Elastic impedance equations for vertically fractured media

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Abstract

Equations for elastic impedance (EI) are derived for vertically cracked rocks. Transverse isotropy with horizontal axis of symmetry (i.e., the HTI model) is used for simulating first–order effects of azimuthal anisotropy caused by vertical small fractures. Two distinct approximations for PP–wave reflection ($\mathrm{R_{PP}}$) coefficients in HTI media are introduced into previous isotropic formulation for the EI equation. The general expression for both approaches is presented as a product of two terms. The resulting equations differ only in the correction to anisotropy, i.e., the term which controls incorporation of the effects of anisotropy into the EI technology. Due to the use of distinct anisotropy parameters and the definition of reference isotropic velocities, inaccuracies are found in computing approximate $\mathrm{R_{PP}}$ coefficients with the derived EI equations. These inaccuracies are mainly attributed to the use of both the Thomsen's parameter δ and the S–wave splitting parameter $\gamma^{(\mathrm{H})}$ by one of the derived EI equations. Assumption of invariability for the squared ratio between the average of S–wave to P–wave reference velocities seems to have negligible impact on the above–mentioned inaccuracies.

Introduction

The analysis of amplitude–versus–offset (AVO) responses is currently performed by using well logs (Ross, 2000). In this context, calibration and inversion of near–offset seismic amplitudes extensively use the rock attribute known as acoustic impedance (AI), which is easily obtained by multiplying (P– or S–wave) sonic and density logs. However, typical classes of AVO responses, depicted for instance in Rutherford and Williams (1989), are impossible to simulate using only the AI attribute. As a result, the EI technology is the leading alternative to replace the AI attribute in investigating far–offset amplitude anomalies.

The EI technology was first proposed by Connolly (1998, 1999) as a way to promote AVO analysis at greater offsets (i.e., regions of high angles of incidence). The EI equation clearly shows dependence on P– and S–wave velocities and density, and on the incidence phase angle. If normal incidence is taken into account, the EI equation reduces to AI. In view of this outcome, EI is currently assumed as a rock attribute analogous to AI but for varying incidences. The benefits of EI for AVO studies using well logs was then rapidly recognized, culminating with the SEG Virgil Kauffman award in 2001 (TLE, November 2001, page 1296).

The EI technology facilitates the integration of seismic data with well (sonic and density) logs and rock laboratory measurements. Additionally, the technology can be used as a pore–fluid discriminator at far offsets since it allows simulating typical seismic amplitude anomalies at a wide range of incidences. However, the EI equation focuses only on isotropic media with small contrasts in rock properties. Despite a strategy can be designed to cope with high contrasts (Martins, 2003), the issue of neglecting anisotropy still persists. As published for instance in Thomsen (1986) and Wright (1987), seismic anisotropy, even with weak strength, represents a non–negligible phenomenon in sedimentary basins. The restriction of the EI equation to weak–contrast isotropic media thus precludes the study of azimuthal AVO responses caused by seismic anisotropy.

In the formulation of the equation for EI, Connolly (1999) used an approximate expression for $\mathrm{R_{PP}}$ coefficients at plane interfaces separating weak–to–moderate isotropic media (Aki and Richards, 1980). Thus, in order to allow the study of seismic anisotropy by means of the EI technology, a suitable approximation for $\mathrm{R_{PP}}$ coefficients in anisotropic media must be introduced into Connolly's (1999) formalism. Consequently, the restrictions imposed on the use of the resulting EI equation will be motivated by the anisotropic symmetries involved in the $\mathrm{R_{PP}}$ coefficient approximation utilized in the formulation.

In order to incorporate seismic anisotropy into the EI technology, Martins (2002) took advantage of the approximation for $\mathrm{R_{PP}}$ coefficients in arbitrary anisotropic media studied in Pšenčík and Martins (2001). The resulting novel EI equation is formed by multiplication of two terms. The first term is recognized as Connolly's (1999) isotropic EI equation, while the second term is interpreted as the first–order correction from isotropy to weak anisotropy. The expression for the correction term is written in terms of the weak anisotropy (WA) parameters (Pšenčík and Gajewski, 1998), which are dependent on arbitrary choice for the reference isotropic velocities. In summary, the correction term incorporates all parameters (i.e., the azimuthal information and the density–normalized stiffnesses) necessary to the study of transverse isotropic and orthorhombic media. The novel anisotropic EI equation thus allows analyzing azimuthal AVO variations caused, for instance, by vertical small fractures (i.e., cracks). If the cause of seismic anisotropy is attributed only to parallel vertical cracks, the transverse isotropic model with horizontal axis of symmetry (HTI) can be used in the investigation. Note that the HTI assumption implies that the vertical cracks are circular and embedded in an isotropic matrix (Rüger and Tsvankin, 1997).

