Estimation of seismic texture orientation

André Gerhardt, Curtin University, Perth, Australia

Abstract

Over the years several orientation estimation techniques have been developed either as variants of some early methods or as part of particular attribute computations or applications. Designed with specific properties in mind, they usually yield different results for complex signals like seismic data. Most known methods generally require relatively high computational effort. Additionally, all methods are sensitive to noise, so additional pre-filtering of the data is usually required, increasing the computational cost. This paper proposes an orientation estimation method based on the autocovariance function of the data. The proposed approach results in robust and geologically meaningful orientation estimates throughout a seismic dataset. By estimating the orientation from a smoothed version of the autocovariance of the data (not the data itself!), I show that it is possible to achieve better results with considerable less computational effort.

Introduction

Classical image processing filters are designed for very simple images where only homogeneous regions are considered. However, in many real-world images, regions are not homogeneous but contain a texture, which can be defined as a form of ordered structure. We are interested in textures which present a pattern of ordered lines, or *oriented textures*. Examples of textures of this type are images of certain types of wood, fingerprints, rocks, and seismic data in general. Textures of this class are characterised by their orientation, scale, anisotropy, and curvature. Analyses of such textures usually involve some sort of *orientation estimation technique* in order to identify different domains in an image.

Besides texture analysis, orientation estimation also plays an important role in image processing applications, like adaptive filtering and image enhancement. In geophysics, despite some initial investigations (Dalley et al., 1989), only recently the orientation itself (in the form of dip and azimuth) became of interest not only for structural interpretation (Marfurt et al., 1998; Steeghs et al., 1998) but also to perform adaptive filtering (Bakker et al., 1999; Hocker and Fehmers, 2002), pattern classification and recognition (Randen et al., 2000), and to compute other seismic attributes (Van Spaendonck et al., 2001), among others.

Unfortunately, there is no strict mathematical interpretation of the notion of "orientation". Currently, for seismic images, "orientation" can be determined either from the correlation structure of the signal (Marfurt et al., 1998), angular distribution of signal power spectrum (Steeghs et al., 1998), or by analysing the joint statistics of the gradient vectors (Randen et al., 2000). Since the first two methods are very expensive computationally, I shall here concentrate on the orientation estimation from gradient vectors. The vast majority of existing orientation estimation techniques belongs to this class.

The development of the initial concepts of orientation estimation (Granlund and Knutsson, 1995; Kass and Witkin, 1987) by different fields of applied science has resulted in a myriad of apparently different (but basically closely related) techniques. Nearly all established techniques are based on the same criterion which leads to an eigensystem problem. Due to a lack of formalism and rigorous referencing, it is difficult to identify genuinely new approaches in the literature. For instance, one such technique, the *structure tensor*, can be performed by several different methods (e.g. the *gradient structure tensor*, the *quadrature tensor*, the *polynomial tensor*, etc.). All of them can be shown to have the same fundamentals and give the "correct" orientation when applied to a simple signal. However, only the gradient structure tensor and the polynomial tensor are *rotationally invariant* (meaning that a rotation of the signal results in corresponding rotation of the tensors), while the quadrature tensor is the only one to show *phase invariance* (meaning that the norm of the orientation tensor does not depend on the signal phase). Those particular properties can have a significant impact on the results when complex signals, like seismic data, are considered. A theoretical analysis of such tensor variants for relatively complex signals is given by (Johansson and Farnebäck, 2002). To add to the confusion, some of the formulations are known by different names in different fields of science. For instance, the gradient structure tensor is also known as the *moment tensor* or *inertia tensor*. It has also been used in the geophysical literature without proper reference or credits, as in (Randen et al., 2000).

The present paper presents a brief review of the current orientation estimation methods based on the structure tensor and then proposes an alternative method based on the autocovariance of the data. Only plane-like linear structures are considered; curvilinear structures like channels are outside the scope of this paper. Additionally, only techniques that estimate a single dominant orientation are considered. Structures with conflicting dominant orientations, like stratigraphic terminations or truncations, require a different theory.

