Induced polarization response to 2-D model in fractal medium

Valcir J. C. Farias and Brígida R. P. da Rocha Universidade Federal do Pará

Abstract

A finite element code to obtain de induced polarization response to 2-D model was developed. The "Fractal Model to Complex Resistivity" was applied as intrinsic electrical property of the geological medium, and the influence of the fractal parameter in the induced polarization response was analyzed. The model considered in this work consists of one block buried in half-space both chargeable with complex resistivity given by "Fractal Model to Complex Resistivity". The results showed that the response in amplitude has a similar behavior of the homogenous half-space and the phase response detect the variation of the fractal parameter.

Introduction

The induced polarization phenomena (IP) has electrochemical origin and generally it is associated to geological (Rocha & habashy,1995) and biological environments (Rocha et al, 1997). As a consequence of this phenomena, the electrical resistivity (or its reciprocal electrical conductivity) is a complex number and frequency-dependent.

The method of induced polarization in geophysical environment uses the complex character of the low-frequency conductivity of rocks to carry out a variety of prospecting activities. The method was originally developed for search of disseminated ores. With the development of this techniques it became possible to apply the IP method in mineral discrimination (Sampaio et al., 1998) and in environment investigation (Kemna et al,1999 e 2000).

A quantitative interpretation of Induced Polarization data in geophysical prospecting is an hard task due to the fractal nature of geologic environment. To carry out this interpretation, it's necessary a physical model to explain the behavior of the polarizable medium within a large frequency range.

Rocha (1995) introduced a physical model that consider the fractal effects of the porous surface and include the bulk response of rocks. The introduction of the fractal roughness factor permits the investigation of the texture of rocks which is very important factor in explaining their electrical properties.

Rocha & Habashy (1995) applied the fractal model of complex resistivity as intrinsic electrical properties of a horizontally stratified media (1-D model) and they analyzed the IP response. They observed that the fractal parameter of the fractal model dominates the phase response of the apparent resistivity.

In this paper we simulate the shaped bodies (2-D geological model) and apply the fractal model to complex resistivity as intrinsic electrical property of this medium to analyze the influence of the fractal parameter of the model in the IP response to this geological geometry. This fractal parameter represents the fractal index which can be related to the rock texture.

The fractal model

Representing the time dependence of the electric field as $e^{i\omega t}$, the expression propose by Rocha (1995) for the complex electrical resistivity $\rho(\omega)$ is:

$$\rho(\omega) = \rho_o \left[1 - m \left(1 - \frac{1}{1 + \frac{1+u}{\delta_r (1+v)}} \right) \right] \gamma_h \qquad (1)$$

where

ρ_o : is the DC resistivity of the material;

m : is the chargeability;

δ_r is the grain percent resistivity;

$\gamma_h = 1/(1+i\omega\tau_o)$;

$u = i\omega\tau\,(1+v)$;

$v = (i\omega\tau_f)^{-\eta}$;

τ: is relaxation time constant related the double layer oscillations;

τ_o:is the sample relaxation time constant;

τ_f: is the fractal relaxation time and it is related with the time involved in the transference of charge and energy in the rough interfaces and;

η: is the parameter directly related to the fractal geometry of the medium.

The maximal, minimal and typical values of the parameters of the model cited by Rocha (1995), are presented in the table 1

Two-dimensional forward modelling

The aim of the forward modelling in geophysical prospecting with electrical method is to calculate the apparent resistivity, with the electrode configuration adopted, for a given subsurface conductivity structure. We assume that the region of interest may be represented as a 2-D complex resistivity distribution. Neglected the electromagnetic induction effects, the forward problem is defined by Poison' s equation for a point source with real current I, thus

$$-\nabla.\left(\frac{1}{\rho^*(x,z)}\nabla V(x,y,z)\right)=I\delta(x)\delta(y)\delta(z)$$

where V(x,y,z) is the potential and ρ*(x,z) is the complex resistivity of the medium which is given by (1).