Studying $\mathrm{R_{PP}}$ coefficients at weak-contrast plane interfaces separating HTI media, Rüger (1996) derived a very useful approximation for the investigation of seismic anisotropy caused by vertical cracks. Although written in terms of Thomsen's (1986) anisotropy parameters δ, ϵ and γ, and for specific reference isotropic velocities, Rüger's first–order $\mathrm{R_{PP}}$ coefficient formula has the same general form of the relation studied in Pšenčík and Martins (2001) for HTI models. It is thus similarly possible to introduce Rüger's (1996) $\mathrm{R_{PP}}$ approximation into Connolly's (1999) formalism in order to incorporate effects of vertical cracks on the EI equation. The resulting equation differs only in the dependence of the correction term to anisotropy of Martins (2002) EI approach. In this work, both approximations for the EI technology in vertically cracked media are investigated in terms of numerical accuracy.

Approximate reflection coefficients

The need for accounting seismic anisotropy into azimuthal AVO analysis has stimulated investigations on scattering coefficients at interfaces separating anisotropic rocks (Banik, 1987; Wright, 1987). Due to the complexity of exact formulas, however, a thorough understanding on the dependence of $\mathrm{R_{PP}}$ coefficients on elastic parameters can hardly be accomplished. Fortunately, small contrasts in velocities and density and weak anisotropy behavior found in most rocks (Thomsen, 1986) support derivation of linearized approximations for the coefficients (Thomsen, 1993; Rüger, 1996; Vavryčuk and Pšenčík, 1998). In general, for a qP–wave incidence on a weak–contrast interface separating weakly anisotropic media, the formulas for linearized coefficients can be condensedly written as

$$\mathrm{R_{PP}}(\varphi,\theta) = \mathrm{R^{iso}_{PP}}(\theta) + \Delta\mathrm{R_{PP}}(\varphi,\theta), \qquad (1)$$

where $\mathrm{R^{iso}_{PP}}(\theta)$ is the well-known approximate formula for reflections at weak–contrast plane interfaces separating isotropic media (Aki and Richards, 1980). The formula for $\mathrm{R^{iso}_{PP}}(\theta)$ depends on the relative contrasts in bulk density ($\Delta\rho/\bar{\rho}$) and in P–wave ($\Delta\alpha/\bar{\alpha}$) and S–wave ($\Delta\beta/\bar{\beta}$) phase velocities. Contrasts across an interface are denoted by Δ (i.e., density contrast: $\Delta\rho = \rho_2 - \rho_1$), and averages by a bar over the corresponding quantity. Subscripts 1 and 2 index upper and lower medium, respectively.

The additional term in Eq. (1) represents first–order correction from isotropy to weak anisotropy. It can be expressed as

$$\begin{aligned} \Delta\mathrm{R_{PP}}(\varphi,\theta) &= \mathrm{A^{ani}}\cos^2\theta + \mathrm{B^{ani}}\sin^2\theta \\ &+ \mathrm{C^{ani}}\sin^2\theta\tan^2\theta. \end{aligned} \qquad (2)$$

To simplify notation, the subscript PP is used henceforth in place of qPqP.

The $\mathrm{R_{PP}}$ approximation studied in Pšenčík and Martins (2001) is quite suitable for the purposes of this work. The dependence of the approximation on the incidence angle θ and on the direction of the measurement profile (i.e., azimuth) φ is clearly observed, as well as linearity of the prevailing terms of the corresponding relationship for Eq. (3). For a plane interface separating HTI media with coinciding horizontal axes of symmetry, the correction to anisotropy $\Delta\mathrm{R_{PP}}(\varphi,\theta)$ of the approximation, which is indeed the linearized version of Eq. (39) of Vavryčuk and Pšenčík (1998), reduces to

$$\begin{aligned} \mathrm{A^{ani}} &= \frac{1}{2}\,\mathrm{A_1}, \\ \mathrm{B^{ani}} &= \frac{1}{2}\,(\,\mathrm{B_1}\cos^2\varphi + \mathrm{B_2}\sin^2\varphi\,), \\ \mathrm{C^{ani}} &= \frac{1}{2}\,(\,\mathrm{C_1}\cos^4\varphi + \mathrm{C_2}\sin^4\varphi \\ &+ \frac{1}{4}\,\mathrm{C_3}\sin^2 2\varphi\,). \end{aligned} \qquad (3)$$

The relationships for A_i, B_i and C_i depend on contrasts and combinations of contrats in WA parameters ϵ_z, ϵ_x, δ_x, γ_x and γ_y (Pšenčík and Gajewski, 1998) and on the ratio involving the reference velocities, $\mathrm{k} = (\bar{\beta}/\bar{\alpha})^2$. Note that derivation of Vavryčuk and Pšenčík's (1998) $\mathrm{R_{PP}}$ coefficient makes no a priori assumption for reference isotropic velocities.

