Calibration procedures of LaCoste & Romberg "G" meters of Observatório Nacional

Marcelo Ferreira de Souza* and Mauro Andrade de Sousa OBSERVATÓRIO NACIONAL, Brazil

This paper was prepared for presentation at the 8th International Congress of The Brazilian Geophysical Society held in Rio de Janeiro, Brazil, 14-18 September 2003.

Abstract

The most effective use of the LaCoste & Romberg "G" gravimeters of Observatório Nacional has been the establishment of the Brazilian Fundamental Gravity Network (Subiza Piña & De Sousa, 2001; Escobar, 1987). Given the metrological aspect of this effort it is absolutely desirable to model the behavior of our LC&R meters and assure that the highest attainable precision with the use of these meters has been reached. Calibration procedures have been devised to provide accurate corrections to the original LC&R Calibration Table, including periodic screw error effects. This paper deals with results of long range gravity measurements between eight absolute gravity stations in Brazil and Uruguay. Significant accuracy increase is obtained when the new calibration terms are taken into account.

Introduction

The LC&R Calibration Tables supplied with each gravity meter are known to be accurate to 10^{-3}-10^{-4} (Krieg, 1981; Kanngieser & Törge, 1981). Residual linear and higher order terms in the meter scale mainly affect large gravity differences and are due to the non-linearity of the lever system. Graduation errors and eccentricities of the measuring screw and the reduction gear train of the "G" meter induce errors in measuring small gravity differences (Valliant, 1991; Wenzel, 1996) . Instrumental repeatability is claimed to be of 0.01-0.02 while its accuracy is 0.04 mGal (LaCoste & Romberg, 1998) and there seems to be ways of improving the LC&R meter sensitivity up to 1 μGal (Ander *et alli*, 1999). The Observatório Nacional owns three ordinary "G" meters: G257, acquired in 1971 and G602 and G622 bought in 1984. Although fully operational, all meters were submitted to long-term service at the factory labs in October 1998, involving disassembly, cleaning, replacement of rubber seals and relubricating. According to LC&R, the measuring screws were not touched and therefore needed no factory recalibration. A first assessment of the residual scale coefficients was carried out in June 1997. Starting and finishing in Brasília, gravity differences were taken between the absolute gravity stations in Brasília, Viçosa, Vassouras, Vinhedo, Curitiba and Santa Maria (Figure 1). Each gravity interval was measured twice with a total number of twenty readings per meter and the total gravity range was about 1212 mGal. In July 1999 and in February and October 2000, always starting and finishing in Vassouras, other long range measurements campaigns were taken to Teresina and Rivera (Uruguay) broadening the gravity range in approximately 115 mGal. This time, each gravity interval was measured at least four times and 72 counter readings were observed with each meter.

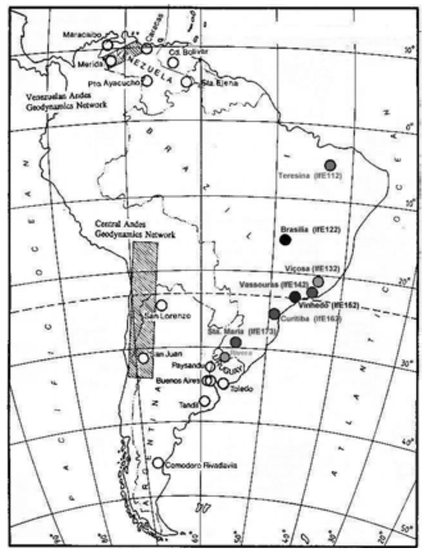


Figure 1 – Absolute gravity stations in South America (Torge *et alli*, 1994). In black, the absolute stations used in this study.

