Are 24 bits enough for the higher frequencies?

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Abstract

Increasingly, today's targets need to be defined with higher frequencies. Vibroseis sweeps are higher (above 100Hz), phones are bunched and many other field practises employed – all in an attempt to record high frequency signal. But what about the recording instrument – can it handle the returned signal?

Amplitudes are affected by factors like spherical spreading, transmission losses and inelastic attenuation. If the ratio of the largest amplitude (from near surface reflectors) to the smallest amplitude (from deep reflectors) exceeds the dynamic range of the recording instrument, then all is lost! Instrument recording today has a dynamic range of 138dB (23 bits plus sign). Losses at typical targets can be in the range of 100dB for frequencies of 100Hz. This means that the 100Hz component of the reflected signal is sampled as a 6 bit number. If we include the effect of noise, then attempts to recover this signal can reduce this to 4 bits or less.

In other words, high frequency signal may not be properly sampled. The cure is to use some amount of pre-amplifier gain which allows sufficient bits to properly sample high frequency signal from depth.

24-bit recording

Today's 24-bit A/D converters take an analog signal as input and create a 24-bit digital output sample. Actually the output sample can be positive or negative and hence there is one sign bit and 23 signal bits. If a maximum input level is assumed to be sampled as full scale (all 23 bits on), then the smallest input signal that can be measured (one bit on) is 138dB below the maximum $20\log_{10}(2^{23})$.

There are three principal causes of signal attenuation as an acoustic wavefront travels through the earth, reflects from an acoustic boundary and travels back to surface:

1. **Spherical divergence.** As an example, consider a target at a depth of 2500m with constant velocity of 3333 m/s from surface to target and, hence, two-way time of 1.5 seconds. A wave that has full scale amplitude (23 bits) at a geophone close to a source will have amplitude of $(100/2500)^2$ less after traveling to the reflector where the initial source-geophone distance was 100m (inverse square law of spherical spreading). The return path (reflector to surface) causes a further equal degradation of amplitude. The total loss of amplitude from full scale at 100m to the reflector at 2500m and back to surface is -68dB.
2. **Transmission loss**. As the acoustic wave travels through each acoustic boundary, some energy is lost due to upward reflection on the downgoing wave – and downward reflection of the upgoing wave. The accumulated effect of these transmission losses is not large (because reflection coefficients are generally small). Roughly half (or more) of the amplitude is lost this way. The loss of amplitude at the reflector depends on the reflection coefficient and may be approximated as a value like 0.25 – or 1/4. Thus total losses for transmission and reflection can be characterized as 1/8 – or -18dB.
3. **Attenuation (Q).** The inelasticity of the earth means that energy is lost as a wave propagates. A generally accepted formulation of this can be expressed as: $\mathbf{A_2 = A_1 \cdot e^{-\pi ft/Q}}$ Where $\mathbf{A_1}$ is the original amplitude, $\mathbf{A_2}$ is the amplitude after traveling for time = **t** seconds and **f** = the frequency of the acoustic wave. **Q** is the constant of attenuation (sometimes called quality factor) – and for typical rock sequences can vary anywhere from a low of 30 to a high of 300 or more. A low value of Q means high attenuation losses (soft rocks – mud etc.) whereas a high value means very small losses (hard rocks). Naturally an acoustic wave is made up of many frequencies, so that each frequency suffers a different amount of attenuation.

If we assume a Q of 300, a target reflector at 2500m, a constant velocity of 3333m/s, then the two-way time to the reflector is 1.5 seconds. If we further calculate attenuation for a frequency of 100Hz, then A_2/A_1 = 0.2 or -14dB. Thus, in this example, the further loss of amplitude (for the 100Hz component of the wave) due to Q is -14dB.

Hence, for the simple example above, total losses are -68dB (spherical divergence) plus -18dB (transmission and reflection loss) plus -14dB (attenuation – Q). This comes to -100dB. Note that if Q=200, attenuation is -20dB and hence the total losses are -106dB, while for Q=100, attenuation is -41dB and total losses are -127dB.

A 6 bit number has a range from 1 to 64 (2^6) – or a dynamic range of 36dB. Hence the original dynamic range of 138dB (full 23 bits) less 100dB (due to losses) comes to 38dB remaining – which is very close to the dynamic range of a 6 bit number. Thus, our example acoustic wave which was recorded full scale (23 bits) at a geophone 100m from the source will be recorded as a 6 bit number for reflections from the 2500m target. In the calculations above, noise is assumed to be zero.

