Gravity inversion and Uncertainty analysis Using Simulated Annealing: An Application over Lake Vostok, East Antarctica

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Abstract

Interpretation of gravity data warrants uncertainty estimation due to its inherent non-uniqueness. Although the uncertainties in model parameters cannot be completely reduced, they can aid in the meaningful interpretation of the results. Here we have employed a simulated annealing (SA) based technique in the inversion of gravity data to derive multi-layered earth models consisting of two and three dimensional bodies. In our approach we assume that the density contrast is known and solve for the coordinates or shapes of the causative bodies, resulting in a non-linear inverse problem. We attempt to sample the model space extensively so as to estimate several equally likely models and use all the models sampled by SA to construct approximate marginal posterior probability density function (PPD) in model space, and several orders of moments. The correlation matrix clearly shows the inter-dependence of different model parameters and the corresponding trade-offs. We also investigate the use of derivative information to obtain better depth resolutions and to reduce the underlying uncertainties. We applied the technique on two synthetic datasets and an airborne gravity data set collected over Lake Vostok, East Antarctica for which *a priori* constraints were derived from available seismic and radar profiles

Introduction

The aim of the inversion of gravity anomalies is to derive density and shape (including depth) of the causative bodies. Ambiguity in gravity inversion is well known; there exists significant trade-off between density and depth. There are basically two general approaches to formulate the inverse gravity problem: linear and non-linear. In a linear approach, the source is considered as an ensemble of a number of blocks of simple geometries, e.g. rectangular blocks, and the density of each block is determined. In the non-linear approach, the density contrast of the source is assumed known and the shape of the body is determined by determining the positions of the corners of arbitrary shaped polygons (Talwani et al, 1959) or by computing the thickness of equal width rectangular prisms (Bott, 1960). In this case, the relation between observed data and the unknown parameters is non-linear. Due to the inherent ambiguity in gravity interpretation, several models can satisfy the data equally well. Therefore, instead of trying to present a single absolute result, it is desirable that we derive the uncertainties in the estimated results and present these in a quantitative manner. The uncertainty analysis helps to visualize the acceptability of the results by showing strong as well as poorly delineated areas of the source volume.

We have used the non-linear approach to invert two- as well as three dimensional gravity anomalies over two layered basins composed of water and sediment respectively. The Very Fast Simulated Annealing (VFSA) technique is used to invert the gravity anomaly and we use a large number of inverted models to construct marginal posterior probability density function (PPD) and different statistical measures, viz., mean, variance and correlation, to analyze the uncertainty. We demonstrate our technique with application to two synthetic data sets and a 3D air-borne gravity data over lake Vostok, East Antarctica.

Method

Formulation of forward problem and objective function

The very first task of inversion is formulating the forward problem in such a way that we can use maximum a-priori information to restrict the models within local geology. For two dimensional inversions, we followed the method of Talwani (1959) and modeled the basin with an arbitrary shaped polygon. For three dimensional inversions, the basin is assumed to consist of a large number of elementary 3D prisms with their tops coincident with the reference plane. The horizontal dimensions of all the prisms are known and restricted to a fixed value.

We solve the nonlinear inverse problem by minimizing a functional named as misfit which can be represented as

$E = \omega_1\varepsilon_1 + \omega_2\varepsilon_2$

Where, ε_1 and ε_1 represents the misfit of data and the corresponding derivatives respectively, where as, ω_1 and ω_1 are the weighting factors.

We use Very Fast Simulated Annealing (VFSA) – a well known global optimization technique, to minimize the objective function. We allow the algorithm to generate several thousand models, selected from a pre-defined model space. Using Bayesian statistics, we use all the models to construct approximate marginal posterior probability density function (PPD) and several orders of moment viz., variance and correlation (Sen and Stoffa 1995). The moments are used to study underlying uncertainties, or in other words, the degree of reliability of the inverted results.

Results

Gravity data over Lake Vostok, East Antarctica

The data over the lake were acquired at a sampling interval of 250m along the profiles across the lake and the profiles were separated by 7.5 km. Figure 1a shows the reduced gravity data, interpolated using a sampling interval of 2.6km. Each observation point is associated with two vertically adjacent prisms, one for the upper layer (ice & water) and another for the lower layer (sediment). Figure 1b shows the computed anomaly obtained from the inverted model without using any derivative information in the objective function. The water column thickness and sediment thickness are shown in figures 1(c) and (d) respectively. The plot of variance (Figure 1e) of water depth reveals that results along the edge of the basin, especially along the eastern boundary, are more ambiguous (or less certain), where as, the lake bottom is fairly well determined. We find that along the lake axis, there exist three structural lows, out of which, one in the southern end is relatively shallow and the middle and northern parts are as deep as 1.2 km.

