Customized a priori information in gravity inversion of a sedimentary basin relief

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Abstract

We present a new 2D gravity inversion approach for estimating the basement relief of a sedimentary basin. It consists in stabilizing the inversion by constraining just a small interval of the estimated relief instead of constraining the whole set of parameters describing the basin relief. The interpreter specifies the shape of a feature he believes to exist in the basement, and the method looks for a relief estimate closest to the specified feature and at the same time fitting the observed anomaly. Test with synthetic data showed that the method correctly identifies the location and size of the specified feature. Besides, an ambiguity region is automatically obtained as a by-product of the method implementation. This approach presents the advantage of allowing the introduction of different kinds of a priori information in the same inversion problem in contrast with traditional approaches which require that the same type of a priori information hold for the whole area. In addition, there is no theoretical limit for the complexity of the specified feature.

Introduction

Gravity inversion of the basement relief of a sedimentary basin is a ill-posed problem requiring the introduction of a priori information to guarantee solution stability. Substantial effort has been directed to the formulation of well-posed interpretations by the incorporation of specific a priori information via Tikhonov's regularization method (Tikhonov, 1977). The a priori information more commonly used so far is that the estimated basement relief be smooth. The degree of smoothness is established by the interpreter, and it must be above the minimum smoothness necessary to produce stable solutions (Pilkington and Crossley, 1986; Leão et al., 1996, Barbosa et al, 1997, for example). An extension of this approach establishes that the estimated basement relief be overall smooth, but may present local abrupt discontinuities (Barbosa et al., 1999). In this case, additional information about the maximum basin depth is also required to guarantee solution stability.

A strong limitation of the above methods is that a single kind of a priori information is introduced under the assumption that it holds for the whole area being interpreted. However, geological environments are too complex to be described by a single qualitative attribute, such as "smoothness", "roughness", or "discontinuities" holding over the whole study area. Therefore, the development of interpretation methods allowing the introduction of a priori information in a more flexible way would be most desirable. Specifically, the method should permit the interpreter to introduce whatever geometric information he has about *local* basement features, and not about the whole area.

We present a new gravity inversion method for estimating the basement relief of a sedimentary basin. The interpretation model consists of a set of vertical, juxtaposed prisms, whose thicknesses are the parameters to be determined. The user specifies the **shape** of any local feature he believes to exist in the basement such as an anticline, a syncline, a step fault or any combination of these structures. The method finds the *x*- and *z*-positions (x_o and z_o) of the structure and a scale factor (β) defining the feature size. This is accomplished by a systematic search, in the x_o - z_o - β space, for the point x_o^*, z_o^*, β^* producing geophysical solutions which minimize the Euclidean norm Φ of the difference between the estimated relief and the relief specified by the interpreter. Only the relief information within the segment specified by the interpreter is used in the stabilization process, but the interpretation model is defined over the entire basin. As a result, the estimated relief outside the segment containing the a priori information is not stable. The result is a 3D function Φ (x_o, z_o, β), whose minimizer x_o^*, z_o^*, β^* estimates both the dimension (β^*) and the position (x_o^*, z_o^*) where the specified feature is expected to occur.

As a by-product of the method, ambiguity regions are obtained by plotting the hypersurface Φ^ε (x_o, z_o, β), where Φ^ε is the value of the objective function Φ associated with an acceptable precision in estimating the basement relief.

Method

Let *B* be the basement relief of a 2D sedimentary basin *S* presenting a constant and known density contrast ρ relative to the basement. We approximate *S* by the volume consisting of a set of *M* vertical, juxtaposed 2D prisms with density contrast ρ, so that the basement relief *B* is approximated by a broken line joining the central points of the bases of all adjacent prisms (Figure 1). We assume that the gravity anomaly over the basin is known at *N* points. Also assume that the interpreter wants to verify whether a given basement feature, defined by a polygon *F* (specified at an arbitrary position and with arbitrary dimension) (Figure 2), and whose geometry is obtained from a priori geological information, is consistent with the available gravity data. To this end, a grid is defined on the x-z plane and the centroid *C* of *F* is made to coincide with each grid point (x^o, z^o). For each fixed point (x^o, z^o), the feature *F* is expanded or contracted by multiplying the line segments joining the centroid *C* with each vertex of *F* by a factor β>0, for a suit of pre-specified values of β. Then, for each point (x^o, z^o,β) the following inverse problem is solved:

