A 3D Discrete Simulation of Secondary Oil Recovery

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Abstract

We propose the simulation of oil recovery by means of a molecular type approach. By using a finite set of particles under the interaction of a Lennard-Jones type potential we simulate the behavior of a fluid in a porous media, and we show under certain conditions that the fingering phenomena appears

Introduction

In this work we propose the simulation of oil recovery by means of a molecular type approach. This means that we consider the materials to be composed of a finite number of particles, which are approximates for molecules. Porous flow is studied qualitatively under the assumption that particles of rock, oil and the flooding flow interact with each other by means of a compensating Lennard- Jones type potential. We also consider the system to be under the influence of gravity. We study miscible displacement in an oil reservoir from various sets of initial data. The velocity and the rate of injection of the ingoing particles proved to be among the most important parameters that can be adjusted to increase the rate of production. It is also noted that the fingering phenomenon is readily detected. This simulation technique has been used in [2-5] to simulate several physical systems. Details of this method applied to the study of porous flow can be founded in [1,6].

Model formulation

Consider a cubic region R, which is a porous medium. We assume that in this region we have a resident fluid or oil. We shall introduce a different kind of fluid which, as a matter of convenience, will be called water, although it is an aqueous solution which could be a polymeric solution, surfactant solution or a brine. The physical system consists of $N=N_1+N_2+N_3$ particles, $P_1, P_2, \ldots, P_N$, with masses $m_1, m_2, \ldots, m_N$. The particles

$P_1, P_2, \ldots, P_{N_1}$ Represent rocks,

$P_{N_1+1}, P_{N_1+2}, \ldots, P_{N_2}$ Oil, and

$P_{N_2+1}, P_{N_1+2}, \ldots, P_N$ Incoming water

For purposes of injection of water and production of oil, two wells are opened, one in one corner of R, for injection, and other in the diagonally opposite corner for production, see Fig. 1and 2.

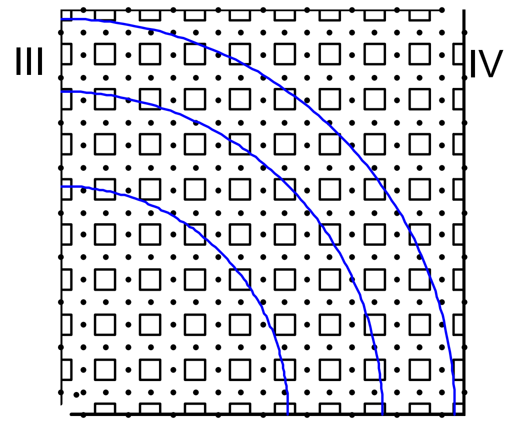

Fig. 1. Initial configuration in two dimensions with four subregions of equal area.

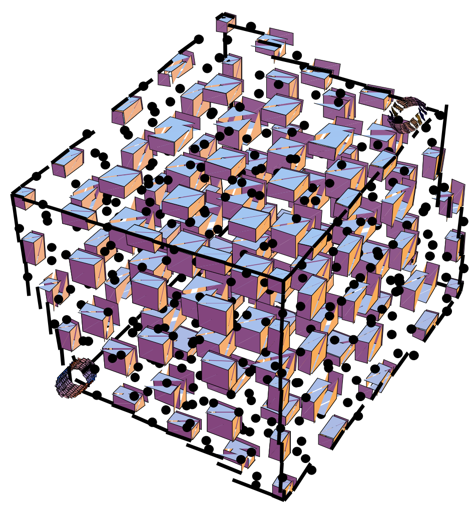

Fig. 2. Initial configuration in three dimensions

The variables at time $t_k = k\Delta t$ are:

$\mathbf{r}_{i,k}$	Coordinates of the ith particle
$r_{i,j,k}$	Distance between the particles P_i and P_j
$\mathbf{v}_{i,k}$	Velocity of the ith particle,
$\mathbf{a}_{i,k}$	Acceleration of the ith particle,
$\mathbf{F}_{i,j,k}$	Local force exerted on P_i by P_j ,
$\mathbf{F}^*_{i,k}$	Local force acting on P_i due to the other particles,

$\bar{\mathbf{f}}_{i,k}$ Long range force acting on P_i (like gravity),

$\mathbf{F}_{i,k}$ Total force on particle P_i

for i=1,2,...,N and k=0,1,...

