Impedance-based indicator for elastic parameters prediction

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Abstract

In the last two decades, many approximations for the P-P reflection coefficient have been proposed in the literature. Almost derived from the cassical aproximation of Aki & Richards, using additional assumptions on the medium parameters. The aim of constructing such aproximations is to establish reliable attributes that can be capable to indicate the presence of oil or gas in rocks. In this work we introduce a new indicator, based on an impedance-type approximation for the reflection coefficent. Such indicator also provides the Lamé parameter λ without invertion.

Introduction

The variation of amplitude with offset (AVO) is a powerful tool to discriminate rocks containing gas and oil. Several approximation of the P–P reflection coefficient (R) have been proposed and different AVO indicators were extracted from them. However, there is no agreement about which is the best attribute and in which situation it would be better applied. The starting point of almost all the approaches is the classic approximation of Aki & Richards (1980), which is based on a weak constrat in the media parameters and small angle of incidence. Recently, impedance-type approximations for the reflection coefficient have been introduced (Connolly, 1999 and Santos & Tygel, 2004). Based on this kind of approximation we introduce a new indicator. Numerical examples demonstrate the ability of this attribute to discriminates gas and oil in sands. Also, we wrote the reflectivity ($\mathcal{R}$) as a function of Lamé parameters (λ and μ) and density. For a especific angle of incidence, $\mathcal{R}$ is equal to the λ reflectivity, so is possible to extract λ directly from this new paremeter.

Aproximation for $\mathcal{R}$ and the associated seismic attributes

Let us consider two semi-infinite isotropic homogeneous elastic media in contact at a plane interface. Each medium has a P-wave velocity α, a S-wave velocity β and a density ρ. Further, let us consider an incident compressional plane wave impinging upon this interface. The reflection coefficient R for a compressional reflected wave has an exact expression knowing as Zooeptriz-Knott formula. This formula is very hard to handle and it is difficult to extract the physical sense of their terms.

For a small contrast between the properties of the two media and a small angle of incidence, the well known linear approximation of Aki & Richards (1980) is given by

$$R \approx \frac{1}{2}\left[1 - 4\frac{\beta^2}{\alpha^2}\sin^2\theta\right]\frac{\Delta\rho}{\rho} + \frac{\sec^2\theta}{2}\frac{\Delta\alpha}{\alpha} - 4\frac{\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\beta}\,, \tag{1}$$

where θ is the incident angle and $u = (u_2 + u_1)/2$, $\Delta u = u_2 - u_1$ for $u = \alpha$, β, and ρ.

Shuey (1985) rewrote the expression equation (1) as a function of θ,

$$R \approx A + B\sin^2\theta + C[\tan^2\theta - \sin^2\theta], \tag{2}$$

where the parameters A (Intercept), B (Gradient) and C are given by

$$A = \frac{1}{2}\left[\frac{\Delta\rho}{\rho} + \frac{\Delta\alpha}{\alpha}\right], \quad B = \frac{1}{2}\frac{\Delta\alpha}{\alpha} - 2\frac{\beta^2}{\alpha^2}\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta\beta}{\beta}\right],$$

$$\text{and} \quad C = \frac{1}{2}\frac{\Delta\alpha}{\alpha}\,. \tag{3}$$

Shuey had shown that for incidence angles smaller than 30°, $\tan^2\theta \approx \sin^2\theta$ and then, equation (2) turns to be

$$R \approx A + B\sin^2\theta\,. \tag{4}$$

Equation (4) is the most popular AVO formula. Castagna &Smith (1994) presented a large study using A and B, $A \times B$ and $(A+B)/2$ as a AVO indicators. In that work they have shown that the difference between the normal incidence P-P and S-S reflection coefficients can be well approximated by $(A+B)/2$. Moreover, it is also a robust indicator for clastic section to separate brine sands to gas sands, as shown in Figure 1 (left). However, for some of their suite of 25 measurements of the Gulf of Mexico and Gulf Coast, this indicator failed.

Smith and Gidlow (1987) used Gardner's relationship, $\rho = \alpha^{1/4}$ (Gardner et al,1974), for water-saturated rocks to obtain other approximation for R

$$R \approx \left[\frac{5}{8} - \frac{1}{2}\frac{\beta^2}{\alpha^2}\sin^2\theta + \frac{1}{2}\tan^2\theta\right]\frac{\Delta\alpha}{\alpha} - 4\frac{\beta^2}{\alpha^2}\frac{\Delta\beta}{\beta}. \tag{5}$$