The resistivity distribution $\rho^*(x,z)$ is 2D. However, the current electrode is a point source (3D problem). By taking the Fourier transform of the equation above in the y-direction, the 3D potential distribution V(x,y,z) is reduced to a 2D transformed potential, thus

$$-\nabla.\left(\frac{1}{\rho^*}\nabla V^*(x,\lambda,z)\right)+\lambda^2\frac{V^*(x,\lambda,z)}{\rho^*}=I\delta(x)\delta(z) \qquad (2)$$

where V* is the transformed complex potential, λ is the Fourier transformation variable.

The Equation (2) was solved using the finite element method for an appropriated boundary condition. The finite element discretization of the (2) gives rise to the system of algebraic equations

$$AV^*= b \qquad (3)$$

where the conductance matrix A is a N × N symmetric sparse banded matrix , with N been the number of nodes; b is the current vector.

The complex potential V*(x,λ,z) obtained in different wavernumbers (generally five to ten) are used to reconstruct the Potential V(x,y,z) by an inverse Fourier Transform. When the potential electrodes are located in the plane y=0, the inverse Fourier transform reduces to the integral

$$V^*(x,0,z)=\frac{1}{\pi}\int_0^\infty V^*(x,\lambda,z)d\lambda \qquad (4)$$

The complex apparent resistivity was calculated with the expression

$$\rho_a=K\frac{V}{I} \qquad (5)$$

where K is the geometric factor related to the electrode configuration.

Table 1 – Minimal, typical and maximal values of the parameter of the fractal model

Parameter	Minimal	Typical	Maximal
ρ_o (Ω.m)	10^{-1}	10^{2}	10^{5}
m	0.05	0.5	0.95
δ_r	10^{-3}	1.0	10^{3}
η	0.05	0.5	0.95
τ(s)	10^{-9}	10^{-6}	10^{-3}
τ_f (s)	10^{-4}	10^{-3}	10^{-2}
τ_o (s)	10^{-15}	10^{-12}	10^{-9}

Results

Consider the synthetic model in Figure 1. It consists of one block buried in half-space both chargeable with complex resistivity given by fractal model presented in (1). The parameters of the model of the half-space are the typical values cited by Rocha (1995) and presented in the Table 1. The block was considered with the same parameters of the half-space with exception of the fractal parameter (η) that was considered with the value 0.25.

The 2-D model was divided into 147 × 20 cells The measured data are generated using a dipole-dipole array with dipole length of 2m, this electrode configuration consist in introduce a current into the medium by pair of electrodes and the voltage is measured in another pair of electrode. The geometric factor to this electrode configuration is:

$$K=\frac{1}{AM}+\frac{1}{BN}-\frac{1}{AN}-\frac{1}{BM}$$

where AM, AN, BM e BM are the distances between the electrodes of current (A and B) and the electrodes of potential (M and N).

The data were collected using 30 electrodes with 16 n-spacing and represented in the form of pseudo-section . The simulations were carried out in five different frequencies 1 Hz, 10 Hz, 100 Hz, 1 kHz and 10 kHz .

The results are showed in Figures 2, 3, 4, 5 e 6. We can observer that the response in amplitude has the behavior of the homogenous half-space and the phase angle response detect the variation of the fractal parameter (η). This result was expected because the fractal exponent dominate the phase response mainly in low frequency (Rocha & Habashy,1995). This is a very important result because in low frequency the parameters carries information about porous roughness. Thus, from field data and with appropriated inversion algorithm it will be possible to make inference about the transport properties of the investigated material.

We observe also that in low and high frequencies the phase angle variation is smaller than in intermediate frequencies.

According to Weller & Börner (1996) subsurface hydrocarbon contamination may be related to a reduction of the phase angle response of the apparent resistivity. Thus, starting from the knowledge of the fractal parameter (η) the hydrocarbon contamination may be detected.