The local force $F_{i,j,k}$ exerted on P_i by P_j , is

$$\mathbf{F}_{i,j,k} = m_i m_j \left[\frac{H_{i,j}}{r_{i,j,k}^{q_{i,j}}} - \frac{G_{i,j}}{r_{i,j,k}^{p_{i,j}}} \right] \frac{\mathbf{r}_{j,k} - \mathbf{r}_{i,k}}{r_{i,j,k}} , \qquad (1)$$

where the values of the parameters $H_{i,j}$, $G_{i,j}$, $q_{i,j}$, and $p_{i,j}$ depend on the particles which are interacting. The total local force $\mathbf{F}^*_{i,k}$ acting on particle P_i due to the other particles is given by:

$$\mathbf{F}^*_{i,k} = \sum_{j=1, j\neq i}^{N} \mathbf{F}_{i,j,k} \ . \qquad (2)$$

Therefore, the total force acting upon the particle P_i is

$$\mathbf{F}_{i,k} = \mathbf{F}^*_{i,k} + \mathbf{f}_{i,k} \qquad (3)$$

The acceleration of P_i is related to the force by Newton's Law :

$$\mathbf{F}_{i,k} = m_i \mathbf{a}_{i,k} \ . \qquad (4)$$

In general, the second order differential equations system (4) can not be solved analytically from given initial positions and velocities, therefore it must be solved numerically. For economy, simplicity and relative numerical stability we use the “leap frog" formulae, which has second-order accuracy in time.

$$\mathbf{v}_{i,1/2} = \mathbf{v}_{i,0} + \frac{1}{2}\mathbf{a}_{i,0}\Delta t$$

$$\mathbf{v}_{i,k+1/2} = \mathbf{v}_{i,k-1/2} + \frac{1}{2}\mathbf{a}_{i,k}\Delta t$$

$$\mathbf{r}_{i,k+1} = \mathbf{r}_{i,k} + \frac{1}{2}\mathbf{v}_{i,k-1/2}\Delta t$$

for i=1, 2,…, N and k=1,2,…

The number of calculations required to evaluate (1) at each iteration is O(N^2). However this number is much smaller if the force is truncated for a distance greater than r_c .

Initial conditions

The rock and oil particles, for two an three dimensions, were set up at the initial time in such a way that they satisfied an equilibrium state, as shown in Fig. 1 and 2.

Boundary conditions

We assume that the particles of the fluids loose energy when they interact with the walls of the region R, therefore it will be necessary to model the hardness of the wall relative to the reflection of the interacting fluid, and this is done by using the following damping factors acting on the velocity of the reflected particles, as shown in Figure 3.

$$\delta_i = 0.4 \quad for \ i = N_1 + 1, ..., N_1 + N_2, \ \ and$$

$$\delta_i = 0.8 \quad for \ i = N_1 + N_2 + 1, ..., N.$$

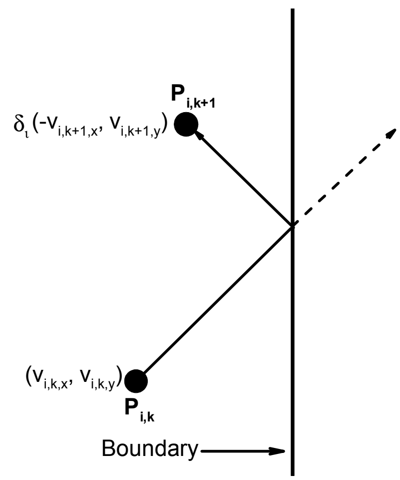

Fig. 3. Reflexion of the position and damping of the velocity

Numerical results in 2D

All the examples were run with time step $\Delta t = E-5$ on a cluster of pc computers, the distance between particles of water before going into the well was d=0.5 and their velocity was v=15. The gravity constant was equal to g=9.8. The Lennard-Jones potential parameters are summarized in table 1.

	Rock	Oil	Water
Rock	*H=0* *G=0*		
Oil	*H=1* *G=3* *E=0.60*F*	*H=1* *G=1* *E=1.3*F*	
Water	*H=1.5* *G=0* *F=F*13/36*	*H=1* *G=0* *E=1.15*F*	*H=1* *G=0* *E=F*

Table1. Parameters for the numerical experiments in two dimensions. In this case *F=0.5*.

Figure 4 shows the advancement of water for different times, the shaded area is the region which has been traveled only by water, this means that no oil particle has been in that area for some time. This figures remind us of the fingering phenomena.