Using the mudrock line of Castagna et al (1985),

$$\alpha = 1.36 + 1.16\,\beta \quad \text{(in km/s)}\,, \tag{6}$$

which relates the P- and S-wave velocities for water-saturated sandstones, siltstones and shales, Smith and Gidlow (1987) defined the "fluid factor" indicator ΔF as

$$\Delta F = \frac{\Delta\alpha}{\alpha} - 1.16\frac{\beta}{\alpha}\frac{\Delta\beta}{\beta}\,, \tag{7}$$

where the α and β contrast can be estimated from equation (5). The second term in the fluid factor is the value of $\Delta\alpha/\alpha$ predicted from $\Delta\beta/\beta$ using the mudrock line. So ΔF will be close to zero for water-bearing and shales rocks and nonzero for other type of rocks or fillings. Figure 1 (right) shows the behavior of the fluid factor for the same types of interfaces and the same data used previously.

Following the simple cases of normal incidence in elastic media and general oblique incidence in acoustic media, two new approaches had appeared recently in the literature (Connolly, 1999, Santos & Tygel,2004). The idea is to write the reflection coefficient as a function of a "angular" impedance,

$$R \approx \frac{I_2 - I_1}{I_2 + I_1}, \tag{8}$$

where I_1 refers to the incident side and I_2 to the transmission side.

Connolly (1999) introduced the *elastic* impedance, $I = EI$,

$$EI = N_0\, \alpha^{\sec^2\theta}\, \beta^{-8K\sin^2\theta} \rho^{1-4K\sin^2\theta}, \tag{9}$$

where $K = \beta^2/\alpha^2$ is assumed constant, and N_0 is a normalization constant (Whitcombe, 2002). In the derivation of EI, the angle θ was considered constant in both side of the interface, which is not true in a physical sense.

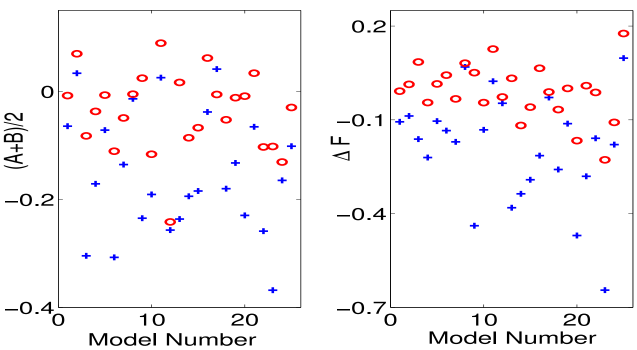

Figure 1. Separation between shale over gassand (+) and shale over brinesand(o). Left: Average of slope(A) and gradient (B). Right: Fluid Factor.

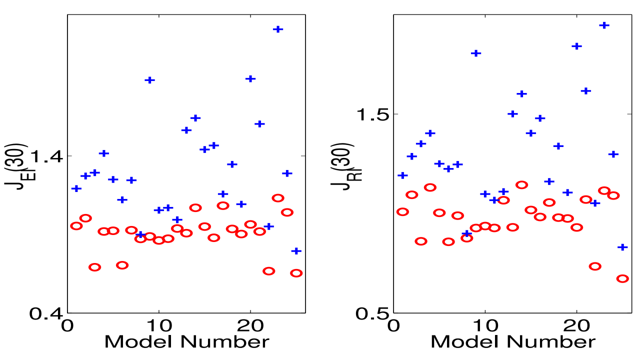

Figure 2. Separation between shale over gassand (+) and shale over brinesand(o). Left: J fator using elastic impedance. Right: J fator using reflection impedance.

Santos & Tygel (2004) has shown that no exact closed-form solution for equation (8) exists. However, under suitable restrictions in the medium parameters (e.g., $\rho = b\beta^{\gamma}$), they introduce the *reflection* impedance, $I = RI$,

$$RI = M_0\, \frac{\rho\alpha}{\sqrt{1-\alpha^2p^2}} \exp\{-4p^2[\beta^2 + f(\beta)]\}\ , \tag{10}$$

where M_0 is a normalization constant, p is the ray parameter, and f is a function, wich is obtained considering a functional relation between ρ and β. For the case $\rho = \beta^{\gamma}$, $\frac{\gamma}{2}\beta^2$. It is important to note that in the derivation of RI, p was considered constant on both sides of the reflector,which follows the actual phisics of the ray.

For any choice of the impedance, $I = EI$ or $I = RI$, from equation(8), it is possible to define a new atribute J

$$J = \frac{I_1}{I_2} \approx \frac{1-R}{1+R}. \tag{11}$$

Clearly, such indicator depends on the angle of incidence. Figure 2 shows the behavior of J for elastic and reflection impedance, taking $\theta = 30°$. We can observe that this new attribute separates well gas sand from brine sand for almost the 25 models presented.