Conclusion

We analyzed the influence of the fractal parameter of the "Fractal Model to Complex Resistivity". It was verified that the fractal parameter has a great influence in the phase angle IP response. This result shows that the IP surveys using the fractal model is a very promising technique for use in environmental investigation.

References

Kemna, A., Räkers, E. and Dresen, L., 1999, "Field application of complex resistivity tomography." Presented at the Society of Exploration of

Geophysicists 69th Annual Meeting, Expanded Abstract.

Kemna, A., Binley, A., Ramírez, A. and Daily, W., 2000, "Complex resistivity tomography for environmental applications." Chemical Engineering Journal, vol. 77, pp. 11 – 18.

Sampaio, E. E. S., Santos, A. B. and Sato, H. K., 1998, "Spectral induced polarization and mineral discrimination.", Presented at the Society of Exploration of Geophysicists 68th Annual Meeting, Expanded Abstract.

Rocha, B. R. P. da, 1995, "Modelo fractal para resistividade complexa de rochas: sua interpretação petrofísica e aplicação à exploração geoelétrica". Tese de doutorado, Centro de Geociências - Universidade Federal do Pará.

Rocha, B. R. P. da, Tejo, A. F., Valle, R. R. M. and Melo, M. A. B., 1997, "Fractal eletromagnetic model for biological systems". In : Progress in eletromagnetic research, proceedings vol. 1, Cambridge, Massachussets, USA, pp. 101 – 101.

Rocha, B. R. P. da and Habashy, T. M., 1995, "Fractal geometry, porosity and complex resistivity I: from rough pore interfaces to the hand specimens". In: M. A. Lovell, Developments in petrophysics, London Geological Society, special publication, pp. 277 – 286.

Rocha, B. R. P. da and Habashy, T. M., 1995, "Fractal geometry, porosity and complex resistivity II: from hand specimens to field data". In: M. A. Lovell, Developments in petrophysics, London Geological Society, special publication, pp. 287 – 296.

Weller, A. and Börner, F. D., 1996, "Measurements of spectral induced polarization for environmental purposes". Environmental Geology, vol. 27, pp. 329-334.

Acknowledgments

We would like to thank Campus Universitário de Abaetetuba and Programa de Pós-Graduação em Engenharia Elétrica of the Universidade Federal do Pará

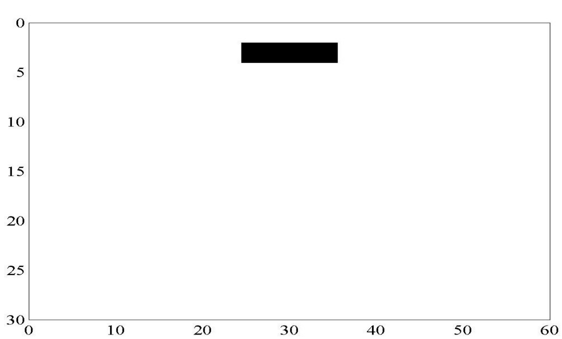

Figure 1. Synthetic model consists of a block embedded in a half-space both chargeable. The dimension of the block is 12×2.