Under same conditions of weak contrast across interface and weak anisotropy for a similar HTI/HTI model given above, Rüger (1996) derived slightly different expressions for $\mathrm{B^{ani}}$ and $\mathrm{C^{ani}}$

$$\begin{aligned} \mathrm{B^{ani}} &= \frac{1}{2}\,(\,\Delta\delta^{(\mathrm{V})} + 8\mathrm{k}\,\Delta\gamma^{(\mathrm{H})}\,)\cos^2\varphi, \\ \mathrm{C^{ani}} &= \frac{1}{2}\,(\,\Delta\epsilon^{(\mathrm{V})}\cos^4\varphi + \frac{1}{4}\,\Delta\delta^{(\mathrm{V})}\sin^2 2\varphi\,). \end{aligned} \qquad (4)$$

As noticed, Rüger's approximation neglects the product $\mathrm{A^{ani}}\cos^2\theta$ in Eq. (3). Moreover, the relation for

8th Intern. Cong. of The Brazilian Geophys. Society, Rio'2003

$\mathrm{B}^{\mathrm{ani}}$ takes into account that, for weak anisotropy, the velocity $\beta^{\perp}$ for the S–wave polarized perpendicularly to the isotropy plane ($\varphi = 90^{\mathrm{o}}$) can be approximated in terms of the shear–wave splitting parameter $\gamma^{(\mathrm{H})}$ as $\beta^{\perp} = \beta^{\|}(1-\gamma^{(\mathrm{H})})$. As a result, the corresponding approximation for the shear–wave modulus holds: $\mathrm{G}^{\perp} = \mathrm{G}^{\|}(1-2\gamma^{(\mathrm{H})})$.

The anisotropy parameters $\delta^{(\mathrm{V})}$ and $\epsilon^{(\mathrm{V})}$ in Eqs. (4) are defined in Thomsen (1986) for transversely isotropic rocks in which the vertical axis corresponds to the axis of symmetry (i.e., VTI media). Furthermore, in contrast to Thomsen's (1986) parameter $\gamma^{(\mathrm{V})}$, which is defined with respect to the vertical, $\gamma^{(\mathrm{H})}$ is defined with respect to the horizontal axis of symmetry of the HTI medium. Rüger (1996, 1997) and Tsvankin (1997) give the relation between $\gamma^{(\mathrm{H})}$ and $\gamma^{(\mathrm{V})}$ as

$$\gamma^{(\mathrm{H})} = -\frac{\gamma^{(\mathrm{V})}}{1+2\gamma^{(\mathrm{V})}}. \tag{5}$$

Inspection of both $\mathrm{R_{PP}}$ approximations given above shows remarkable differences in the relation for $\Delta\mathrm{R_{PP}}(\varphi,\theta)$. Clearly, linear anisotropy measures are used in relations (3), while distinct anisotropies are introduced in equations (4). Further observation refers to the different choice for the reference isotropic velocities in each given $\mathrm{R_{PP}}$ approximation. As shown below, these differences are transferred into the corresponding EI equations derived by using Connolly's (1999) formulation.

EI approaches for HTI media

Following Connolly's (1999) formulation, the function EI is assumed analogous to AI but for varying incidence phase angles θ. Reflection coefficients are thus approximated as

$$\mathrm{R_{PP}}_n \approx \frac{\mathrm{EI}_n - \mathrm{EI}_{n-1}}{\mathrm{EI}_n + \mathrm{EI}_{n-1}} \approx \frac{1}{2}\frac{\Delta\mathrm{EI}}{\overline{\mathrm{EI}}}. \tag{6}$$

Since anisotropy is incorporated into the formulation, EI must also be dependent on the measurement profile. Hence, in Eq. (6), $\mathrm{R_{PP}} \equiv \mathrm{R_{PP}}(\varphi,\theta)$ and $\mathrm{EI} \equiv \mathrm{EI}(\varphi,\theta)$.