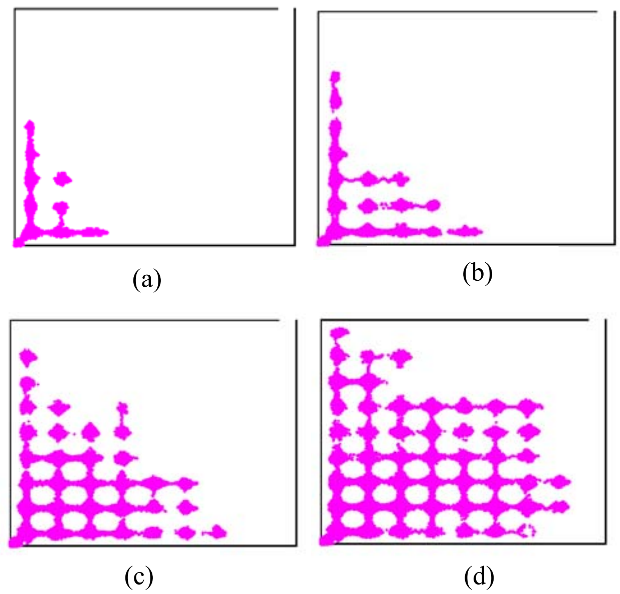

Fig. 4. Advancement of water for *d=1, v=5*, at different times. (a) *Iter=3E5*, (b) *Iter=4.25E5*, (c) *Iter=7.8E5* and d) *Iter=1.2E6*.

Figure 5 shows the number of particles of oil out and the number of particles of water out versus time. We can see from the graph that for t small, the rate of oil production is higher when v is higher. We can also observe that water comes out of the production well sooner for *v=100* than for *v=15*.

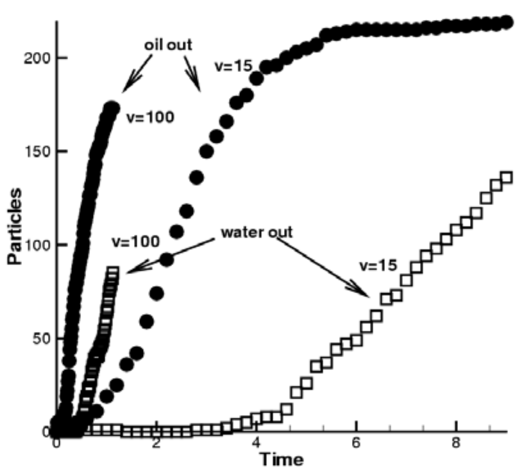

Fig. 5. Comparison of the effect of the velocity of the water particles on the oil and water particles.

Figure 6 shows the effect on the oil and water production in two different cases: first, the angle of water injection is fix to π/4. In the second case the angle is aleatory with a uniform distribution in the range *[π/4-π/6, π/4+π/6]*. In both cases the oil production is very similar.

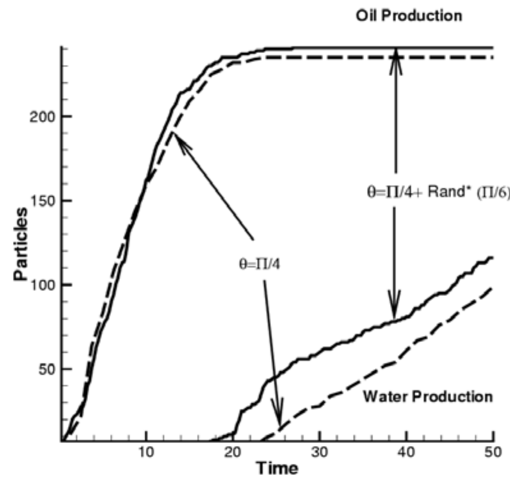

Fig. 6. Effect of the injection angle on oil and water production.

In figure 1 we showed the region R, subdivided in four subregions of the same area, and in figure 7 we observe the amount of oil and water particles in each one of the subregions. We see that the behavior of the oil and water is similar in all of them. Also we notice that the subregions tend to have a constant number of particles in each one.

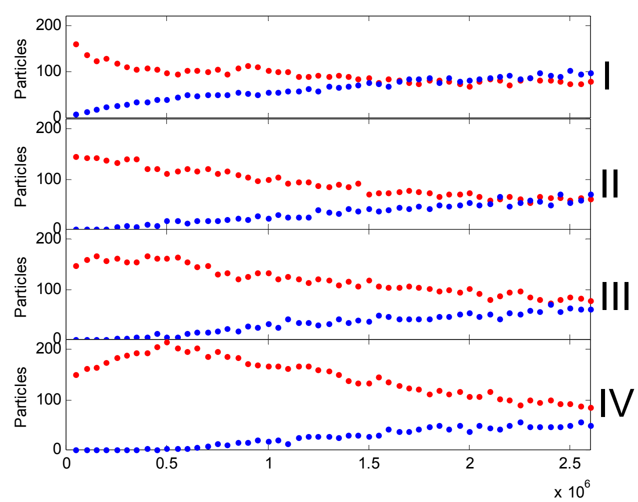

Fig. 7. Comparison of oil and water particles in each subregion. Oil in red and water in blue. d=1, v=5.

