Semi-analytical logs of electrical focusing tools in basic subsurface models

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Abstract

This work presents a procedure to obtain synthetic logs for the electrical tools as LLS, LLD and LL7 in subsurface models: plane-parallel layers and cylindrical shells. The synthetic focusing used to acquire semi-analytical logs is based on the superposition principle applied to all transmitter-receiver pair of electrodes. Forward models can be created for investigations of real electrical resistivity data trough inversion procedures. Also, correction charts can be constructed for these tools or any other similar configuration.

Introduction

Following Maute (1992) the first electrical profile was accomplished by Doll in 1927 and still alive nowadays in petroleum industry and water research. Basic electroresistivity tool, known by normal an lateral arrangement, was extensively used and studied as the years.

Dool (1951) proposed a new focused probe, LL7 (laterolog 7), smaller and lighter than its predecessor LL3 (laterolog 3) composed by guard electrodes. Dool exemplified trought real experiments the superiority of LL7 tool over the normal and lateral tools (without focusing). The automatic focusing technique was used for development of new tool as DLL (dual laterolog), composed by nine electrodes, allowing simultaneous measurement of shallow and deep radial investigation of the formation apparent resistivity in borehole environment.

Numerical investigations as finite difference method (FDM) and finite element method (FEM) were performed in evaluation of a tool response for normal, lateral, LL7 and DLL, for study of different influences generate by mud, borehole diameter, invades zones and shoulder effects. Such study can be found in Towle et al. (1998), Dutta (1997), Roy and Dutta (1997), Liu et al. (1999), Cozzolino (2004).

Short theory

The normal arrangement is illustrated in Figure 1a and is composed by four electrodes, being two on the body sonde: current electrode A and potential electrode M. A current of intensity I_o is emitted from A returning to the electrode B, at the surface (or on cable, more close to the sonde body). The potential difference between M and N (ΔV_{MN}) is registered and converted into apparent resistivity formation by

$$R_N = K_N \frac{\Delta V_{MN}}{I_o}, \tag{1}$$

where $K_N = 4\pi\overline{AM}$ is the tool constant and $\overline{AM}$ is the transmitter-receiver spacing (Ellis, 1987). For practical studies the electrode N is assumed to be at the electrical infinity and so $\Delta V_{MN}=V_M$.

The lateral arrangement is illustrated in Figure 1b and is composed three electrodes on body sonde: one of current (A) and two of potentials (M and N). The potential difference ΔV_{MN} is measured and converted into apparent resistivity by

$$R_N = K_L \frac{\Delta V_{MN}}{I_o}, \tag{2}$$

where $K_L = 4\pi\overline{AM}\,\overline{AN}/\overline{MN}$ is the tool constant and $\overline{MN}$ is the distance between the potential electrodes.

The DLL focused device is illustrated on Figure 1c and has five current electrodes (A_1 to A_5) plus four potentials electrodes (M_1 to M_4). The electronic operation used in LLD (left) assure that the potential difference between M_1 and M_2, as well as between M_3 and M_4, goes to zero. This is obtained by applying different currents at A_1, A_2, A_4 and A_5 that varies continuously. This procedure forces the central current I_o emitted by A_3 into surrounding formation in a disc of height O_1O_2, approximately. With the value of the potential at M_1 is possible to obtain apparent resistivity R_{LLD}.

The apparatus LLS (Figure 1c, right) works in a different way. The electrodes A_2, A_3 and A_4 emits different currents that return to the electrodes A_1 and A_5, witches have an inverse polarity. When the potential difference between M_1 and M_2, as like as, M_3 and M_4, are zero the potential at M_1 is measured and converted to apparent resistivity R_{LLS}.

With an extension of the relationship proposed by Roy and Dutta (1997) to LL7 device is possible to obtain the apparent resistivity of DLL tool (Cozzolino, 2004) by

$$R_{LL(D,S)} = K_{LL(D,S)} \frac{V_{M1}}{I_o}, \tag{3}$$

with the tool constant defined as

$$K_{LL(D,S)} = \frac{4\pi}{\left(\frac{\alpha}{\overline{A_1M_1}} + \frac{\beta}{\overline{A_2M_1}} + \frac{1}{\overline{A_3M_1}} + \frac{\gamma}{\overline{A_4M_1}} + \frac{\lambda}{\overline{A_5M_1}}\right)},$$

where $\overline{A_iM_1}$ represents the distance between the current electrode I and the potential electrode M_1, $\beta = I_2/I_o$ and