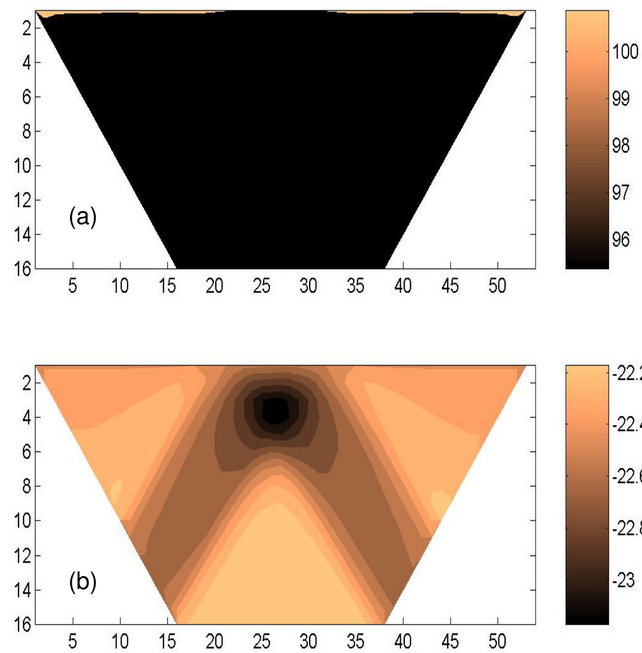

Figure 2. Pseudosections of the synthetic model, collected by dipole-dipole array. a) e b) amplitude and phase angle of apparent resistivity, respectively to frequency 1Hz.

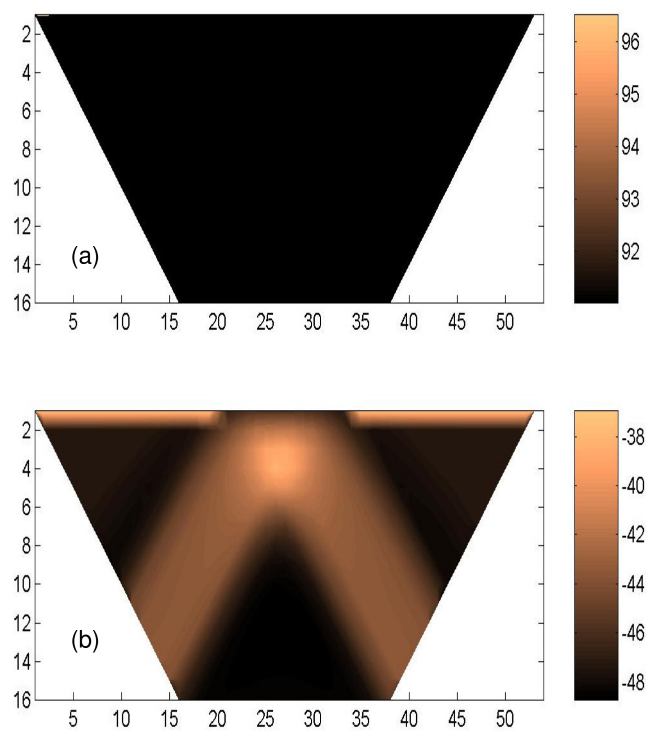

Figure 3. Pseudosections of the synthetic model, collected by dipole-dipole array. a) e b) amplitude and phase angle of apparent resistivity, respectively to frequency 10Hz.

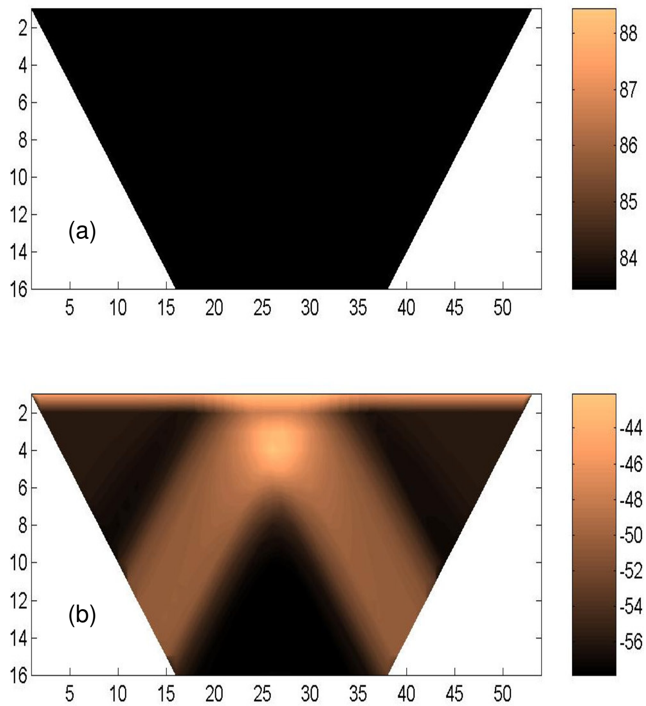

Figure 4. Pseudosections of the synthetic model, collected by dipole-dipole array. a) e b) amplitude and phase angle of apparent resistivity, respectively to frequency 100Hz.