Q Filters

Attenuation increases with depth. The frequency spectrum of the downgoing wavelet (source pulse) is continuously changed. The effect is more pronounced on high frequencies than low. A forward Q filter may be computed from the simple expression $\mathbf{A_2 = A_1 \cdot e^{-\pi ft/Q}}$. This will typically take the form of a matrix (the matrix is formed from the expression above) multiplied by a vector (trace samples). Similarly an inverse Q filter can be determined as the inverse of the forward Q matrix.

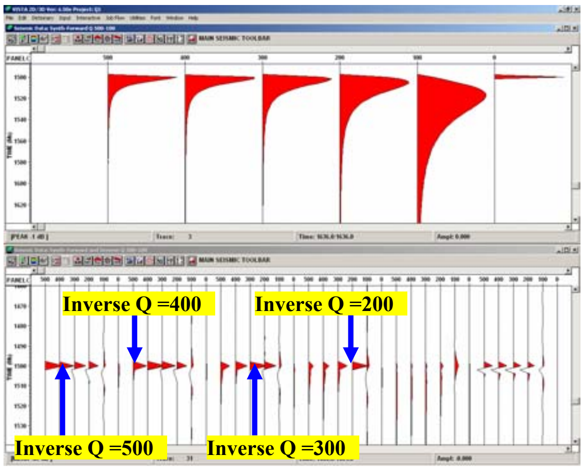

Figure 1

In Figure 1 we see an input pulse (spike) at 1500ms (upper right). On the left of Figure 1, a forward Q has been applied to the spike – values of 500, 400, 300, 200 and 100. Clearly the low values of Q (100) have delayed and stretched the spike more than higher values of Q. In the lower part of Figure 1 we see an inverse Q filter applied to these 6 wavelets. Inverse Q values of (500, 400, 300, 200, 100 and 0) were applied to each of the forward Q traces – creating 6 x 6 = 36 traces. The first 6 traces are the 5 forward Q traces (plus the original spike) with an inverse Q filter of 500 applied.

- The next 6 traces are the same 5 forward Q traces (plus spike) with inverse Q of 400 applied. Then the next 6 have inverse Q of 300, etc.
- Thus trace 1 is the original spike with forward Q=500 and inverse Q=500.
- And trace 8 is the original spike with forward Q=400 and inverse Q=400.
- And trace 15 is the original spike with forward Q=300 and inverse Q=300.
- And trace 22 is the original spike with forward Q=200 and inverse Q=200.
- And trace 29 is the original spike with forward Q=100 and inverse Q=100.

Note that these traces have a stronger spike than the traces beside them – indicating that applying the correct inverse Q maximizes the amplitude of the result. In these other traces, a different inverse Q was used than the forward Q.

Q Filters applied to synthetic traces from well logs

A synthetic well log trace was forward Q filtered with 5 different values of Q (500, 400, 300, 200, 100). The resulting 5 traces (plus the original unfiltered trace) are shown in Figure 2. The section from 1500ms to 1600ms is zoomed on the right of Figure 2. Then noise was added to these 6 traces such that the S/N ratio = 10. Next these 6 traces were inverse Q filtered – again with inverse Q values of (500, 400, 300, 200, 100, 0).

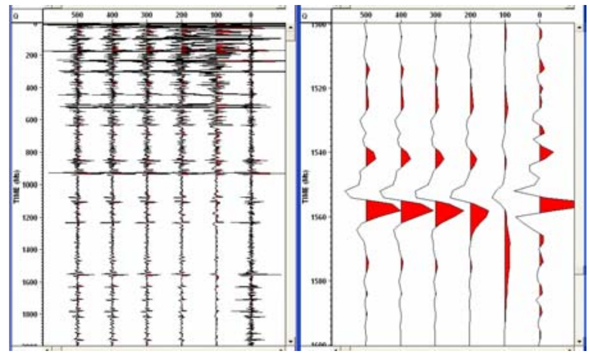

Figure 2

The resulting 36 traces derived from the original synthetic were interspersed with 36 copies of the original synthetic trace to create a 72 trace data set. Each pair of traces was then auto-correlated and cross-correlated to derive a S/N value for each pair.

The first part of the 72 trace data set is shown in Figure 3 with a color background which represents the local S/N value for each trace pair (a moving window of 2 traces and 500ms was used to calculate S/N).

- Trace 1 is the original spike with forward Q=500 and inverse Q=500.
- And trace 15 is the original spike with forward Q=400 and inverse Q=400.
- And trace 29 is the original spike with forward Q=300 and inverse Q=300.
- And trace 43 is the original spike with forward Q=200 and inverse Q=200.