Basic concepts

A linear structure in the N-D space is shift invariant in at least *one* orientation, but not in *all* orientations. Therefore, a plane-like feature in 3-D can be defined as shift invariant along two orientations. A local vicinity with *ideal orientation* can be defined by (Granlund and Knutsson, 1995)

$$s(\vec{x}) = f(\vec{x}.\vec{n}) ,$$

with f being a scalar function of a scalar argument and $\vec{n}$ the unit vector perpendicular to the lines of constant grey values. However, in real images textures are almost never oriented locally in such an ideal way. Besides the intrinsic stochastic variation in local orientation, real signals are always corrupted by various types of noise or non-stationary perturbations (Figure 1). Another problem with such representation is that it does not indicate a method for estimating the orientation axis and its level of confidence. In a seminal paper (Knutsson, 1989) found a tensor mapping, which is suitable for further processing, defined by

$$\mathbf{T} \equiv \frac{\mathbf{x}\mathbf{x}^T}{\|\mathbf{x}\|} .$$

The major advantage of the tensor mapping is that it is independent of the dimension of the image. This representation was expanded by (Haglund, 1992) as

$$\mathbf{T} \equiv \overline{\left(\frac{\mathbf{x}\mathbf{x}^T}{\|\mathbf{x}\|^n} \right)}, \qquad (1)$$

with $\overline{(\;)}$ indicating some sort of local averaging. Equation (1) is also known as the *structure* tensor. The computation of the structure tensor involves estimation of the orientation for each point in the image followed by the tensor mapping and averaging. The normalization factor determines how the averaging step is weighted with respect to the local intensity contrast. Various implementations of the structure tensor can be found in the literature, the earliest being (Haglund, 1992; Kass and Witkin, 1987; Rao and Schunck, 1991).

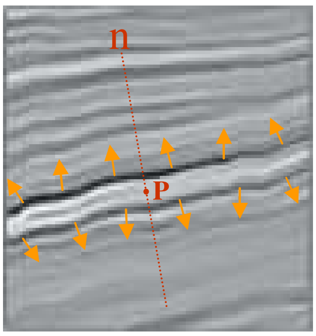


Figure 1 – Local orientation axis $\mathbf{n}$ at point $\mathbf{P}$ of a 2-D seismic image. Notice the variance in orientation caused by stochastic geometry variation and noise along the seismic reflector.

Gradient Structure Tensor (GST)

The GST is defined by

$$\mathbf{T} \equiv \overline{\mathbf{g}\mathbf{g}^T} , \qquad (2)$$

which is equivalent to equation (1) with n=0, meaning that the elements of the GST can be interpreted as gradient energies.

The GST is an efficient implementation of the structure tensor. It also consists of two steps:

1. Estimate the gradient $g = \nabla I$ of the image I at a given scale σ_g. This is accomplished by convolving the image with the first order derivative of a Gaussian $G(x, \sigma_g)$ according to $g_i = I(x) \otimes \frac{\partial}{\partial x_i} G(x, \sigma_g), i \in \{1, \ldots, N\}$, where N is the dimension of the image;
2. Map the gradient to the structure tensor using the dyadic product equation (2) and average the tensor components at a given scale σ_T (usually three times the gradient scale).

In the 3-D case the structure tensor (or covariance matrix) $\mathbf{T}$ of $\mathbf{g}$ takes the form

$$\mathbf{T} = E \begin{bmatrix} \left(\frac{\partial s}{\partial x_1}\right)^2 & \frac{\partial s}{\partial x_1}\frac{\partial s}{\partial x_2} & \frac{\partial s}{\partial x_1}\frac{\partial s}{\partial x_3} \\ \frac{\partial s}{\partial x_1}\frac{\partial s}{\partial x_2} & \left(\frac{\partial s}{\partial x_2}\right)^2 & \frac{\partial s}{\partial x_2}\frac{\partial s}{\partial x_3} \\ \frac{\partial s}{\partial x_1}\frac{\partial s}{\partial x_3} & \frac{\partial s}{\partial x_2}\frac{\partial s}{\partial x_3} & \left(\frac{\partial s}{\partial x_3}\right)^2 \end{bmatrix} . \qquad (3)$$

In practice, the expectation operator $E\{\,\}$ is replaced by a windowed local estimate, typically a low-pass filter. The orientation is represented by the axis $\vec{n}$, which is on average perpendicular to the lines of constant grey values (seismic amplitudes). The GST leads to an optimisation problem that must be solved for each point $\mathbf{P}$ in the image (Figure 1). It can be demonstrated (Jahne, 1997) that $\vec{n}$ is equivalent to the eigenvector of $\vec{n}^T \mathbf{T} \vec{n} \rightarrow \max$ corresponding to the largest eigenvalue.

