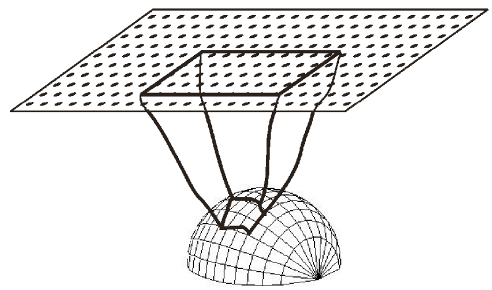

Figure 6: An image patch on the hemisphere at the image point arising from a regular area patch on the acquisition surface. The ratio of these differential patches is the right factor in the Beylkin determinant in equation (1).

the hemisphere, then this second factor on the right side of (1) is simple, but the source/receiver pair on the upper surface must be determined by ray tracing. If we use polar coordinates, the triple scalar product in (1) is simply the sine of the polar angle; for cylindrical coordinates, (such as used in the grid depicting the hemisphere in Figure 6), the triple scalar product is equal to one! Returning to the upper surface options, when the grid points of the figure are the midpoints of a common-offset data set, the triple scalar product is particularly difficult to compute and the angle θ varies from point-to-point, as well. In contrast, recall that in a common-opening angle m/i, θ is constant. Thus the trade-off between upper surface coordinates and coordinates at depth is a difficult Beylkin determinant in the former case and a difficult data access problem in the second case. Common-opening angle m/i in sources and receivers adds some complexity to the computation of the latter, but is still easier than computing the former.

3D common-opening angle m/i.

The common-opening angle inversion formula in 3D is

$$\beta(\boldsymbol{x},\theta,\psi) = \frac{1}{4\pi^2}\left[\frac{2\cos\theta}{c(\boldsymbol{x})}\right]^2 \int \frac{D_3(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r)}{A(\boldsymbol{x},\boldsymbol{x}_s)A(\boldsymbol{x},\boldsymbol{x}_r)} dS. \qquad (3)$$

In this equation the A's are Green's function amplitudes, dS is the differential surface area on the hemisphere of $\hat{\nu}$ introduced in equation (2), and $\boldsymbol{x}_s$ and $\boldsymbol{x}_r$ are the source and receiver point that define a trace. The function D_3 is the filtered observed data given by

$$D_3(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r) = \frac{1}{2\pi}\int u_S(\boldsymbol{x}_s,\boldsymbol{x}_r,\omega)F_3(\omega)e^{-i\omega\tau(x,x_s,x_r)}d\omega \tag{4}$$

with $\boldsymbol{u_S}$ being the observed data on the trace define by the source and receiver point, $\boldsymbol{\tau}$ being the traveltime from source to image point to receiver and

$$F_3(\omega) = i\omega e^{isign(\omega)K(x,x_s,x_r)\pi/2}. \tag{5}$$

Here, K is the *KMAH index,* which is the count of the changes in sign of the ray Jacobians along the rays from source and receiver to the image point. When there are no caustics of the rays—no multipathing—$K = 0$ and the filter reduces to the more familiar $i\omega$, which is the negative of the time derivative. For $K = 1$, this filter is the Hilbert transform of the first derivative. K-values of 2 or 3 yield just the negatives of K = 0 or 1. Thereafter, the values of the filter simply repeat. Thus, we need to compute only two sets of filtered data, the derivative of the observed data and the Hilbert transform of the derivative, in order to have the required filtered data for the processing formula in (3).

We now describe the transformation of the formula (3-5) into an integral over all sources and receivers. We remark that the source and receiver points on the upper surface are each described by two coordinates. Thus, we must transform a two-fold integral into a four-fold integral. We do this through the device of introducing integration over the opening angle and azimuth angle into (3) as follows:

$$\beta(\boldsymbol{x},\theta,\psi) = \frac{1}{4\pi^2}\left[\frac{2\cos\theta}{c(\boldsymbol{x})}\right]^2 \int \frac{D_3(\boldsymbol{x},\boldsymbol{x_s},\boldsymbol{x_r})}{A(\boldsymbol{x},\boldsymbol{x_s})A(\boldsymbol{x},\boldsymbol{x_r})}\left|\frac{\partial\hat{v}}{\partial\alpha_1}\times\frac{\partial\hat{v}}{\partial\alpha_2}\right| \cdot\delta(\theta'-\theta)\delta(\psi'-\psi)d\alpha_1 d\alpha_2 d\theta' d\psi'. \tag{6}$$

Here, we have used the explicit representation of $\boldsymbol{dS}$ in (3), as defined by (1) and (2). Given a pair of dip angles, α_1, α_2, an opening angle, θ', and an azimuthal angle, ψ', it is possible to trace rays to the upper surface thereby mapping these angle variables to the surface source and receiver points $\boldsymbol{x_s}$, $\boldsymbol{x_r}$. We proceed as if the upper surface is flat, so that these vectors are defined by horizontal coordinates at $z = 0$. Then, the integral in (6) can be recast as an integral over the surface coordinates at the cost of the introduction of another Jacobian that we will denote by J. That result is

$$\beta(\boldsymbol{x},\theta,\psi) = \frac{1}{4\pi^2}\left[\frac{2\cos\theta}{c(\boldsymbol{x})}\right]^2 \int \frac{D_3(\boldsymbol{x},\boldsymbol{x_s},\boldsymbol{x_r})}{A(\boldsymbol{x},\boldsymbol{x_s})A(\boldsymbol{x},\boldsymbol{x_r})}\left|\frac{\partial\hat{v}}{\partial\alpha_1}\times\frac{\partial\hat{v}}{\partial\alpha_2}\right| \cdot\delta(\theta'-\theta)\delta(\psi'-\psi)J dx_r dy_r dx_s dy_s \tag{7}$$