$$\text{minimize} \quad \Phi = \sqrt{\frac{\sum_{L}^{K}\left[p_j - p_j^o(x^o, z^o)\right]^2}{K-L+1}} \qquad (1)$$

subject to

$$\Psi = \sqrt{\frac{\sum_{1}^{N}\left[g_i^o - g_i(\mathbf{p})\right]^2}{N}} = \delta, \qquad (2)$$

where $\mathbf{p} \equiv \{p_j\}$ is an M-dimensional vector of parameters defining the prisms thicknesses, g_i^o is the *i*th gravity observation, $g_i(\mathbf{p})$ is the theoretical gravity anomaly at the *i*th position, produced by the set of *M* adjacent vertical prisms, δ is the rms of the noise realizations contaminating the observations, and p_j^o are the depths to the specified feature *F* centered at (x^o, z^o). These depths are interpolated at the same *x*-coordinates of the center of the prisms defining the interpretation model. Note that, in general, the summation in equation (1) is not over all *M* prisms, but over a subset of *M*, defined by *K*-*L*+1 prisms (Figure 3).

The problem formulated in equations (1) and (2) is solved via Lagrange multipliers, by obtaining the unconstrained minimum of

$$\tau = \Psi(\mathbf{p}) + \mu\Phi(\mathbf{p}) \qquad (3),$$

where μ is the inverse of the Lagrange multiplier. It is selected as the smallest positive value still producing stable solution. The solution stability is inferred by inverting theoretical anomalies corrupted with different pseudorandom noise sequences.

The proposed method may be described as a systematic mapping of the minima $\Phi_{min}(x_o, z_o, \beta)$ in parameter space x_o - z_o - β, where Φ_{min} is the minimum of Φ resulting from the minimization of τ given in equation (3). The basis of this space define the dimension (β) and the position (x_o, z_o) where the specified feature is expected to occur. The minimizer x_o^*, z_o^*, β^* of $\Phi_{min}(x_o, z_o, \beta)$ is the point where the solution is closest to the specified feature *F* and produces an anomaly consistent with the observations. In addition, mapping Φ_{min} (x_o, z_o, β) provides an appraisal of the confidence on each possible solution (if any), simply by inspecting the hypersurface $\Phi_{min}^{\varepsilon}(x_o, z_o, \beta)$, where $\Phi_{\min}^{\varepsilon}$ is the value of Φ_{min} associated with an acceptable precision in estimating the basement relief.

Example with synthetic data

Figure 4a shows the gravity anomaly produced by the basement relief of a homogeneous sedimentary basin presenting a density contrast of –0.3 g/cm^3 relative to the basement (Figure 4b). The theoretical anomaly was corrupted with pseudorandom Gaussian noise with zero mean and standard deviation of 0.3 mGal. We want to verify if and where feature *F* shown in Figure 5 may be considered a geophysical solution. If the centroid of F is centered at x=16 km and z=2 km, the feature will coincide with the central basement uplift shown in the central part of Figure 4b. To apply the proposed method, a grid of 15 × 15 × 5 points along x, z, and β, respectively, was established. Figure 4b shows the grid nodes on plane x - z. The grid nodes values assigned to β were 0.8, 0.9, 1, 1.1, and 1.2.

The proposed method was applied to the anomaly of Figure 4a using a stabilizing parameter μ = 0.3 (the smallest value found to produce stable solutions). A set of 62 vertical, juxtaposed prisms were used as interpretation model. The results are shown in Figure 6 where five slices of the function Φ_{min} (x_o, z_o, β) are shown, one slice for each value assigned to β. The region between the minimum value of Φ_{min} and Φ_{min} + 0.2 km is displayed in green. This region represents an ambiguity region within which the solution explains the observations and fits the feature *F* within an average precision of 200 m. Reasonably well-defined minima occur on all slices about *x*=16 km and *z*=2 km, but the slice corresponding to the correct value (β=1) presents the more precise minimizer as indicated by the smallest ambiguity region (green region in Figure 6).

To confirm the above analysis, we show in Figure 7 the estimated solution (red), the true basement (blue), and the feature *F* (black), computed at x_o=16 km, z_o=2 km, and β=1. In the interval [12 km, 22 km], where the a priori information is constraining the estimated relief, it is very close to the true relief. Outside this interval, there is no a priori information constraining the estimated relief, so it is very unstable.

