Wavefront construction in weakly anisotropic medium using isotropic ray tracing with velocity perturbation

Rodrigo S. Portugal, State University of Campinas, Alcides Aggio and Eduardo Filpo, PETROBRAS S/A

This paper was prepared for presentation at the 8th International Congress of The Brazilian Geophysical Society held in Rio de Janeiro, Brazil, 14-18 September 2003.

Abstract

We propose a correction of an isotropic wavefront construction method in order to be used in quasi-isotropic media. In anisotropic media the wavefronts are no longer orthogonal to the rays. However when the anisotropy is weak and has some symmetry, such as VTI or HTI media, the isotropic wavefront construction method can be corrected giving the expect result of a fully anisotropic ray tracing. Finally, we show a synthetic example of sampled wavefront and timetables of a VTI media, comparing the proposed method against a complete anisotropic ray tracing.

Introduction

According to Thomsen (1986), the main anisotropic models present in the exploration geophysics are the weak ones, more specifically the transversely isotropic (TI) media. Therefore is quite natural the extension of isotropic media methods to the weakly anisotropic media ones. Applying a first-order perturbation theory, using an isotropic medium as reference, it is possible to compute accurately the qP (quasi-P) waves in an arbitrary weakly anisotropic medium, according to Pšencík & Gajewski (1998).

The wavefront construction (WFC) method was introduced by Vinje et al. (1993). Although it is mainly a modeling method, it has great use in the Kirchhoff-based imaging process, basically because it can generate traveltime tables as well as the dynamic quantities necessary to perform true amplitude computations. Concerning the imaging process, one particular advantage of WFC over other methods is that later arrivals can be provided, improving the image quality.

The WFC method can be seen as a stepwise (or numerical) form of Huygens' principle. Its main idea is to consider the wavefront as a sampled surface that propagates in a medium. This propagation is performed by means of ray tracing algorithm parameterized by traveltime. In this way, each sampled point of the wavefront is a point source of a short duration ray. The set of all end points of all those rays defines a new sampled wavefront. This process can be repeated as many times as necessary.

Another feature of WFC is a sampling analysis of each newly constructed wavefront, in order to keep the wavefront regularization. If a minimal sampling criteria, such as minimum distance between neighbor points, is not satisfied then new interpolated points are inserted in the wavefront. These procedure not only guarantees a minimum sampling of the wavefront but also avoids undersampling in potential shadow or caustic areas.

Originally this method was devised to work for isotropic media, i. e., the ray equations were chosen to be isotropic ones. However the structure WFC method has another interesting feature: the ray equations kernel and the sampling monitoring module are independent of each other. This means that improvements or changes in one part do not affect the other. This characteristic enable us to change at our will the rays equation.

In this work we present a correction on the isotropic WFC method, in order to construct wavefronts in weakly anisotropic media, such as vertical transversal isotropic (VTI). This correction was proposed by Schneider (2002) and it is a first order perturbation of the velocity field. More specifically, we propose modified ray equations that generate weakly anisotropic wavefront points. In other words, at each sampled point we deceive the internal ray-tracing procedure letting it to think that it is a isotropic velocity field, when actually it is a corrected one. As a result, the output wavefront will present a controllable deformation, given by the anisotropy.

In this paper we specified the application for TI medium, constructing a perturbation anisotropic timetables that has a good fit with exact ones. The perturbation methods have a low computation cost to construct a weakly anisotropic medium for using in modeling/inversion & migration.

Method

Inside the isotropic WFC method, the kernel which comprises the isotropic ray tracing is performed by a numerical integration of the ray equations

$$\frac{d\boldsymbol{x}}{dt} = v^2 \boldsymbol{p} \; ; \qquad \frac{d\boldsymbol{p}}{dt} = -v^{-1}\nabla v \; , \tag{1}$$

where v is the isotropic P- or S-velocity field, $\boldsymbol{x}$ is the ray position and $\boldsymbol{p}$ is the slowness vector. The important point here is that the velocity field is a function of ray position only, simply following isotropy definition.

In order to extend the isotropic WFC to weakly anisotropic models, we simply change the kernel represented by equation (1) by the following ray equations

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}_a \; ; \qquad \frac{d\boldsymbol{p}}{dt} = -v^{-1}\nabla v \; ; \tag{2}$$

where v is a reference isotropic velocity and $\mathbf{v}_a$ is the anisotropic group velocity, defined by

$$\mathbf{v}_a = |v_a|(\cos f, \sin f);$$
$$|v_a|^2 = f(\mathbf{p}, v; e, d)^2 + \left(\frac{\partial f(\mathbf{p}, v; e, d)}{\partial q}\right)^2. \quad (3)$$

Here $f(\mathbf{p},v;e,d)$ is a function that comprises the first order perturbation of velocity and e?and d are the parameters of Thomsen. In the case of VTI medium, this function is

$$f(\mathbf{p}, v; e, d) = \left[1 + \sin^2 q(d\cos^2 q + e\sin^2 q)\right]v, \quad (4)$$

where

$$\sin q = p_x / |\mathbf{p}|; \quad \cos q = p_z / |\mathbf{p}|. \quad (5)$$

The angles Therefore, at each time iteration, the algorithm numerically integrates equation (2), producing new wavefront points. We can see that the corrected ray equations (2) extend the original equations (1), because if the Thomsen's parameters are set to zero, then the system (2) reduces to (1).