Extracting λ from R

Fundamental rock properties such as Lamé parameter or module of Poisson are better understood than velocities or impedances, so is desirable to get it from the data. In that direction, Whitcombe et al (2002) define a new function called extended elastic imepdance. From this function they obtain λ, μ and κ, assuming the ratio β^2/α^2 and A/C in equations (1) and (2) are constant .

Our work follows this idea to obtain the Lamé parameters but trying to be a little more general. To obtain λ, directly we rewrote the equation(1) in function of λ, μ, ρ and θ.

$$\mathcal{R} = \left[1 - \frac{1}{2}\sec^2\theta\right]\frac{1}{2}\frac{\Delta\rho}{\rho} + \frac{(1-2K)}{2}(\sec^2\theta)\frac{1}{2}\frac{\Delta\lambda}{\lambda} + + \frac{K}{1-K}(\sec^2\theta - 4\sin^2\theta)\frac{1}{2}\frac{\Delta\mu}{\mu}. \tag{12}$$

where $K = (\beta/\alpha)^2$.

We can observe that if we fixed the incidence angle θ in $45°$ before solving, equation (12) remains

$$R = (1-2K)\frac{1}{2}\frac{\Delta\lambda}{\lambda}. \tag{13}$$

Like R is approximately $\Delta I/2I$ (Santos & Tygel, 2004) and considering K constant we can combine equation (11) and (13) to get

$$J(45°) \approx \left(\frac{\lambda_1}{\lambda_2}\right)^{1-2K} = r_\lambda \tag{14}$$

Thus, sorting the data for angles and searching the section of $\theta = 45°$ we can obtain λ carrying just one invertion for J.

Numerical Experiments

To test the validity of our approximation (equation 14) we used synthetic well logs (Figura 3). The correlation between the elastic parameter(r_λ) and $J(45)$ was ≥ 0.98 for the whole well. The two curves are almost indistinguishable (Figure 4). This high correlation can be seen in detail in Figure 5.

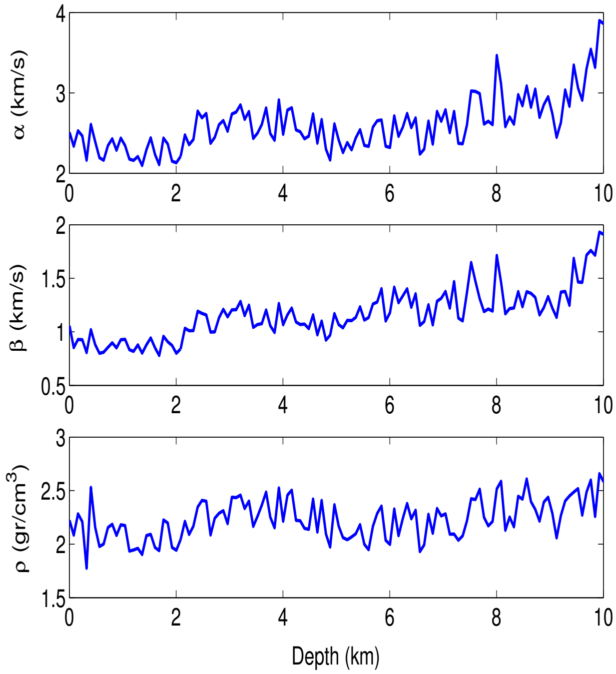

Figure 3. Synthetic well logs.

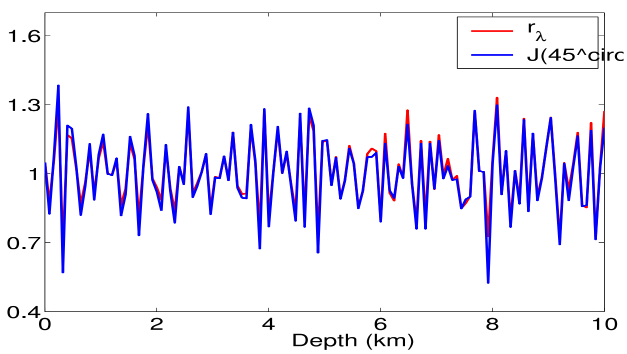

Figure 4. Comparison of a $(\lambda_1/\lambda_2)^{1-2k}$ curve with a $J(45^\circ)$ curve for the entire synthetic well log.

Using the two different approaches for J, the elastic or the reflection impedance, we will also have two different approaches for r_λ. In Figure 6 we can see that J_{RI} fits better r_λ.

In our deduction we consider that K is constant but this is not true, however $J(45^\circ)$ was a good aproximation for r_λ. To test how the value of K (left constant) affect the good correlation between J and r_λ we disturbed the well log data (multiplying α or β for a constant). We computed the correlation for $\theta = 45$, the resalt is in Table 1. We can observe that the correlation remanins high. However, in same cases, the highest correlation happened for other angles. We are investigating which was the cause.