Figure 8 shows a similar plot for x_o=16 km, z_o=1 km, and β=0.8. Although the estimated relief is stable in the interval [12 km, 22 km], it is very far from the true relief and cannot even be considered a geophysical solution because the observations are not fitted within the experimental errors.

Conclusions

We presented a new gravity inversion method for interpreting structural features of the basement of a sedimentary basin in a stable and operational way. The interpreter may introduce a priori information about any structural shape he knows (or believes) to exist on the basement surface. The method returns the structure location and factor scale compatible with the gravity data, and possible ambiguity regions.

The interpretation model used extends itself along the whole horizontal extent of the basin whereas the a priori information is introduced on just a restricted horizontal window. As a result, only part of the estimated basement relief is stabilized. This allows the incorporation of more complex a priori information because this information is not required to hold for the whole area.

A practical limitation of the presented method is that there is a loss of information at the top and at both horizontal borders equal to one half the respective maximum dimension prescribed for the feature being analyzed. In this way this method is more effective when applied to basins with large horizontal extent relative to the size of the specified feature.

The method can be easily adapted to take into account the increase of the sediment density with depth.

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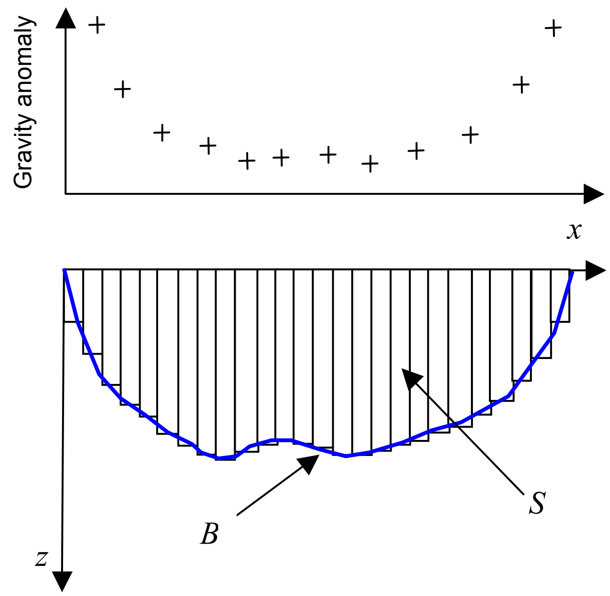

Fig. 1 – Gravity observations (above) produced by a sedimentary basin S (below), which is approximated by a set of juxtaposed vertical prisms with constant and known density contrasts. The basement B is approximated by a line joining the central points of the bases of all adjacent prisms.

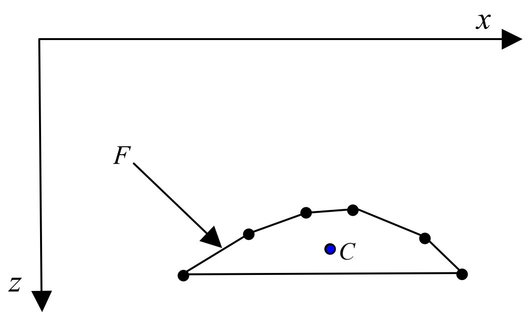

Fig. 2 – Feature *F* specified by the interpreter at an arbitrary position and with arbitrary dimension.

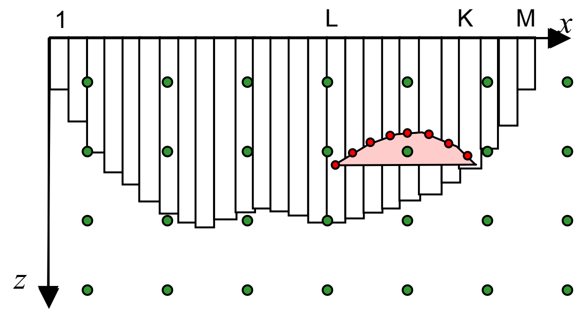

Fig. 3 – Grid (green dots) where the centroid of the specified structure (pink) is positioned. The depths to the structure (red dots) are interpolated at the horizontal positions coinciding with the centers of prisms L, L+1, … K .