The GST is a positive semi-definite tensor, meaning that all eigenvalues are real and positive. Additionally, the tensor is also symmetric, so only N(N+1)/2 elements have to be processed. In the 2-D and 3-D cases those eigenvalues can be found analytically. Higher dimensions require numerical solutions. The computation of the GST in the 3-D case requires six convolutions, two for the gradient components and four for the tensor averaging, for each point in the volume.

Interpretation of the eigenvalues

The degree of contrast between the computed eigenvalues defines the local structure model. Assuming that the eigenvalues are sorted, i.e. $\lambda_i > \lambda_{i+1}$, four possibilities exist for 3-D images:

λ_1	λ_2	λ_3	Local structure
0	0	0	Constant amplitude with no measurable structure
>0	0	0	Plane-like linear structure
>0	>0	0	Line-like linear structure
>0	>0	>0	Non-linear structure. If $\lambda_1 = \lambda_2 = \lambda_3$ the structure is isotropic

In practice, the eigenvalues should be checked against a threshold level. Since we are interested in plane-like structures (i.e. $\lambda_1 > \lambda_2 \approx \lambda_3$), it is possible to define a contrast independent confidence measure for this model as

$$C = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2},$$

which takes values between 0 (non-planar, i.e., isotropic or line-like) and 1 (totally planar).

Some problems with the GST technique

The most important aspect of the GST is the calculation of the joint distribution of gradients, the associated errors, and the perturbation they propagate to the subsequent tensor mapping. The computation of gradients of discrete signals constitutes a challenging problem in itself. The problem of optimisation of gradient filters has been extensively investigated in the literature (Bentum, 1996; Marschner and Lobb, 1994; Möller et al., 1998; Neumann et al., 2002), but there seems to exist no consensus on which metrics to use. Additionally, little is known about the influence of noise on the final optimisation. Since real data, and particularly seismic data, usually display moderate to high noise levels, the gradient estimation step must be preceded by a low-pass filtering procedure. In practice, the gradient and the low-pass filters are combined into a single operator, resulting in a longer operator. Longer operators suffer from precision problems near image borders, which usually results in less pronounced contrast in the eigenvectors of the tensor. Furthermore, longer operators require more machine operations, degrading the overall performance. For complex structures, like the ones commonly found in seismic data, a better approach is necessary in order to achieve higher accuracy in the estimated orientation.

Orientation from the autocovariance of the data

Instead of developing new gradient optimisation techniques, I propose a non-differential approach to the estimation of the eigenvalues based on the autocovariance of the data. The autocovariance function (ACF) ϕ of the signal $s(\vec{x})$ can be approximated in the origin of the coordinate system by a Taylor series as (Mester, 2000)

$$\phi_{ss}(\vec{x}) \approx \phi_{ss}(\vec{0}) - \frac{1}{2}\vec{x}^T \mathbf{H}\vec{x} + \ldots,$$

with $\mathbf{H}$ being the Hessian of $\phi_{ss}(\vec{x})$ at the origin as defined by

$$\mathbf{H} = -\left.\begin{pmatrix} \frac{\partial^2 \phi_{ss}}{\partial x_1^2} & \frac{\partial^2 \phi_{ss}}{\partial x_1 \partial x_2} & \frac{\partial^2 \phi_{ss}}{\partial x_1 \partial x_3} \\ \frac{\partial^2 \phi_{ss}}{\partial x_1 \partial x_2} & \frac{\partial^2 \phi_{ss}}{\partial x_2^2} & \frac{\partial^2 \phi_{ss}}{\partial x_2 \partial x_3} \\ \frac{\partial^2 \phi_{ss}}{\partial x_1 \partial x_3} & \frac{\partial^2 \phi_{ss}}{\partial x_2 \partial x_3} & \frac{\partial^2 \phi_{ss}}{\partial x_3^2} \end{pmatrix}\right|_{\vec{x}=\vec{0}}.$$

The elements of the Hessian $\mathbf{H}$ can be shown (Granlund and Knutsson, 1995) to be equal to

$$h_{11} = E\left[\left(\frac{\partial s}{\partial x_1}\right)^2\right],\; h_{22} = E\left[\left(\frac{\partial s}{\partial x_2}\right)^2\right],\; h_{33} = E\left[\left(\frac{\partial s}{\partial x_3}\right)^2\right],$$

$$h_{12} = h_{21} = E\left[\frac{\partial s}{\partial x_1}\frac{\partial s}{\partial x_2}\right],$$