Method

Usually (Kanngieser & Törge, 1981; Gemael & Rosier, 1991; Schueler, 2000), calibration equations are formed following the so called Δ*g model*, where gravity differences are obtained from successive gravimeter readings in stations *i* and *j*:

$$g_i - g_j = \left(\sum_{k=1}^{m} z_i^k E_k + \sum_{k=1}^{m} z_j^k E_k\right) - \left(\sum_{n=1}^{p} A_n \cos(\omega_n z_i - \varphi_n) + \sum_{n=1}^{p} A_n \cos(\omega_n z_j - \varphi_n)\right). \quad (1)$$

Here, z is the tide and drift corrected reading, g is the known absolute gravity value, E_k is the k-th coefficient of the polynomial calibration function and m is the highest degree, A_n is the amplitude of the n-th periodic term, ω_n is the frequency, φ_n is the phase of the n-th periodic term and p is the maximum degree of the Fourier series. Sometimes, the meter drift model is included in the calibration equation and the counter readings are corrected for Earth tides only.

This study deals with the polynomial calibration function only. Periodic terms will be estimated in the near future.

Counter readings of all three meters were corrected for Earth tides according to the well known Longman´s formulas as well as for static and dynamic meter drifts. Meter temperature, air pressure and temperature and relative humidity were recorded for all readings. These data have not been used in the present study.

Application of the calibration equation (1) to pairs of readings in stations *i* and *j* leads to an overdetermined system of condition equations solved by the linear least squares method. Note that It is implicitly assumed that the Gauss-Markov theorem and maximum likelihood estimation are met by observations and associated errors. After the polynomial coefficients are computed a new Calibration Table can be produced for each gravimeter.

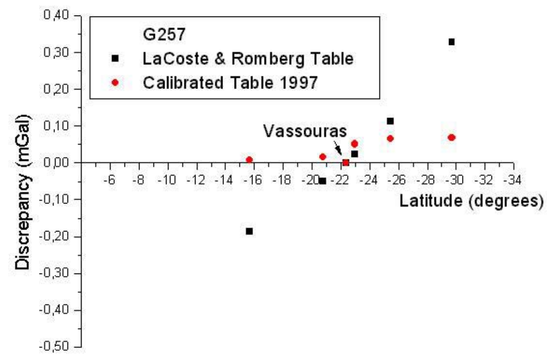
Results

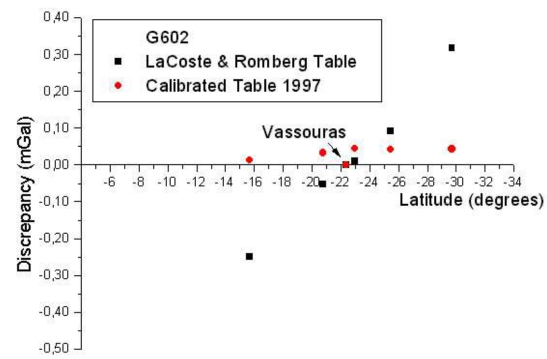
The best degree of the polynomial calibration function was found after the best fit of the LC&R Calibration Table. Reduced chi-square tests pointed out a second degree polynomial as the appropriate choice for the three meters. Residual linear and quadratic scale coefficients are shown in Table 1. Considering two standard deviations, note that the 1997 and the 1999/2000 coefficients are statistically indistinguishable, while both sets differ from those coefficients describing the LC&R Calibration Table. Table 2 shows for the 1997 campaign the gravity intervals computed either with the original LC&R Calibration Table and the Calibration Table obtained as in the previous section, compared to the known gravity differences obtained from absolute gravity stations. Use of the LC&R Table usually produces larger discrepancies, with the systematic exception of the Vassouras-Vinhedo gravity interval.

Table 3 shows the gravity intervals measured during the 1999/2000 campaigns, after the three meters returned from the LC&R labs. As before, discrepancies are usually higher when the original LC&R Table is used and the Vassouras-Vinhedo gravity interval is again the notable exception.

The final goal of this calibration study is to make gravity measurements taken with differential LC&R "G" meters consistent with the absolute datum and scale provided by the absolute gravity stations. This objective seems to have been reached when one examines Figures 2 and 3. Taking the absolute gravity station in Vassouras as the reference station and considering the 1997 dataset, Figure 2 shows the discrepancies found in the absolute gravity values in Brasília, Viçosa, Vinhedo, Curitiba and Santa Maria when the gravity intervals were computed using the LC&R Table and with the Calibrated Table obtained from coefficients of Table 1. The clear latitude dependency seen in the discrepancies computed with the LC&R Table are due to the cumulative error introduced by LC&R meters systematically measuring gravity intervals smaller than their true values. Such latitude dependency is not seen in the discrepancies computed with the 1997 Calibrated Table.