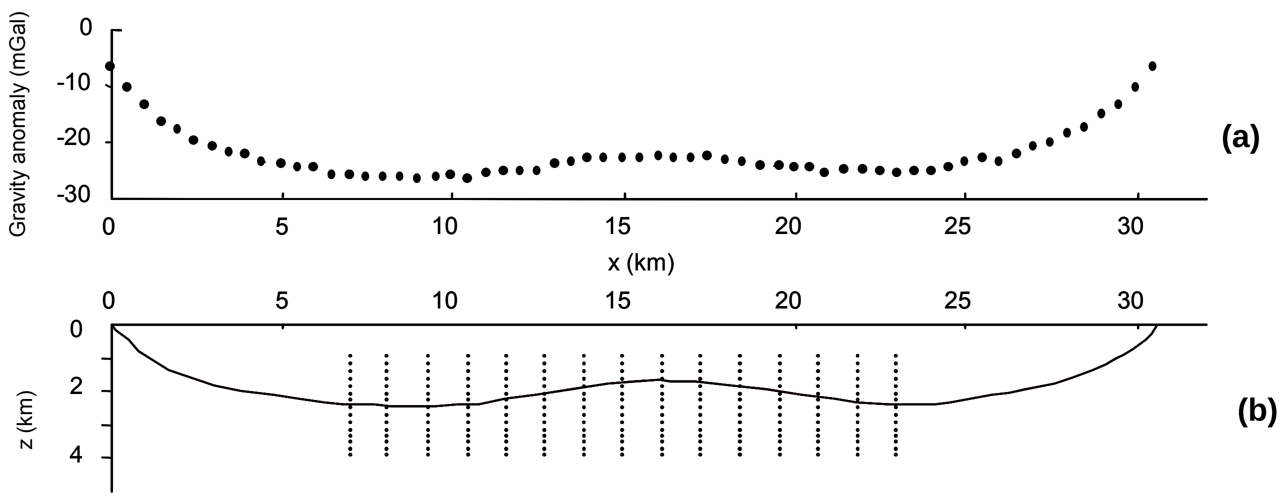

Fig. 4 - Gravity anomaly (a) produced by the basement relief (b) with a density contrast of –0.3 g/cm^3, and corrupted with pseudorandom Gaissian noise with zero mean and standard deviation of 0.3 mGal. Dots in (b) represent the grid where the centroid of feature *F* will be positioned.

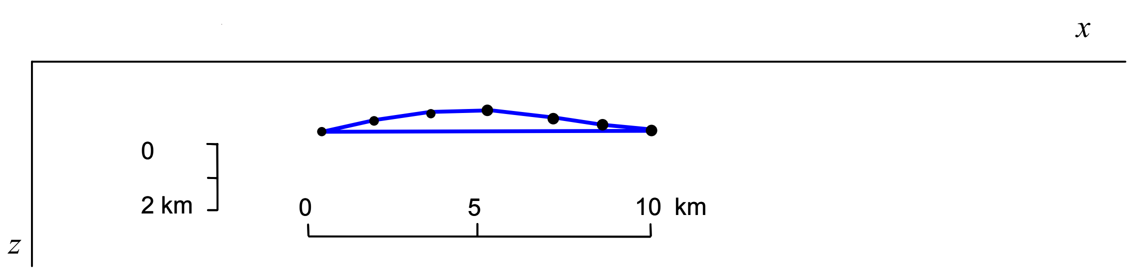

Fig. 5 - Feature *F* (blue) employed in the synthetic tests.