Numerical results in 3D

The evolution of the system, in three dimensions, is shown in Fig 8. All the examples were run with time step $\Delta t = E-5$ on a cluster of PC computers.

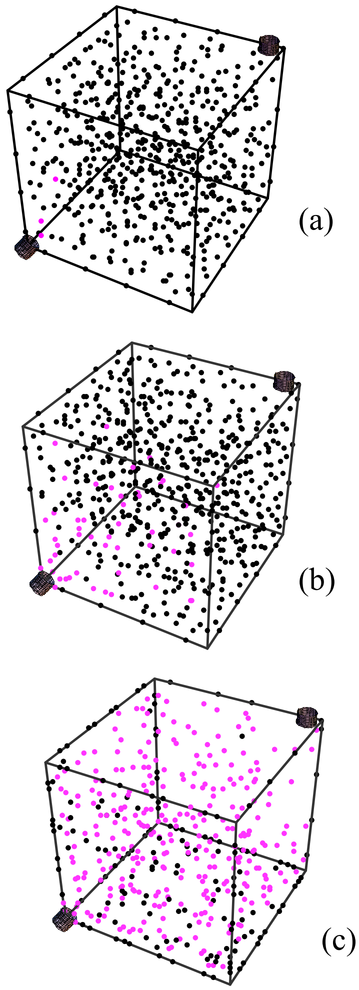

Fig. 8. Evolution of the oil and water particles. (a) Iter=50, 000, (b) Iter=1E6, and (c) Iter=8E6

The distance between particles of water before going into the well, was $d=0.5$ and their and velocity was $v=5.0$. The gravity constant was equal to $g=9.8$. The Lennard-Jones potential parameters are summarized in table 1, but in this case $F=1.0$.

Figure 9 shows the effect of the oil and water production, when the velocity of the water particles is increased. An increment in the velocity produced an increment on the oil and water production.

In figure 10 we have the effect of the water particles separation on the oil and water production. The production of both fluids increases when the particle separation decreases

The effect of the injection angle of water particles on the oil and water production can be observed in figure 11. We consider two cases, first the injection angle is fix and the vector velocity of the water particles is given by $\mathbf{v}=v\mathbf{n}$, where $\mathbf{n}=(1,1,1)/3^{1/2}$ is a unit vector. On the second case the velocity vector is choosen as $\mathbf{v}=v\mathbf{n}_1$, where $\mathbf{n}_1$ is an aleatory unit vector and the angle between $\mathbf{n}$ and $\mathbf{n}_1$ has a uniform distribution in the interval [0, $\pi/6$].

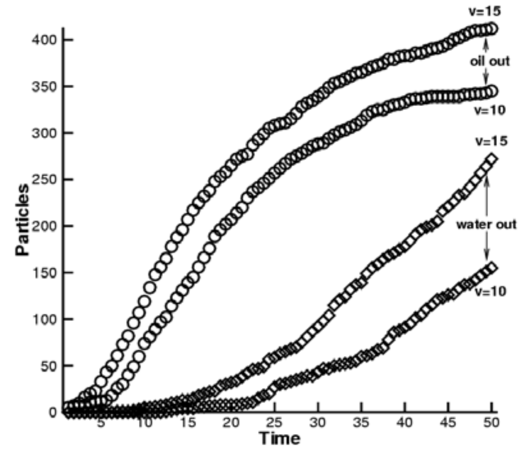

Fig. 9. Comparison of the effect of the velocity of the water particles and the oil and water production

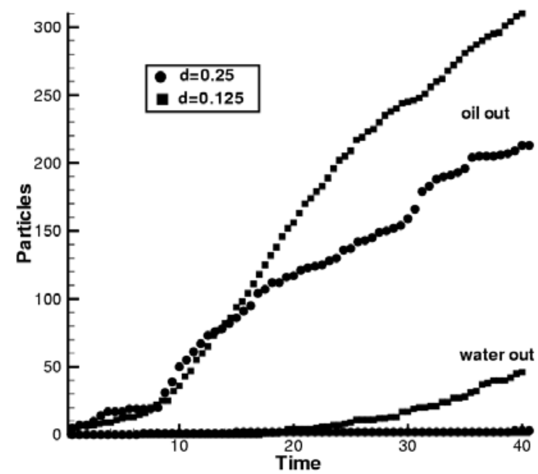

Fig. 10. Comparison of the effect of the water particle separation on the oil and water production.