$\gamma = I_4/I_o$. The current emitted by the central electrode A_3 is I_o and the potential registered on the electrode M_1. The α and λ account extra focalization current and are defined as

$$\begin{aligned} LLD:&\quad \alpha = \beta + c \quad , \quad \lambda = \gamma + c \\ LLS:&\quad \alpha = -(\beta + c), \quad \lambda = -(\gamma + c) \end{aligned} \quad . \qquad (4)$$

In this work the value c=1.75 was fixed for LLD, representing an additional of current emitted by the electrodes A_1 and A_5 with respect to the adjacent electrodes A_2 and A_4, respectively. For LLS the value c=0.5 was fixed, implying in an additional return of current (0.5 I_o) for each of one electrode A_1 and A_5.

Synthetic focusing of DLL

In heterogeneous media is necessary impress different focalization currents I_1, I_2, I_4 and I_5 to ensure the nullity of potential differences $\Delta V_{21}= V_{2,T}-V_{1,T}$ and $\Delta V_{34}= V_{3,T}-V_{4,T}$ where the subscript T denote total.

With an experimental work Shattuck et al. (1987) suggest a synthetic focusing for LLS device ia a laboratory scale. This technique requires a stopped tool in each depth until that all transmitter electrodes have been activated and the respective potentials measured.

Although this procedure is far away from the real operation of the focused tool it is useful to the numeric calculations proposed in this work. Using the idea proposed by authors, but with some modifications, it is possible to set out the total potentials in each electrode M_i of the DLL tool using the superposition principle, e.g.

$$\begin{aligned} V_{1T} &= \alpha V_{11} + \beta V_{12} + V_{13} + \gamma V_{13} + \lambda V_{14} \\ V_{2T} &= \alpha V_{21} + \beta V_{22} + V_{23} + \gamma V_{23} + \lambda V_{24} \\ V_{3T} &= \alpha V_{31} + \beta V_{32} + V_{33} + \gamma V_{33} + \lambda V_{34} \\ V_{4T} &= \alpha V_{41} + \beta V_{42} + V_{43} + \gamma V_{43} + \lambda V_{44} \end{aligned}$$

(5) where the potential V_{ij} is measured on electrode M_i due to current intensity I_o acting on electrode A_j with other sources disable. After some manipulation is possible to obtain the following system of equations

$$\begin{bmatrix} (V_{22}-V_{12}) \pm (V_{21}-V_{11}) & (V_{24}-V_{14}) \pm (V_{25}-V_{15}) \\ (V_{32}-V_{42}) \pm (V_{31}-V_{41}) & (V_{34}-V_{44}) \pm (V_{35}-V_{45}) \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} = -\begin{bmatrix} (V_{23}-V_{13}) \pm c\{(V_{21}-V_{11}) + (V_{25}-V_{15})\} \\ (V_{33}-V_{43}) \pm c\{(V_{31}-V_{41}) + (V_{35}-V_{45})\} \end{bmatrix}, \qquad (6)$$

that was written in a compact form with the aid of both relationship in Eq. (4) being plus and minus signal related to the LLD and LLS operations, respectively. Assuming a system of the type Ax=b, the solution is

$$\begin{aligned} \beta &= (A_{22}b_1 - A_{12}b_2)/\Delta \\ \gamma &= (A_{11}b_2 - A_{12}b_1)/\Delta \end{aligned} , \qquad (7)$$

where $\Delta=A_{11}A_{22}-A_{12}A_{21}$. This procedure guarantees the equality of the potential, e.g., $V_{1,T}=V_{2,T}$ and $V_{3,T}=V_{4,T}$. Just in a few situations β and γ will be the same, as like as the potentials at M_1 to M_4: one in a homogeneous media, and other in a media composed by cylindrical shells. To outline this asymmetry the apparent resistivities will be calculated through the average potential registered at electrodes M_1 and M_4, e.g.

$$R_{LL(D,S)} = K_{LL(D,S)} \frac{V_{M1} + V_{M4}}{2I_o} \qquad (8)$$

instead of the expression proposed previously (Eq. 2). Supposing the tool immersed in a homogeneous medium, with some calculations here omitted, is possible to obtain $\beta = \gamma = 1.0133$, $\alpha = \lambda = 2.763$ and a tool constant K_{LLD}=0.999m $\approx$ 1m, with the following group of spacing among electrodes

$$A_1\, 0.89\, A_2\, 0.2\, M_1\, 0.2\, M_2\, 0.2\, A_3\, 0.2\, M_3\, 0.2\, M_4\, 0.2\, A_4\, 0.89\, A_5$$

with all values expressed in meters (Drahos, 1984).