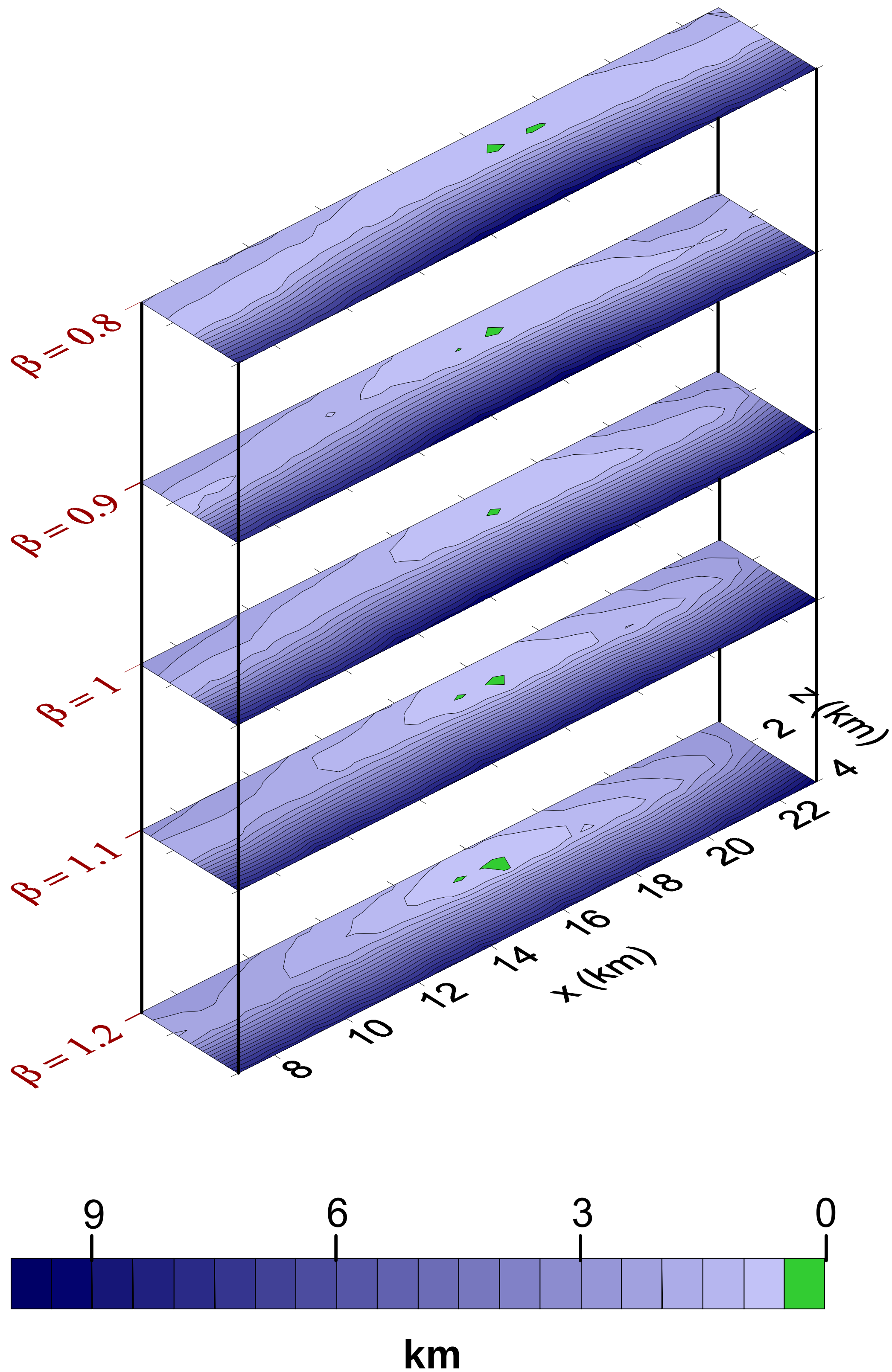

Fig. 6 – Function $\Phi_{min}(x_o, z_o, \beta)$ for five different values of β.