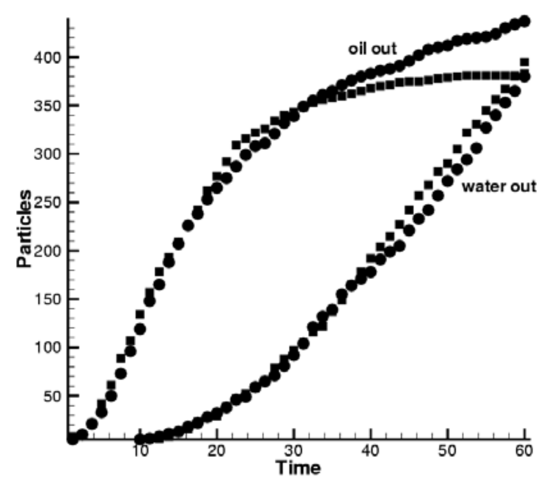

Fig. 11. Comparison of the effect of the injection angle of the water particles on the oil and water production. a) circles: $\mathbf{v}=v\mathbf{n}$, b) squares: $\mathbf{v}=v\mathbf{n}_1$.

Figure 12 shows the cubic region R, subdivided in four regions of the same volume each.

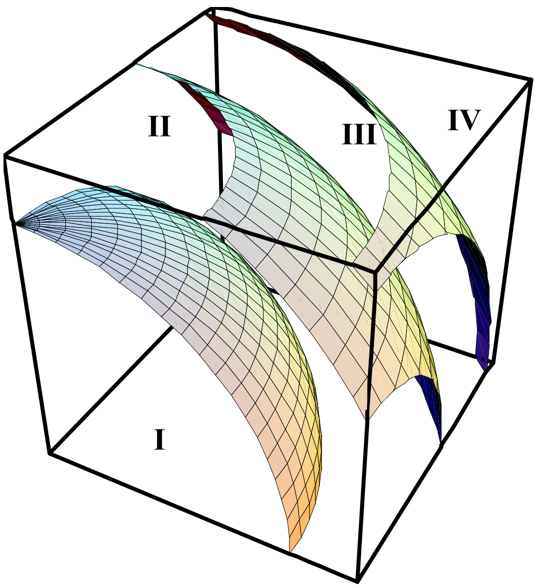

Fig. 12. Well with four subregions of equal volume

In Figure 13 and 14 we have the amount of oil and water particles un each one of the subregions. We observe that the number of particles tends to a constant in each one of them and higher injection velocities the equilibrium is reached in a shorter time.

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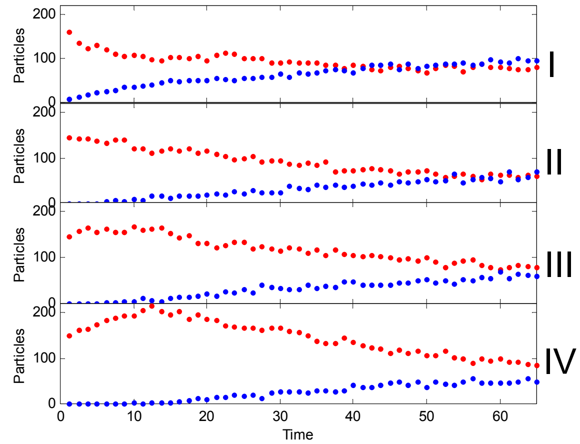

Fig. 13. Comparison of oil and water particles in each subregion. Oil in red and water in blue. d=1, v=5.

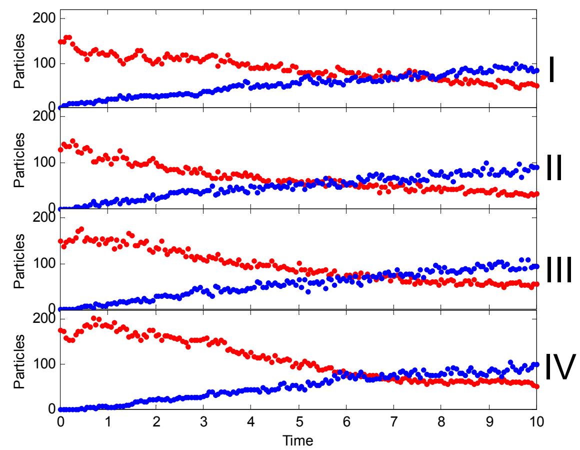

Fig. 14. Comparison of oil and water particles in each subregion. Oil in red and water in blue. d=1, v=50.

Acknowledgments

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