To reduce the uncertainties along the lake-boundary, we incorporate horizontal gradient information in our objective function by using $\omega_1 = \omega_2 = 0.5$. Further, from the magnetic data over that area (Studinger et al., 2002), it can be inferred that there exists a sharp fault along the eastern boundary of the lake and the bed rock density is higher across the fault. We constrain our model to have density contrasts of -1.6 and -0.6 gm/cc for the upper and the lower layers for the prisms present in the western side of the fault and -1.8 and -0.8 gm/cc for those across the fault. The results obtained using this new density distribution as well as derivative information show sufficient improvement in computed anomaly (Figure 1f), as well as, in edge-uncertainties (Figure 1h). The basic structures of the mean models for water and sediment thickness (Figure 1g & 1i) are similar to the previous one, though the maximum depth of the basement is greater in comparison to the previous one as the density contrast is considered less in most of the part of the model. We further construct an approximate marginal posterior probability density function (PPD) for each parameter based on the inverted models following a method outlined by Sen and Stoffa (1996). The plot of PPD over a profile across and another one along the lake elongation (Fig. 2a and 2b) shows wider spread of PPD along the sharp edges of the lake in comparison to its bottom area.

References

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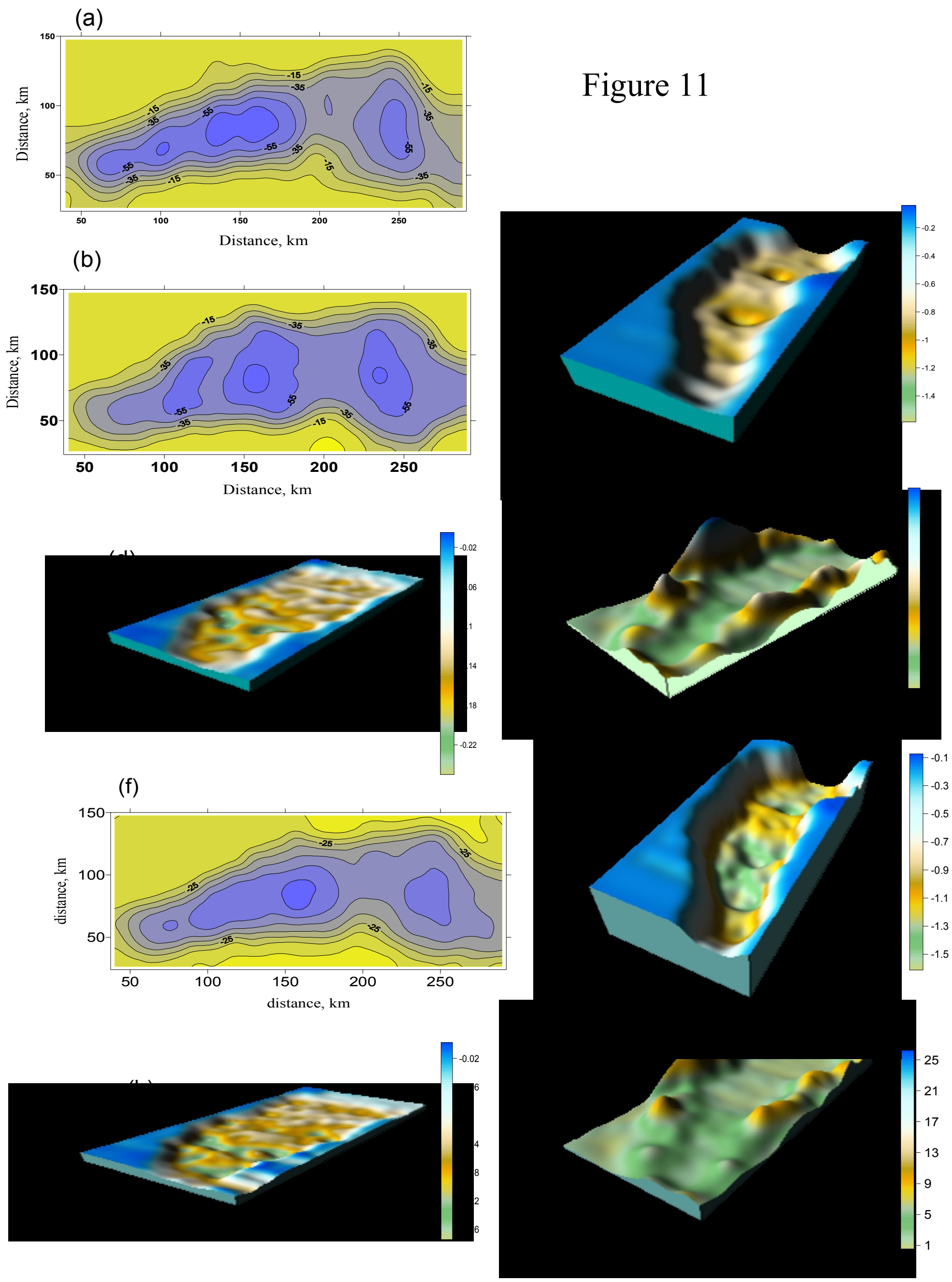