To the LLS tool inside a homogeneous medium we obtain $\beta = \gamma$ =1.1702, $\alpha = \beta$ = -1.6702 and a tool constant given by K_{LLS} = 1.7686m.

Must be observed that if c is null the above calculations provide the focusing of LL7 tool with three current electrodes and four potential electrodes (Doll, 1951).

Semi-analytical logs in cylindrical layered model

Cylindrical composed media are of interest for investigation of the influence from borehole and invasion on logs (say, apparent resistivity). The expression for electrical potential due to a dc point source for this models was given by Drahos (1984),

$$V(r,z) = \frac{I_0 \rho_j}{4\pi} \left\{ \frac{1}{z} + \int_0^\infty \left[C_i e^{\lambda z} + D_i e^{-\lambda z} \right] J_0(\lambda r) d\lambda \right\} \qquad (9)$$

where the coefficients C_i and D_i must be obtained recursively for a source in layer *j* and potential electrode in layer *i*. The J_0 is the zeroth order Bessel function of first kind.

With a model composed by two cylinders of different resistivities R_m (mud) and R_t (formation) is possible to construct a borehole correction chart for DLL sonde. Figure 2 shows the results for borehole with 0.1m and 0.20m of radius. So the corrected log R_{COR} is obtained with the knowledge of the ratios R_{LLD}/R_m and R_{LLS}/R_m.

Invaded models can be approximated by a sequence of cylindrical layers, with different saturations (or resistivities), for inhomogenneous radial profiles. To exemplify the effect of invaded zones on apparent resistivity a single piston-like displacement was used: this model assumes trhee properties (R_m, R_i and R_t) for borehole, invaded and uninvaded zones. Some results are presented in Figure 3.

Logs in horizontally layered model

The synthetic normal and lateral resistivity logs of buried electrode in layered earth model was described by Daniels (1978). The potential on z axis and radial coordinate $r \le r_{bh}$ (inside borehole) is obtained by

$$V(r,z) = \frac{IR_m}{2\pi^2} \int_0^\infty \left[K_0(\lambda r) + C_1(\lambda) I_0(\lambda r)\right] \cos(\lambda z) d\lambda \,, \tag{10}$$

where the coefficient C_1 is obtained recursively, R_m is the mud resistivity and K_0 and I_0 are and zeroth order of the modiffied Bessel function of first and second kind, respectivelly.

To ilustrate the synthetic DLL tool response, in a case of buried electrodes, the simple Oklahoma resistivity model was used with results shown in Figures 4a and 4b. The the main observation is the shoulder effect on LLD apparent resistivities if compared against LLS ones.

The shoulder effect are better visualized in Figure 5 where was used a single layer between two semi-spaces with different resistivities. The differences in apparent resistivities due to shoulder effect are present in both cases for resistive and conductive internal layer.

Conclusions

The synthetic focusing thecnique was sucecciful applied to acquire synthetic logs in simple cases of infinite cylindrical shells and horizontally layered beds. Some interesting situations were present as correction charts for borehole effect.

Despite the one dimensional applications here presented it is possible to make extensions to other semianalytical studies: exponential and linear resistivity dependence on the coordinate r (radial) or z (axial).

Case studies can be presented for beginer students and forward models used for both inversion tests and mesh refinements for optimizations of finite element and finite difference methods.

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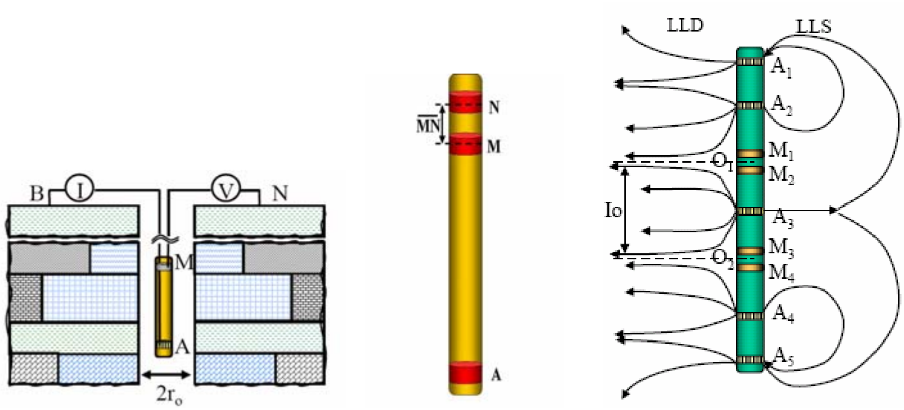