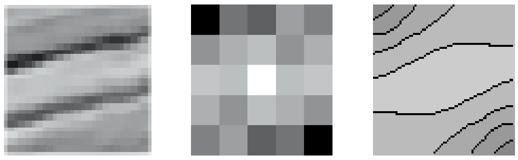
$$h_{13} = h_{31} = E\left[\frac{\partial s}{\partial x_1}\frac{\partial s}{\partial x_3}\right],$$

$$h_{23} = h_{32} = E\left[\frac{\partial s}{\partial x_2}\frac{\partial s}{\partial x_3}\right].$$

Comparison of these equations with equation (3) shows that tensor $\mathbf{T}$ can be computed directly from the autocorrelation function. In other words, the formation of tensor $\mathbf{T}$ can be reduced to the problem of estimating the curvature of the *continuous* ACF near the origin from a large but limited number of (interpolated) samples. This can be accomplished by estimating the curvature near the origin of an interpolating spline function. Thus, all elements necessary for the tensor mapping can be obtained without the troublesome computation of gradient filters and associated optimisation, resulting in great savings in computational effort. Another advantage of this approach is that it can use very small windows, thus it does not require the padding fringe which are necessary near image borders when gradient operators are used. Therefore, the costly GST scheme and the subsequent eigensystem computation can now be replaced by the following steps:

1. Estimate the ACF in the vicinity of point $\mathbf{P}$;
2. Apply a low-pass filter (interpolating spline);
3. Estimate the Hessian $\mathbf{H}$ at the origin;
4. Compute the eigensystem and analyse the relationship between eigenvalues.

Figure 2 illustrates the steps of the technique using a patch near point $\mathbf{P}$ of Figure 1.



(a) (b) (c)

Figure 2 – Some of the steps of the method proposed in this paper: (a) a patch of the image in Figure 1 near point **P**; (b) autocovariance (reduced resolution) of (a); and (c) smoothed version of (b) near the centre of the image patch.

Results

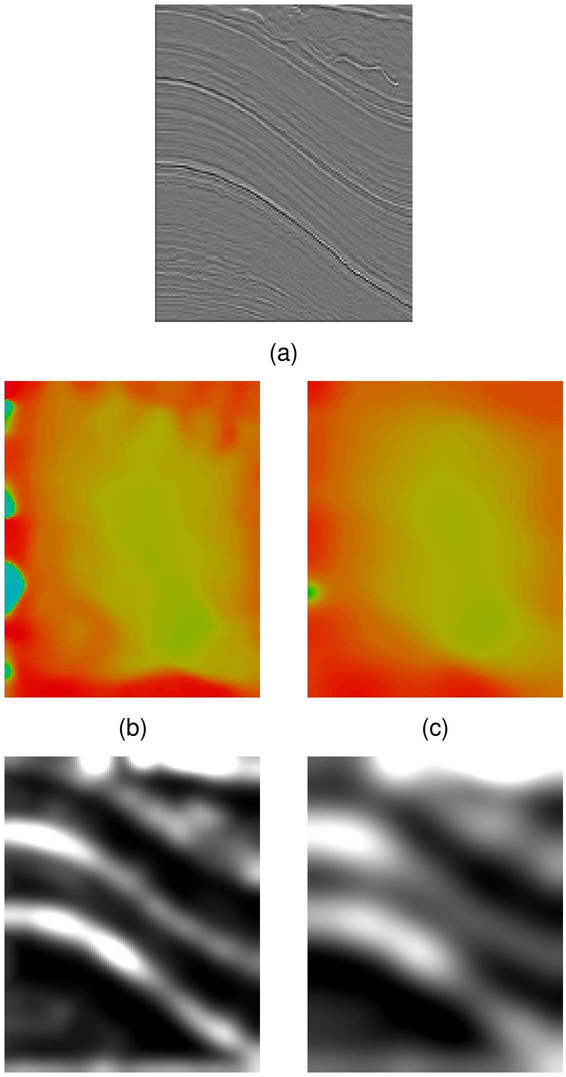
Figure 3(a) shows a seismic section over a geologic fold. Dips along the fold vary approximately between 0 and 30 degrees in a smooth way. Figures 3(b) and 3(d) show the results obtained using the GST. Notice the border problem evident on the left border of the image. Figures 3(c) and 3(e) show the results obtained with the method proposed in this paper. Notice the much smoother dips obtained and the more consistent and smoother eigenvalues. The method proposed here took 30% less computing time than the GST considering only 2-D computations. Whilst a highly optimised GST code was used, no attempts were made at optimising the autocovariance code. Notice that for the 3-D case the gain in performance is expected to be even greater.