As noted in the introduction, we have derived a new result for the Jacobian $\boldsymbol{J}$ appearing here. This Jacobian is made up a product of three separate Jacobians. One of these describes the transformation of the four variables of integration in (6) to the pairs of polar coordinates that define the direction of the rays from the source and receiver point to the image point. The relation between the direction of the rays at depth and the Cartesian coordinates on the upper surface is accomplished through a ray Jacobian, exactly the Jacobian that appears in the WKBJ Green's functions. For ray parameters, we use the polar angles of the vectors $\hat{\alpha}_s$, and $\hat{\alpha}_r$ of Figure 5 and the traveltime along the ray. The ray Jacobians are denoted by J_s and J_r, respectively. Thus, we write

$$J = \mathcal{J}_g\mathcal{J}_s\mathcal{J}_r,$$

$$\frac{1}{\mathcal{J}_g} = 2\sin2\theta'\sin\alpha_2(\cos^2\alpha_2\cos^2\theta' - \sin^2\alpha_2\sin^2\psi'\sin^2\theta'),$$

$$\mathcal{J}_s = \frac{\cos\gamma_s}{c(\boldsymbol{x}_s)J_s}, \qquad \mathcal{J}_r = \frac{\cos\gamma_r}{c(\boldsymbol{x}_r)J_r}. \tag{8}$$

In the last equation, γ_s and γ_r are the angles between the rays to source and receiver and the surface normal at the source and receiver points, respectively. Thus, as stated, the 4X4 Jacobian of transformation of coordinates in (7) has been written in terms of factors that are relatively easily accessible. The ray Jacobians arise from the ray tracing needed for computation of the Green's functions. The geometrical factors are required for description of the neighborhood of the image point. If the acquisition surface were not flat, then we need only add two other Jacobians to describe the three dimensional surface coordinates in terms of two parameters describing the surface.

Discussion

In discrete computation, the delta functions in (6) and (7) become box functions or windows of a prescribed width around central values in θ and ψ. Since we are proposing integrations over all sources and receivers, this computation only makes sense when computation for a suite of choices of θ and ψ is carried out in parallel. CPU time for such a process should be compared to computation for a suite of common-offset values in 2D on the upper surface with $\boldsymbol{h} = (h_1,h_2)$. We believe that the weight factors in (7-9) are simpler than the computation of the 3D Beylkin determinant for common-offset data. Thus, when migrating in parallel to obtain a suite of outputs (opening angle gathers or offset gathers), the computation of angle gathers may prove to be the more efficient of the two methods.

2.5D Common opening angle m/I

The 2.5D and 2D m/i formulas share the same geometrical factors, but require different filters on the data. The 2D formula is given in Xu et al [2001], except that they claim that the geometrical factors are approximate when, in fact, they are exact. Here, we provide the 2.5D formula.

$$\beta(\boldsymbol{x},\theta) = \frac{1}{\left[2\pi\right]^{3/2}} \frac{2\cos\theta}{c(\boldsymbol{x})} \int \frac{\sqrt{\sigma_s + \sigma_r}\, D_{2.5}(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r)}{A_2(\boldsymbol{x},\boldsymbol{x}_s) A_2(\boldsymbol{x},\boldsymbol{x}_r)} \delta(\theta' - \theta) J dx_r dx_s \quad (9)$$

In this equation, $D_{2.5}$ is the 2.5D filtered data given by

$$D_{2.5}(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r) = \frac{1}{2\pi} \int u_S(x_s, x_r, \omega) F_{2.5}(\omega) e^{-i\omega\tau(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r)} d\omega \quad (10)$$

with

$$F_{2.5}(\omega) = \sqrt{|\omega|}\, e^{i\,\mathrm{sign}(\omega)[K(x,x_s,x_r)\pi/2 + \pi/4]}. \quad (11)$$

As above, τ is the traveltime for source to output point to receiver and K is the KMAH index, counting the caustics that the rays pass through on the ttrajectory from source to image point to receiver. Finally, the Jacobian J for this 2.5D (and 2D) m/i is

$$J = \frac{1}{2} \frac{\cos\gamma_s}{c(\boldsymbol{x}_s) J_s} \frac{\cos\gamma_r}{c(\boldsymbol{x}_r) J_r} \quad . \quad (12)$$

Here, as above, γ_s and γ_r are the angles between the rays to source and receiver and the surface normal at the source and receiver points, respectively; J_s and J_r are the 2D ray Jacobians with respect to the dip angles of $\hat{\alpha}_s$ and $\hat{\alpha}_r$ of Figure 1. The factors σ_s and σ_r are out-of-plane geometrical spreading factors and the A_2's are hybrid 2D Green's function amplitudes that arise when the factors of σ_s or σ_r, respectively, are factored out of the 3D Green's function amplitudes. They are characterized by their limits near zero distance by

$$\lim A_2(\boldsymbol{x}_s,\boldsymbol{x}) \sqrt{|\boldsymbol{x} - \boldsymbol{x}_s| / c(\boldsymbol{x})} = \frac{1}{4\pi}, \quad |x - x_s| \to 0. \quad (13)$$

Conclusions

We have proposed a common-opening angle inversion formula as an integral over sources and receivers that samples all traces for traveltimes that contribute to the inversion at every image point. This result starts with an inversion in dip angle that fits the general form of Kirchhoff inversion. The computation as an integral over sources and receivers requires inclusion of weights depending on the geometry at the image point, the source point and the receiver point. We believe that these are more straightforward than the weight for 3D common-offset inversion.

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