Figure 3 shows the discrepancies found in the absolute gravity values of Teresina, Brasília, Viçosa, Vinhedo, Curitiba, Santa Maria and Rivera when the gravity intervals were computed using the LC&R Table and with the Calibrated Table obtained from coefficients of Table 1. The Vassouras absolute gravity station was again taken as the reference station. Comments to Figure 2 also apply to Figure 3.





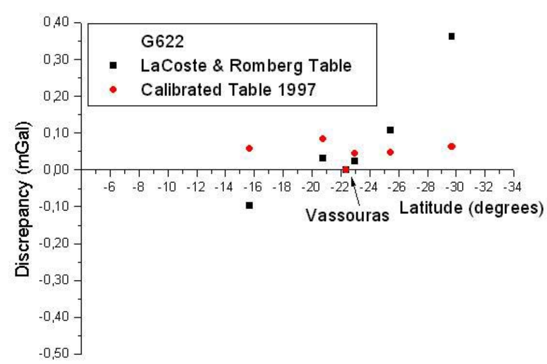
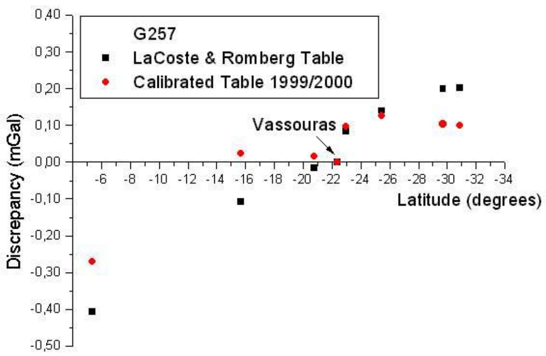
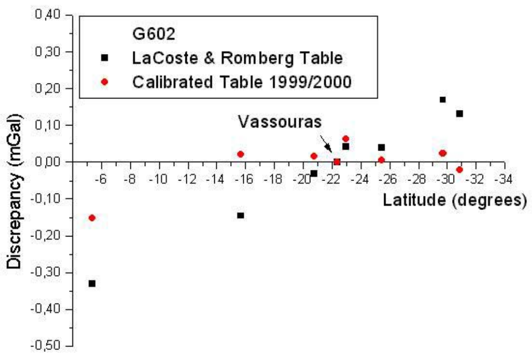


Figure 2 – Discrepancies in absolute gravity values in Brasília, Viçosa, Vinhedo, Curitiba and Santa Maria derived from the 1997 campaign. (a) G257 (b) G602 (c) G622.





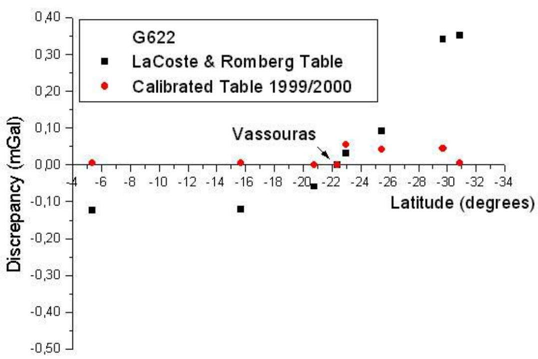


Figure 3 – Discrepancies in absolute gravity values in Teresina, Brasília, Viçosa, Vinhedo, Curitiba, Santa Maria and Rivera derived from the 1999/2000 campaign. (a) G257 (b) G602 (c) G622.

Discussion and conclusions

Results obtained so far regarding the calibration of LaCoste & Romberg "G" meters of Observatório Nacional demonstrated the ability to obtain absolute gravity values consistent with the datum and scale provided by the absolute gravity stations. The calibration model should be refined by including terms of a Fourier series describing the circular errors and more precise Earth tide model. Upon noticing that the matrix of coefficients of the overdetermined system of normal equations involves observational errors and, therefore, the Gauss-Markov theorem is not strictly satisfied (Branhan Jr., 1990), orthogonal (total) least-squares should be used in the minimization procedure.