Note that these traces have a much stronger S/N than the neighboring pairs – indicating that applying the correct inverse Q maximizes the S/N of the result. Note also that the value of S/N decreases with time. Thus at the “target” time of 1.5 secs, the S/N of the best inverted result (inverse Q and forward Q are the same) is approximately half that of the added S/N. Thus Q inversion has increased the noise at the target by a factor of 2.

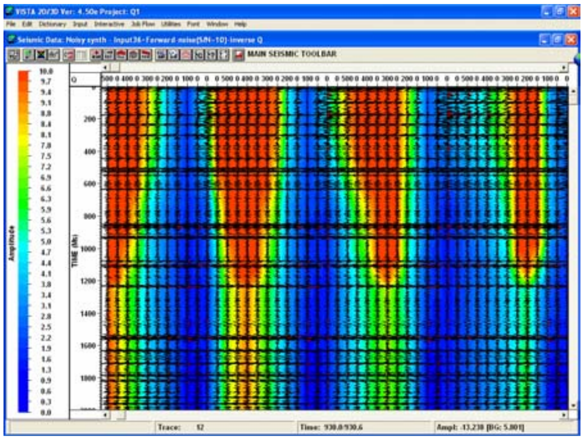
Figure 3

Conclusions to Q applied to synthetic traces.

At the deeper (target) times, the S/N of the result (after forward and inverse Q) has much lower S/N than was applied (10:1) and (5:1). The S/N of the result is also worse for lower Q values.
This means, in practice, that for lower values of Q and for lower values of S/N it becomes doubly difficult (or worse) to find signal above 100Hz.

Quantization of the recorded signal

The 23 bit recorded signal, particularly for high frequencies above 100Hz, may have a dynamic range as small as 38dB for Q=300, 32dB for Q=200 and 11dB for Q=100 at the example target depth of 2500m (1.5 secs). Thus the signal above 100Hz may have 6 bits (Q=300), or 5 bits (Q=200), or 2 bits (Q=100). Noise will normally occupy some of those bit positions.

Thus, for example, if the S/N = 1.0 (not uncommon for raw data), then a 6 bit sample will be the result of adding a 5 bit number (noise) to another 5 bit number (signal). Hence one more bit will be lost. A 5 bit number can only take values in the range of -16 to +16. Hence any change in the actual signal will be represented by a “step” equal to 1/32 of the signal. This “step” will not be accurate – and hence yet further noise is added to that already present. This situation rapidly deteriorates as the number of bits available falls below 5.

The only way to recover the high frequencies if they have suffered any “quantization” loss as described above is to increase the instrument early gain (Pre-amplifier gain). The highest gain (36dB) means that the same signal is amplified by 2**6 = 64 times approximately. This has the effect of possibly clipping strong events (e.g. first breaks) at shallow times, whereas deep events will have more significant bits (out of the 23 bits available) devoted to recording the signal strength.
The effect described above can be demonstrated on real data. In Figure 4 we see a shot which was repeated three times using different levels of pre-amplifier gain. It is possible to see better definition of the deep events (note the two way times run from 1.8 to 2.8 seconds) – and higher frequency signal energy, showing that the higher pre-amp gain can indeed recover more signal.

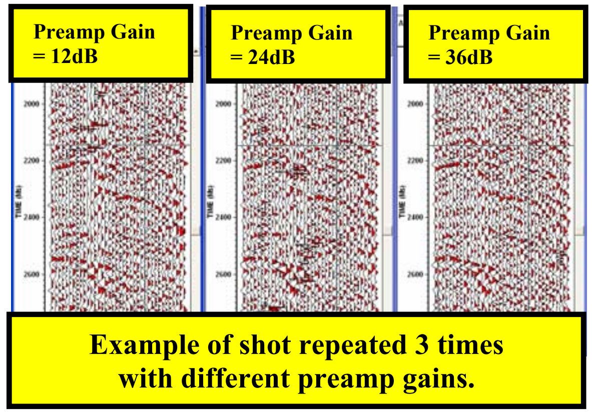

Figure 4

With a Vibroseis source it is important to note that pre-amp gain must be kept below a level which causes clipping. Any clipped energy will be “spread” throughout the record when Vibroseis correlation is performed and will seriously distort all of the samples in the trace. Such clipping will only occur on traces near the source (small offset). It will be necessary to make a compromise between throwing away near traces and finding a suitable level of pre-amp gain.

Conclusions

To ensure proper recording (quantization) of signals above 100Hz, it will likely be necessary to use some amount of pre-amplifier gain. This depends on the local conditions of spherical divergence, transmission losses and also the attenuation Q. To find the right gain level, field testing is mandatory. Note that any comparisons of data recorded with gain changes must be done after a certain amount of inversion such as inverse Q-filtering or deconvolution.