After algebraic manipulations, the general expression for the EI technology in anisotropic media can then be stated in terms of the following product

$$\mathrm{EI}(\varphi,\theta) = \mathrm{EI}(\theta)\ \Delta\mathrm{EI}(\varphi,\theta). \tag{7}$$

The first term represents the isotropic approach for EI given in Connolly (1999). It is written below in a slightly different form

$$\mathrm{EI}(\theta) = \rho\alpha\left(\alpha^{\tan^2\theta}\,\mathrm{G}^{-\eta}\right), \tag{8}$$

where $\eta \equiv \eta(\theta) = 4\mathrm{k}\sin^2\theta$ and $\mathrm{k} = (\bar{\beta}/\bar{\alpha})^2$. Note that application of Eq. (8) to well logs implies invariability for the ratio k.

The correction term of the product in Eq. (7) is related as

$$\Delta\mathrm{EI}(\varphi,\theta) = \exp\left[2\int \Delta\mathrm{R_{PP}}(\varphi,\theta)\right]. \tag{9}$$

Consequently, the EI equation has a correction term to anisotropy which is fully dependent on the correction term for the corresponding $\mathrm{R_{PP}}$ approximation.

Inserting Vavryčuk and Pšenčík's (1998) approximation for HTI media into Connolly's (1999) EI formulation yields an expression for the argument of the exponential function in Eq. (9). On the other hand, using Rüger's (1996) $\mathrm{R_{PP}}$ approximation leads to a rather distinct relation for $\Delta\mathrm{EI}(\varphi,\theta)$. Analysis of both resulting EI equations reveals that main differences are in the choice for the anisotropy parameters and for the reference isotropic velocities. Moreover, since a first–order perturbation methodology was used to derive the corresponding $\mathrm{R_{PP}}$ expression, it is remarkable that the EI equation obtained via Rüger's $\mathrm{R_{PP}}$ approximation depends on the nonlinear Thomsen's $\delta^{(\mathrm{V})}$. In the numerical tests below, the computation of $\mathrm{R_{PP}}$ coefficients using the approximation in Eq. (6) is investigated. The main purpose is to verify the accuracy of both EI equations obtained for vertically cracked media.

Numerical tests

By using the approximation in Eq. (6), $\mathrm{R_{PP}}$ coefficients were calculated for the model shown in Figure 1

$$\alpha_1 = 2.260\ km/s$$
$$\beta_1 = 1.430\ km/s$$
$$\rho_1 = 2.700\ g/cm^3$$

$$\begin{pmatrix} 5.00 & 1.82 & 1.82 & 0.00 & 0.00 & 0.00 \\ & 6.25 & 1.75 & 0.00 & 0.00 & 0.00 \\ & & 6.25 & 0.00 & 0.00 & 0.00 \\ & & & 2.25 & 0.00 & 0.00 \\ & & & & 1.87 & 0.00 \\ & & & & & 1.87 \end{pmatrix}$$

$$\alpha_2 = 2.500\ km/s \qquad \beta_2 = 1.500\ km/s \qquad \rho_2 = 2.700\ g/cm^3$$

$$\Delta\alpha/\bar{\alpha} = 0.101 \qquad \Delta\beta/\bar{\beta} = 0.048 \qquad \Delta\rho/\bar{\rho} = 0.000$$

Figure 1: Model consisting of an isotropic medium over a vertically fractured (HTI) material. Upper and lower elastic parameters indicate weak contrast across the interface. The [weak anisotropy] HTI medium is defined by the matrix of density-normalized stiffnesses in $\mathrm{km^2/s^2}$.

(Rüger, 1997). The model consists of an isotropic overburden and a reflecting vertically cracked medium simulated by an HTI elastic matrix. The weak contrast

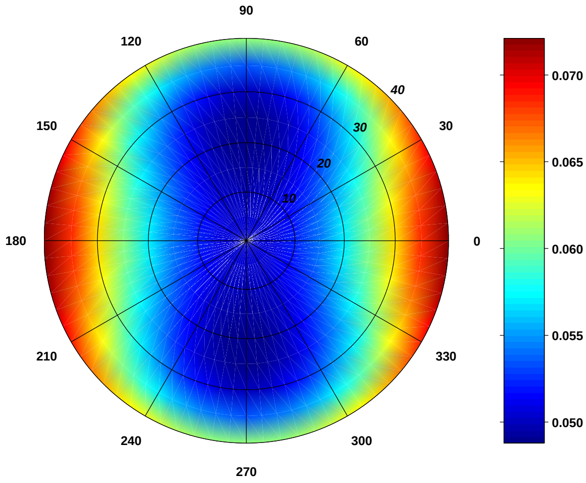

Figure 2: Exact reflection coefficients calculated over the model in Figure 1, via raytracing. Incidence from 0^o to 40^o.

assumption is supported by the small relative contrasts in α, β and ρ.