	K	corr
2α	0.05	0.99
1.2α	0.15	0.99
α	0.22	0.98
0.9α	0.24	0.97
0.95α	0.27	0.94
0.8α	0.34	0.85
1.2β	0.32	0.92
1.5β	0.49	0.85

Table 1. In this table we can see the correlation between $J(45^\circ)$ and r_λ^{1-2K}, for different K. The value of correlation is quite high.

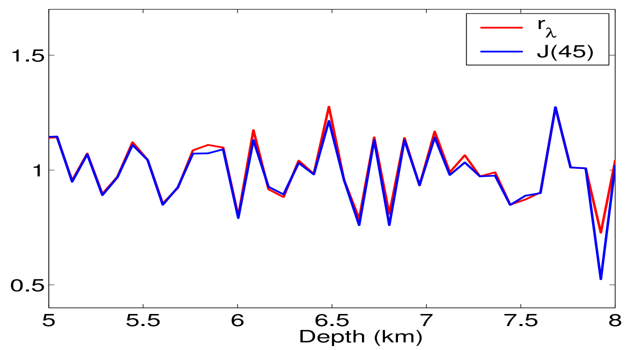

Figure 5. A zoom of Figure 4.

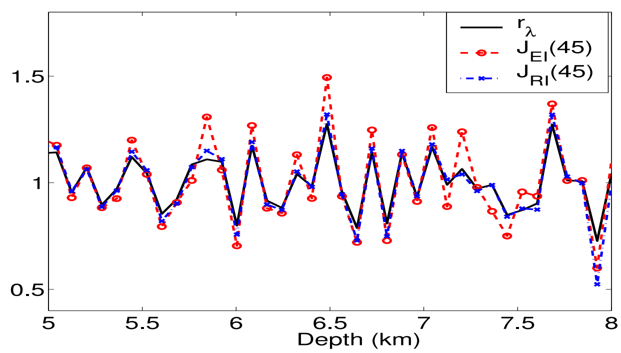

Figure 6. Comparison of a $(\lambda_1/\lambda_2)^{1-2k}$ curve with a $J_{EI}(45^\circ)$ curve and a $J_{RI}(45^\circ)$ curve.

Now we will introduced an experimental relationship between the Poisson relation, that it is known as a fluid indicator, and a kind of near-far relation with J_RI. From Figure 7 we observe that $J_RI(40^\circ)/J_RI(5^\circ)$ fits $r_\sigma = \sigma 1/\sigma_2$ quite well for the whole well. The correlation between both was 0.94. A detail is observed in Figure 8.

Conclusions

We presented a new seismic parameter J that is obtained directly from the data through the reflection coefficient. It allows to separate gas from brine sand for one interface. Furthermore, it is possible to extract the λ parameter and the Poisson relationship from it. We hope to be able to obtain other parameters such as compressibility or shear rigidity to get a complete set physical parameters.

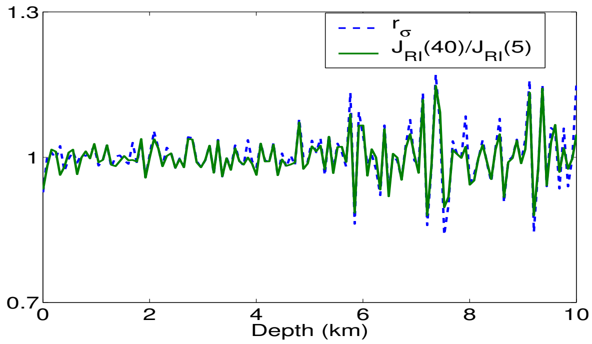

Figure 7. Comparison of a σ_1/σ_2 curve with $J_{RI}(40^\circ)/J_{RI}(5^\circ)$ curve.

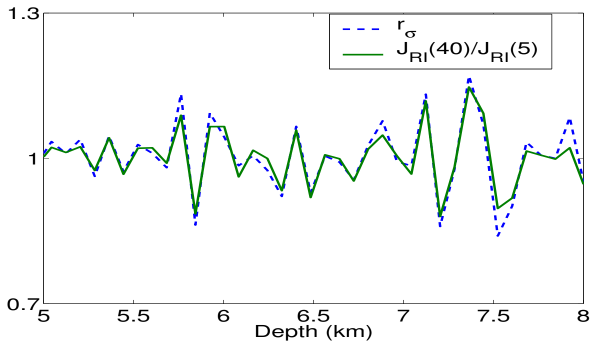

Figure 8. A zoom of Figure 7.

Acknowledgements

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