Example

In this example we consider a homogeneous vertical transversal isotropic (VTI) medium. The reference isotropic velocity is 2.93 Km/s and the Thomsen's parameters are $\varepsilon = 0.2$ and $\delta = 0.18$. The point source is locate at origin and the time step is 50 ms.

Our comparison is performed in two parts: firstly we compare directly the sampled wavefronts and secondly we compare the traveltime tables. Our results were obtained by four methods: (a) analytical solution, (b) isotropic wavefront construction (WFISO), (c) full anisotropic ray tracing (ANRAY software) and (d) our proposed wavefront construction for quasi-isotropic medium (WFQI).

Figure 1 shows three sets of wavefronts: (a) the wavefronts generated by the WFISO (blue squares), (b) the wavefronts computed analytically (green line) and (c) the wavefronts constructed with WFQI (red crosses). As expected, we can observe that the WFISO and WFQI produce quite different results in the horizontal direction.

Figure 2 also depicts three sets of wavefronts: (a) the wavefronts computed analytically (blue line), (b) the wavefronts computed analytically (green line) and (c) the wavefronts constructed with WFQI (red crosses). We can see that there almost is no visual difference among the wavefronts.

Figure 3, 4 and 5 show the traveltime tables generated by the ANRAY, WFISO and WFQI methods, respectively. Observing those figures we can see that, at least visually, the traveltime tables generated by ANRAY and WFQI have little difference. Moreover, as expected, the WFISO and ANRAY are quite different.

In order to evaluate quantitatively the error among the traveltime tables, we simply compute the normalized difference error, taking as basis the traveltime generated by ANRAY. The percent error scale is depicted in Figure 8. Figures 6 and 7 show the errors between traveltime tables generated by WFISO and ANRAY (Fig. 6) and WFQI and ANRAY (Fig. 7). As we can see, the maximum error between WFQI and ANRAY is below 3 percent, while the difference between WFISO and ANRAY reaches 25 percent error.

Summary and Conclusions

In this paper, we present a correction in wavefront construction which allows the construction of wavefront in quasi-isotropic media.

Although it is a simple correction, its consequences could be sensed in Kirchhoff migration which are based on traveltime tables built with wavefront construction methods.

Also, it was shown that this correction has almost no computational cost, therefore it can be easily included in isotropic WFC codes, therefore, providing the user an opportunity to perform a low cost anisotropic WFC, which can be used for a low cost anisotropic migration.

Acknowledgements

The authors wish to thank PETROBRAS for supporting and granting permission to publish this work. Also R. S. Portugal thanks National Agency of Petroleum (PV/PRH15) for financial support.

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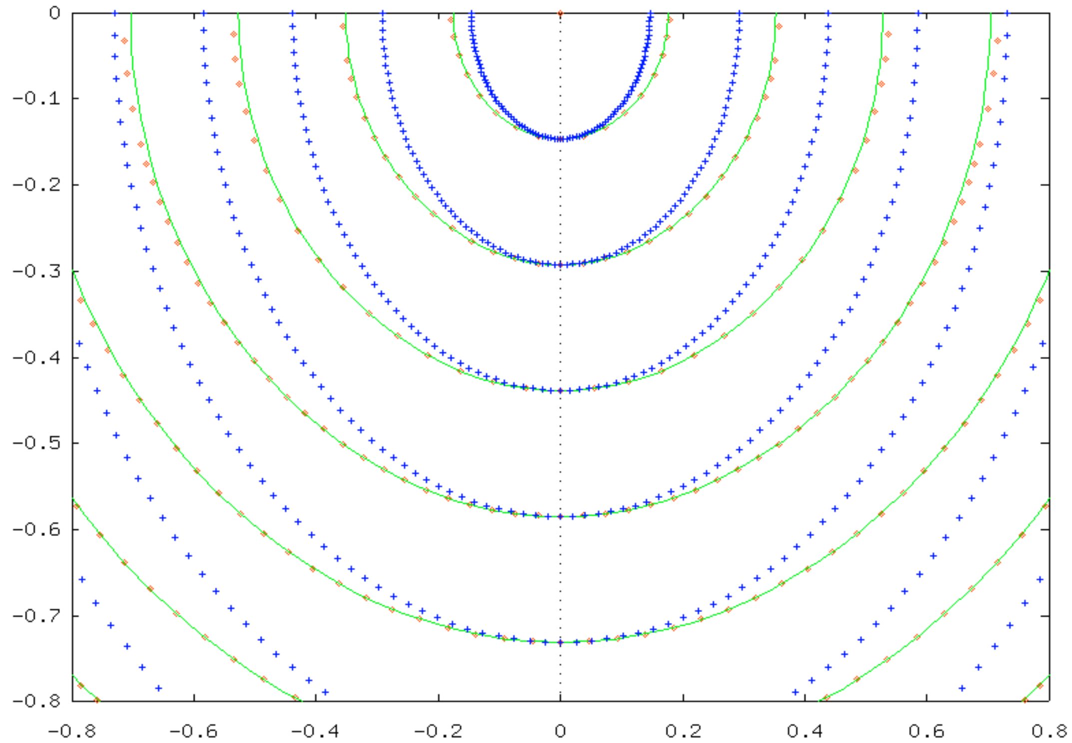