Figure 1. Results of 3D inversion of gravity data over Lake Vostok: (a) Observed data interpolated to a regular grid, (b) best fit computed anomaly without using derivative information in objective function, (c) corresponding water depth and (d) sediment thickness, (e) normalized percentage variance of water depth, (f) computed anomaly using derivative information in objective function, (g) corresponding water depth and (h) sediment thickness, (i) normalized percentage variance of water depth.

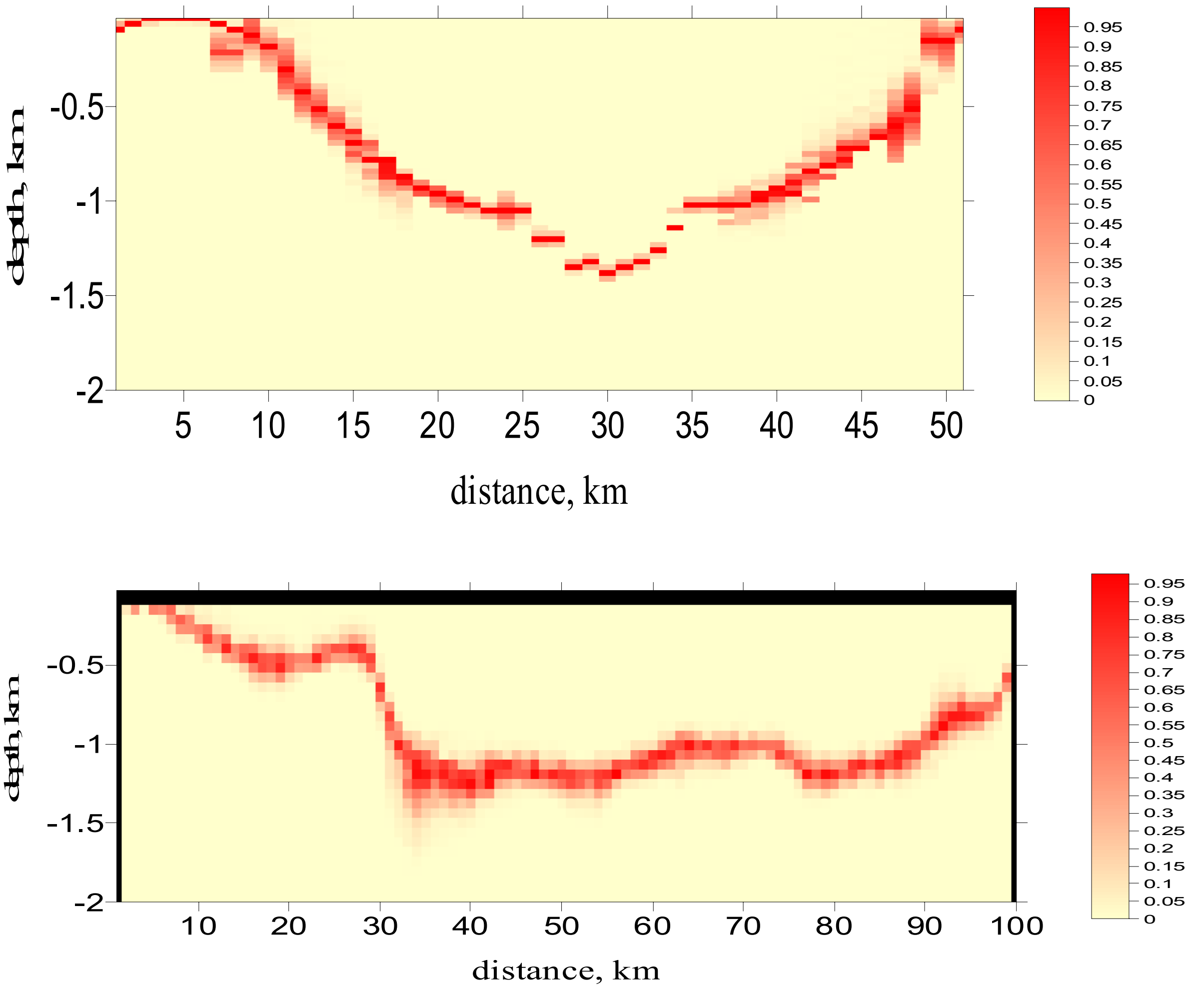

Figure 2. Plot of posterior probability density (PPD) for each depth poi over a profile across the lake and (b) over a north –south profile alon lake.