Acknowledgments

We thank CAPES for providing a M.Sc. scholarship to M.F. de Souza.

References

Ander, M.E., Summers, T. and Gruchalla, M.E. 1999**,** LaCoste & Romberg gravity meter: system analysis and instrumental errors, *Geophysics*, **64**(6): 1708-1719.

Branhan Jr., R.D. 1990, Scientific data analysis: An introduction to overdetermined systems, *Springer-Verlag,* 237 pp.

Escobar, I.P. 1987, A Rede Gravimétrica Fundamental Brasileira, *Departamento de Geofísica*,*Observatório Nacional*, 2 vv.

Gemael, C. and Rosier, F. 1991, Calibração de gravímetros usando a RENEGA, *Annals II Congresso Internacional Geofísica*, **GRM 3**: 120-123.

Kanngieser, E. and Törge, W. 1981, Calibration of LaCoste & Romberg gravity meters, *I.A.G. Gravity Workshop of SSG 3.37 and 3.40*, Paris.

Krieg, L.A. 1981, Mathematical modeling of the behavior of the LaCoste & Romberg "G" gravity meter for use in gravity network adjustment and data analyses, *Report no. 321 Department of Geodetic Science and Surveying*, *Ohio State University*, 172 pp.

LaCoste & Romberg, Inc. 1998, Manual of model "G" gravity meters, http://www.lacosteromberg.com.

Nakagawa, I., Nakai, S., Shichi, R., Tajima, H., Izutuya, S., Kono, Y., Higashi, T., Fujimoto, H., Murakami, M., Tajima, K. and Funaki, M. 1983, Precise calibration of scale values of LaCoste & Romberg gravimeters and international gravimetric connections along the Circum-Pacific Zone (Final Report), *Geophysical Institute*, *Kyoto University*, 117 pp.

Schueler, T. 2000, Introduction to gravimetry: Conducting and processing relative gravity surveys. A brief tutorial, http://ifen/Lehre/ExerciseUniBW/Erd1Gravimetrie/RelGravTutorial.PDF, 15 pp

.**Valliant, H.D.** 1991, Gravity meter calibration at LaCoste and Romberg, *Geophysics*, **56**(5): 705-711.

Subiza Piña, W.H. and De Sousa, M. 2001, The gravity database of the Observatório Nacional and the current status of the Brazilian Fundamental Gravity Network, *Annals VII Congresso Internacional Geofísica*, Salvador.

Törge, W., Timmen, L., Röder, R.H. e Schnüll, M. 1994, The IfE absolute gravity program 'South America´ 1988-1991, *Deutsche Geodätische Komission*, B229, 44 pp.

Wenzel, H.-G. 1996, The vertical gravimeter calibration line at Karlsruhe, *Bull. d'Information Bureau Gravimetrique International*, **78**: 47-56

Table 1 – Coefficients of the polynomial calibration functions.

Gravimeter	Calibration coefficients			
	Degree	**LaCoste & Romberg Table**	**1997**	**1999/2000**
G257	**Linear**	1.06837 ± 0.00001	1.06875 ± 0.00004	1.0686 ± 0.0001
	Quadratic	0.00000044 ± 0.00000001	0.00000053 ± 0'.00000005	0.00000037 ± 0.0000001
G602	**Linear**	1.03796 ± 0.00001	1.03842 ± 0.00004	1.03823 ± 0.00005
	Quadratic	0.00000071 ± 0.00000001	0.00000069 ± 0.00000005	0.00000066 ± 0.00000008
G622	**Linear**	1.02004 ± 0.00001	1.02044 ± 0.00005	1.02040 ± 0.00005
	Quadratic	0.00000062 ± 0.00000001	0.00000077 ± 0.00000007	0.00000082 ± 0.00000008

Table 2 - Gravity intervals calculated with the LC & R Table and with the Calibrated Table of 1997. Figures are in mGal.