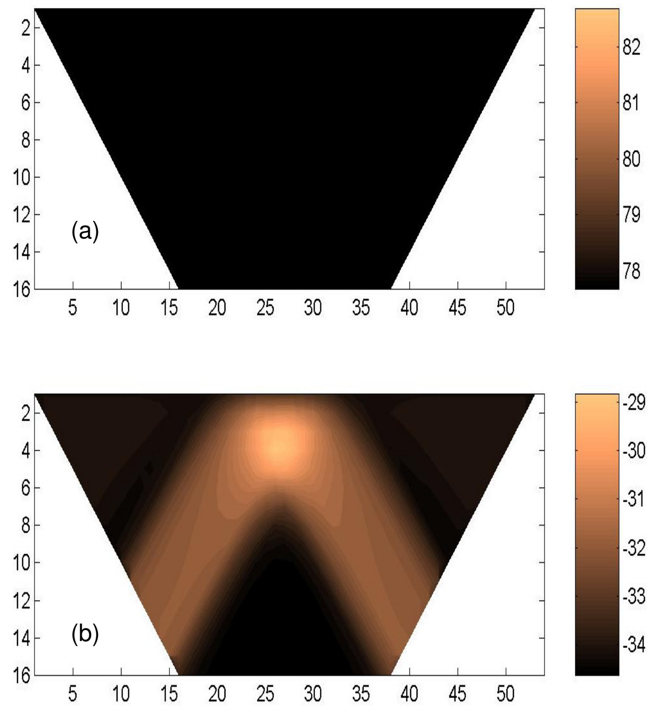

Figure 5. Pseudosections of the synthetic model, collected by dipole-dipole array. a) e b) amplitude and phase angle of apparent resistivity, respectively to frequency 1kHz.

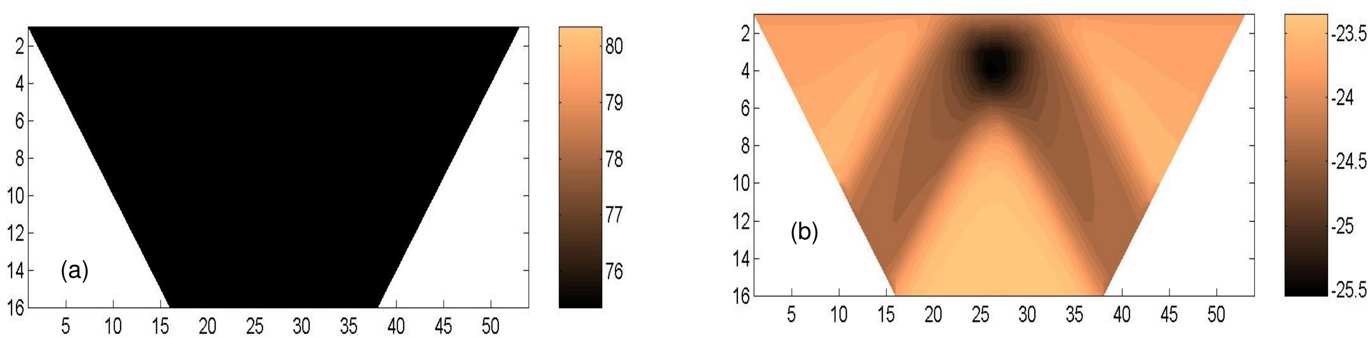

Figure 6. Pseudosections of the synthetic model, collected by dipole-dipole array. a) e b) amplitude and phase angle of apparent resistivity, respectively to frequency 10kHz.