The display of exact coefficients in Figure 2 shows that, although anisotropy is weak for the model in Figure 1, reflections vary significantly along azimuths near the symmetry axis, $\varphi = 0^o$. Comparison of plots for approximate coefficients, calculated by applying each resulting expression for $\Delta\mathrm{EI}(\varphi, \theta)$, revealed good agreement with exact computations. Calculation of constant k took into account the following reference velocities: $\sqrt{A_{33}}$ for P waves, and $\sqrt{A_{44}}$ for S waves.

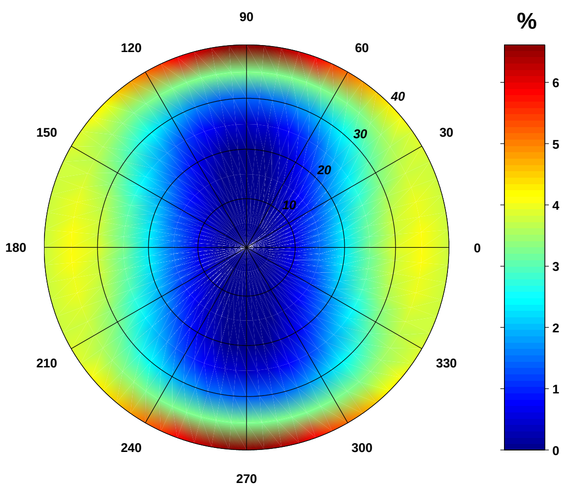

Figure 3: Absolute relative errors computed with the exact coefficients in Figure 2 and the approximate coefficients calculated with derivation of $\Delta\mathrm{EI}(\varphi, \theta)$ using Vavryčuk and Pšenčík's (1998) $\mathrm{R_{PP}}$ equation.

In computing approximate coefficients, the EI equation obtained by using Rüger's (1996) $\mathrm{R_{PP}}$ approximation is far less accurate. This can be verified by plotting the corresponding absolute relative errors. The displays in Figures 3 and 4 show similar magnitudes within the azimuthal range from 60^o to 90^o, i.e., approaching the isotropy plane. On the other hand, approaching the symmetry axis, the errors increase significantly. As shown in Figure 4, if the correction to anisotropy in the EI equation obtained via Rüger's $\mathrm{R_{PP}}$ approximation is chosen, the errors in the computation of approximate coefficients via Eq. (6) will be even higher along the symmetry axis. In computing $\mathrm{R_{PP}}$ coefficients with the derived EI equations, negligible contribution to the verified errors will be obtained if same reference isotropic velocities are used.

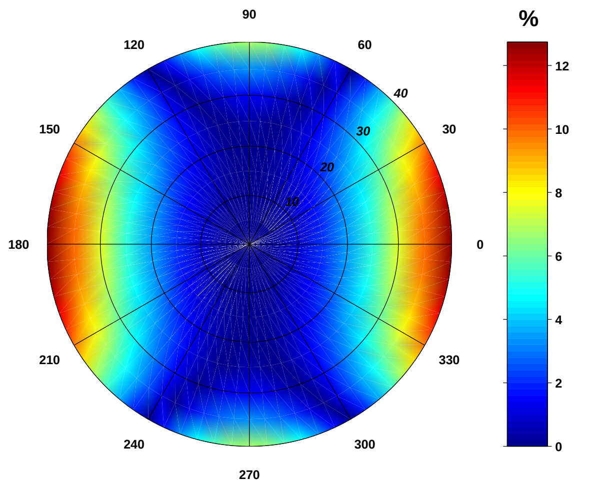

Figure 4: Absolute relative errors computed with the exact coefficients in Figure 2 and the approximate coefficients calculated with derivation of $\Delta\mathrm{EI}(\varphi, \theta)$ using Rüger's (1996) $\mathrm{R_{PP}}$ equation.

Conclusions

The derived EI equations can facilitate the study of vertical cracks in rocks. The equations reinforce the integration aspect of using well logs and laboratory measurements in AVO analysis. Now, by applying an appropriate EI equation, azimuthal amplitude anomalies can be investigated far away around the borehole. The critical point, however, is the determination of the stiffnesses for the rock under assumption. To sort out this issue, the need for laboratory measurements is unquestionable. The inaccuracies found by testing the novel EI equations in computing $\mathrm{R_{PP}}$ coefficients were probably caused by the use of the Thomsen's parameters derived for VTI media, i.e, in respect with the vertical axis of symmetry. Further tests have shown that use of Thomsen–like parameters defined for HTI media, i.e., in respect with the horizontal axis of symmetry, decreases inaccuracies in computing approximate coefficients by means of corresponding EI equation.

8th Intern. Cong. of The Brazilian Geophys. Society, Rio'2003

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