Discussion

I have demonstrated a method for the estimation of local seismic orientation which is both robust to noise and computationally efficient. This method avoids explicit computation and optimisation of gradients associated with the standard techniques based on the structure tensor. Additionally, this approach offers further possibilities for considering the amount of noise present in the image (in the Wiener sense) and to compute more robust statistical measures of the estimated orientation.

Acknowledgments

Financial support of the Interactive Virtual Environments Centre of Western Australia is gratefully acknowledged. The author thanks Boris Gurevich for useful discussions and advice.



(a)

(b) (c)

(d) (e)

Figure 3 - A section of a seismic line (a), the computed orientation (b), and largest eigenvalue (d) computed using the GST. Results using the method proposed here are shown in (c) and (e), respectively.

References

Bakker, P., van Vliet, L.J. and Verbeek, P.W., 1999. Edge preserving orientation adaptive filtering, Proc. IEEE-CS Conf. Computer Vision and Pattern Recognition. IEEE Computer Society Press, Fort Collins, CO, pp. 535-540.

Bentum, M., 1996. Interactive visualization of volume data. Ph.D. Thesis Thesis, University of Twente, Enschede, The Netherlands.

Dalley, R.M. et al., 1989. Dip and azimuth displays for 3D seismic interpretation. First Break, 7(3): 86-95.

Granlund, G.H. and Knutsson, H., 1995. Signal processing for computer vision. Kluwer Academic Publishers, 437 pp.

Haglund, L., 1992. Adaptive multidimensional filtering. PhD thesis Thesis, Linkoping University, Linkoping, Sweden.

Hocker, C. and Fehmers, G., 2002. Fast structural interpretation with structure-oriented filtering. The Leading Edge, 21(3): 238-243.

Jahne, B., 1997. Digital image processing. Spring-Verlag.

Johansson, B. and Farnebäck, G., 2002. A theoretical comparison of different orientation tensors, Proceedings SSAB02 Symposium on Image Analysis, Lund, Sweden, pp. 69-73.

Kass, M. and Witkin, A., 1987. Analyzing oriented patterns. Computer Vision, Graphics and Image Processing, 37: 362-385.

Knutsson, H., 1989. Representing local structure using tensors, 6th Scandinavian Conference on Image Analysis, pp. 244-251.

Marfurt, K.J., Kirlin, R.L., Farmer, S.L. and Bahorich, M.S., 1998. 3-D seismic attributes using a semblance-based coherency algorithm. Geophysics(63): 1150-1165.

Marschner, S.R. and Lobb, R.J., 1994. An evaluation of reconstruction filters for volume rendering, Proceedings of Visualization '94, Washington, DC, pp. 100-107.

Mester, R., 2000. Orientation estimation: conventional techniques and a new non-differential approach, European Signal Processing Conference (EUSIPCO 2000), Tampere, Finland.

Möller, T., Mueller, K., Kurzion, Y., Machiraju, R. and Yagel, R., 1998. Design of accurate and smooth filters for function and derivative reconstruction, Proceedings of the 1998 Symposium on Volume Visualization. IEEE, pp. 143-151.

Neumann, L., Csébfalvi, B., Viola, I., Mlejnek, M. and Gröller, E., 2002. Feature-preserving volume filtering, Proceedings of Joint Eurographics - IEEE TCVG Symposium on Visualization 2002.

Randen, T. et al., 2000. Three-dimensional texture attributes for seismic data analysis, 70th Ann. Internat. Mtg: Soc. of Expl. Geophys. SEG, Calgary, Canada.

Rao, A.R. and Schunck, B.G., 1991. Computing oriented texture fields. Computer Vision, Graphics and Image Processing, 53(2): 157-185.

Steeghs, P., Fokkema, J.T. and Diephuis, G., 1998. 3-D local radon power spectra for seismic attribute extraction, 68th Ann. Internat. Mtg: Soc. of Expl. Geophys. SEG.

Van Spaendonck, R.L.C., Steeghs, T.P.H., Overeem, I., Fernandes, F.C.A. and Fokkema, J.T., 2001. Wavelet-based volume attributes for seismic interpretation, 63rd Mtg.: Eur. Assn. Geosci. Eng. EAGE.