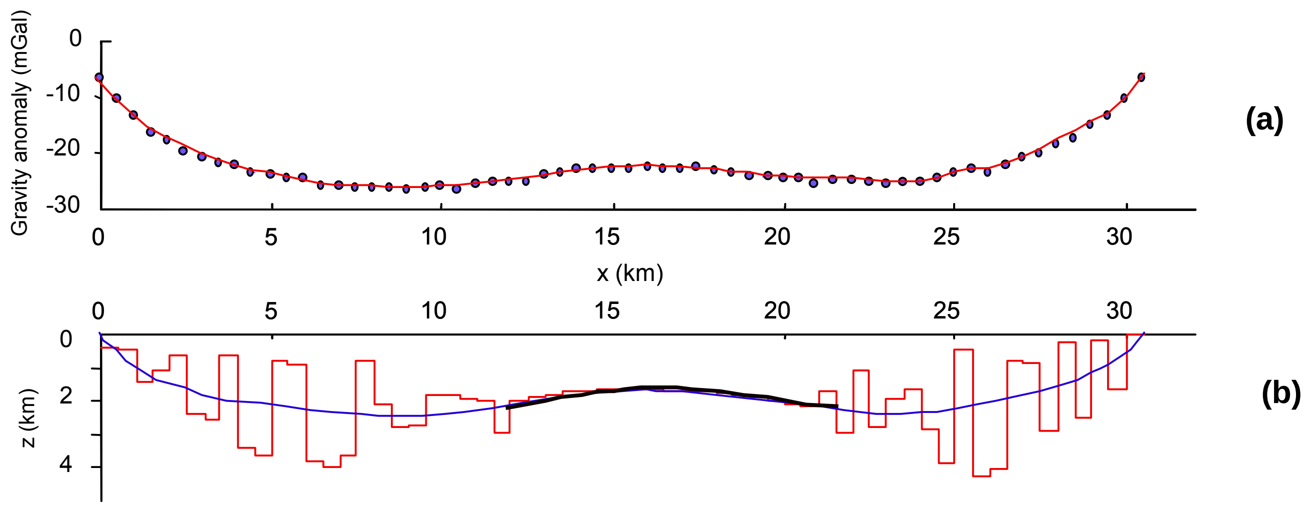

Fig. 7 - (a) Observed (dots) and fitted (solid red line) gravity anomalies. (b) True (blue) and fitted (red) reliefs produced when the centroid of feature *F* (solid black line in b) is centered at x_o =16 km and z_o =2 km and assuming β=1.

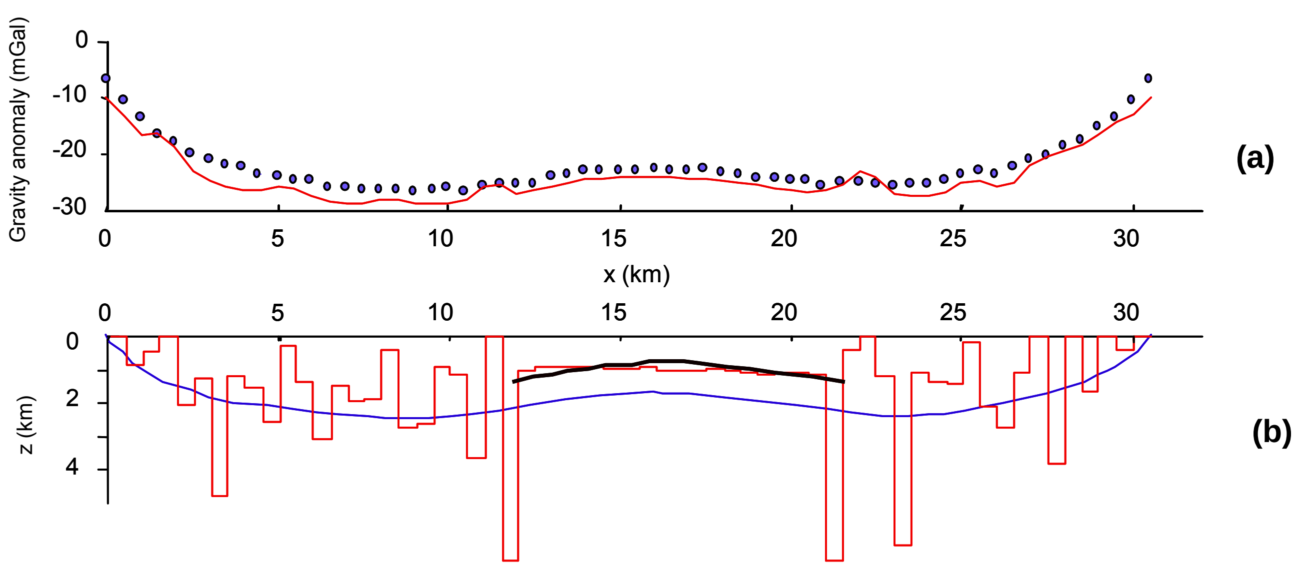

Fig. 8 - (a) Observed (dots) and fitted (solid red line) gravity anomalies. (b) True (blue) and fitted (red) reliefs produced when the centroid of feature *F* (solid black line in b) is centered at x_o =16 km and z_o =1 km and assuming β=0.8