Figure 1: Sampled wavefronts for VTI medium: analytical (green line), WFISO (blue crosses) and WFQI (red squares).

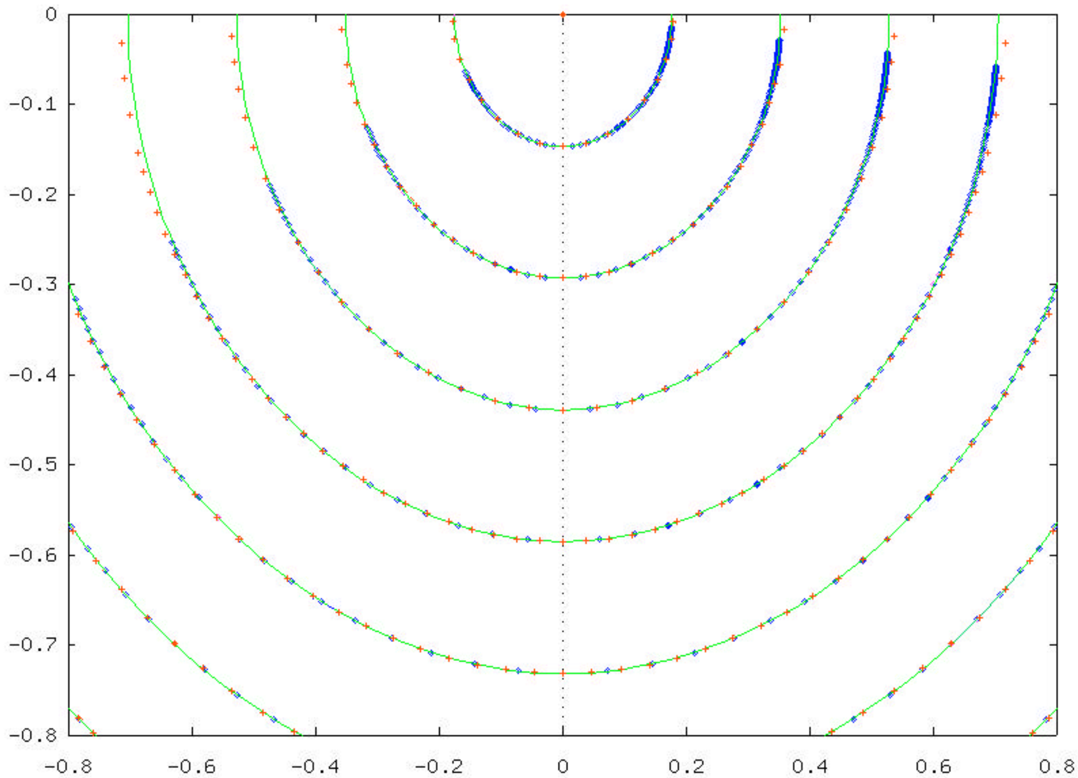

Figure 2: Sampled wavefronts for VTI medium: analytical (green line), ANRAY (blue squares) and WFQI (red crosses).

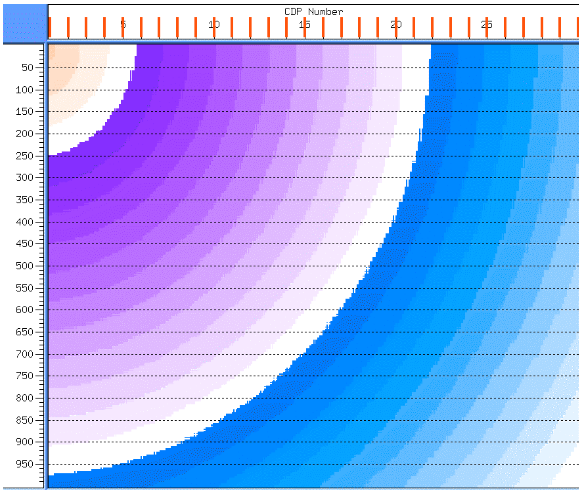

Figure 3: Traveltime table generated by ANRAY.