Known gravity interval	G257		G602		G622	
	LC & R Table	**Calibrated Table**	**LC & R Table**	**Calibrated Table**	**LC & R Table**	**Calibrated Table**
Brasília–Viçosa -411.432	-411.296 ± 0.058	-411.425 ± 0.058	411.417 ± 0.008	411.417 ± 0.008	-411.405 ± 0.062	-411.400 ± 0.062
Discrepancy	-0.136	-0.007	0.015	0.015	-0.027	-0.032
Viçosa–Vassouras -177.351	-177.300 ± 0.055	-177.366 ± 0.055	-177.396 ± 0.046	-177.377 ± 0.046	-177.437 ± 0.019	-177.442 ± 0.019
Discrepancy	-0.051	0.015	0.045	0.026	0.086	-0.091
Vassouras–Vinhedo 73.803	73.827 ±0.032	73.854 ±0.032	73.867 ± 0.037	73.847 ± 0.037	73.845 ± 0.013	73.850 ± 0.013
Discrepancy	-0.024	-0.051	-0.064	-0.044	-0.042	-0.047
Vinhedo–Curitiba -196.609	-196.520 ± 0.012	-196.605 ± 0.012	-196.689 ± 0.076	-196.623 ± 0.076	-196.602 ± 0.022	-196.603 ± 0.022
Discrepancy	-0.089	-0.004	0.080	0.014	-0.007	-0.006
Curitiba–Sta. Maria -501.249	-501.035 ± 0.011	-501.253 ± 0.011	-501.382 ± 0.007	-501.241 ± 0.007	-501.201 ± 0.010	-501.234 ± 0.010
Discrepancy	0.214	0.00 4	0.133	-0.008	-0.048	-0.015

Table 3 – Gravity intervals calculated with the LC & R Table and with the Calibrated Table of 1999/2000. Figures are in mGal.

Known gravity interval	G257		G602		G622	
	LC & R Table	**Calibrated Table**	**LC & R Table**	**Calibrated Table**	**LC & R Table**	**Calibrated Table**
Vassouras–Vinhedo -73.803	-73.873 ± 0.016	-73.888 ± 0.016	-73.844 ± 0.018	-73.866 ± 0.016	-73.846 ± 0.017	-73.871 ± 0.016
Discrepancy	0.070	0.085	0.041	0.063	0.043	0.068
Vinhedo–Viçosa -103.548	-103.449 ± 0.023	-103.467 ± 0.022	-103.473 ± 0.014	-103.501 ± 0.012	-103.458 ± 0.008	-103.492 ± 0.006
Discrepancy	0.099	-0.081	0.075	-0.047	0.090	-0.056
Viçosa–Brasília -411.432	-411.341 ± 0.016	-411.439 ± 0.015	-411.317 ± 0.003	-411.437 ± 0.003	-411.368 ± 0.022	-411.439 ± 0.021
Discrepancy	-0.091	0.007	-0.115	0.005	-0.064	0.007
Vassouras–Curitiba 122.806	122.667 ± 0.028	122.680 ± 0.026	122.767 ± 0.022	122.800 ± 0.062	122.715 ± 0.009	122.765 ± 0.009
Discrepancy	0.139	0.126	0.039	0.006	0.091	0.041
Curitiba–Sta. Maria 501.249	501.188 ± 0.021	501.272 ± 0.022	501.117 ± 0.022	501.231 ± 0.057	500.996 ± 0.024	501.245 ± 0.022
Discrepancy	0.061	-0.023	0.132	0.041	0.253	0.004
Sta. Maria–Rivera 82.741	82.733 ± 0.012	82.737 ± 0.013	82.778 ± 0.004	82.786 ± 0.033	82.730 ± 0.003	82.781 ± 0.003
Discrepancy	0.008	0.004	-0.037	-0.045	0.011	-0.040
Vinhedo–Curitiba 196.609	196.553 ± 0.059	196.580 ± 0.059	196.581 ± 0.049	196.636 ± 0.054	196.595 ± 0.033	196.669 ± 0.027
Discrepancy	0.056	0.029	0.028	-0.027	0.014	-0.060
Vassouras–Teresina -621.238	-620.831 ± 0.034	-620.967 ± 0.021	-620.907 ± 0.045	-621.085 ± 0.027	-621.114 ± 0.020	-621.244 ± 0.012
Discrepancy	0.407	-0.271	0.331	-0.153	0.124	0.006