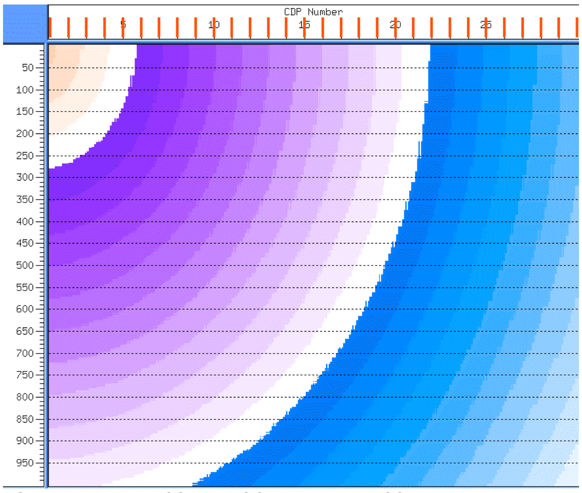

Figure 4: Traveltime table generated by WFISO.

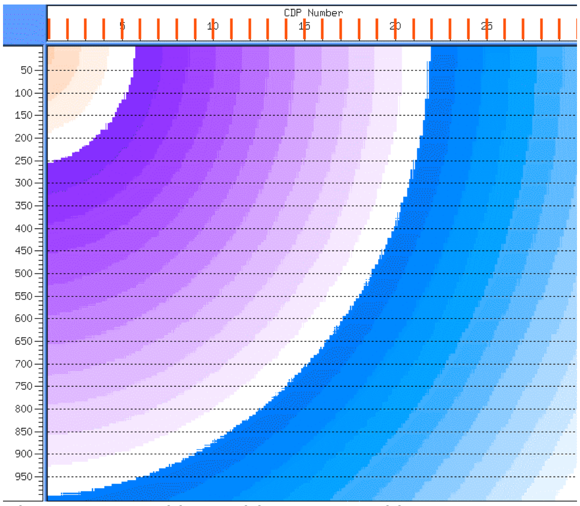

Figure 5: Traveltime table generated by WFQI.

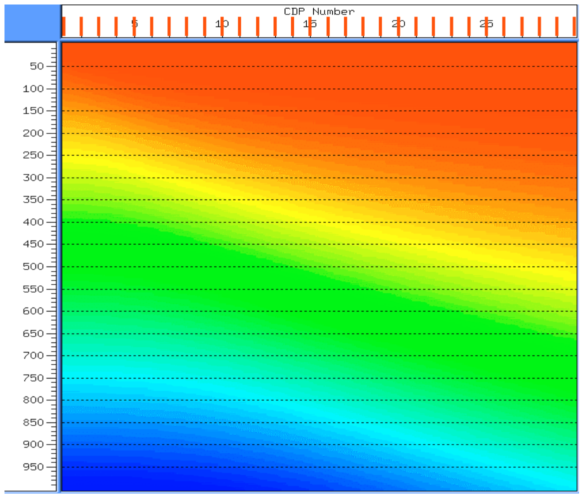

Figure 6: Relative error between traveltime tables generated by ANRAY and WFISO.

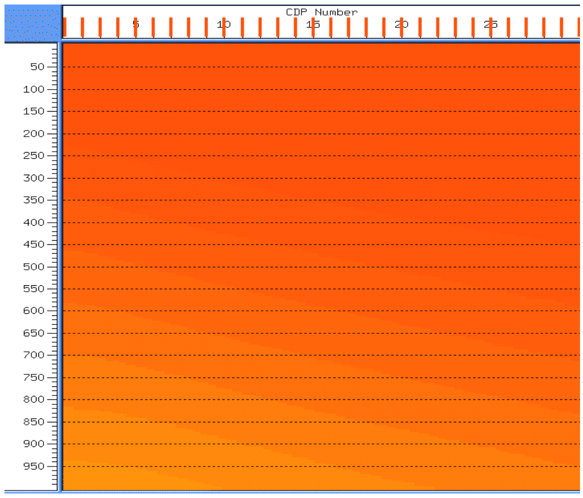

Figure 7: Relative error between tables generated by ANRAY and WFQI.

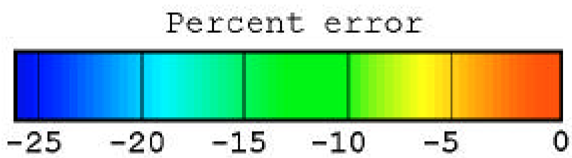

Figure 8: Relative percent error scale used in Figures 6 and 7.