(a) (b) (c)

Figure 1: (a) scheme of the normal tool and (b) scheme of the lateral electrode configuration (c) DLL tool with five current electrodes $A_1 \ldots A_5$ and four potential electrodes $M_1 \ldots M_4$ (arrows indicate current path).

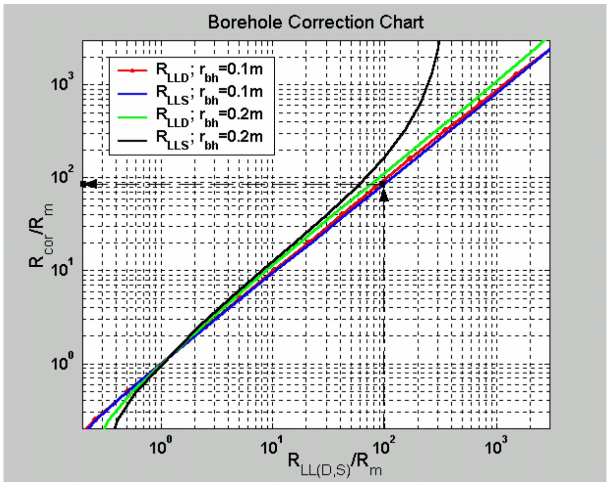

Figure 2: Curves for correction of apparent resistivity caused by different mud resistivity and borehole radius.

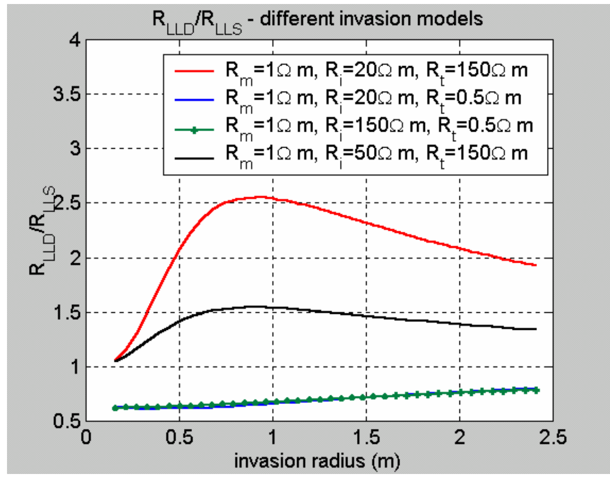

Figure 3: influence of different invasion radius on ratio R_{LLD}/R_{LLS}. Four different models were used (see legend) to show the differences when the invasion zone are more resistive than uninvaded zone.

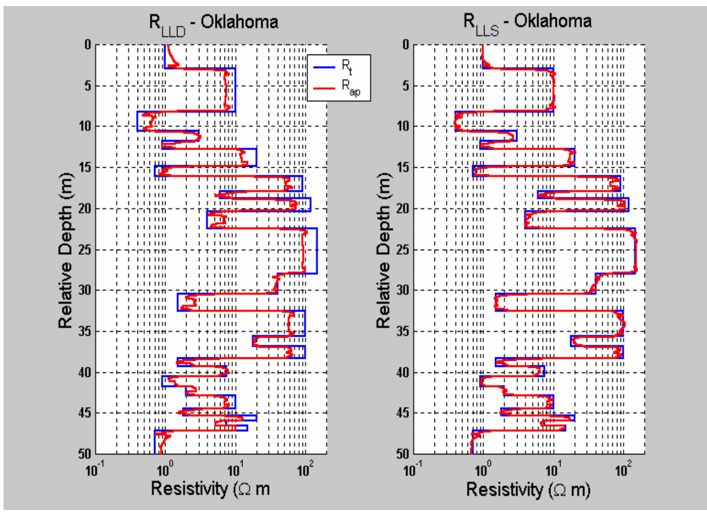

Figure 4: (a) Synthetic LLD log and (b) LLS log for Oklahoma model without borehole and invasion.

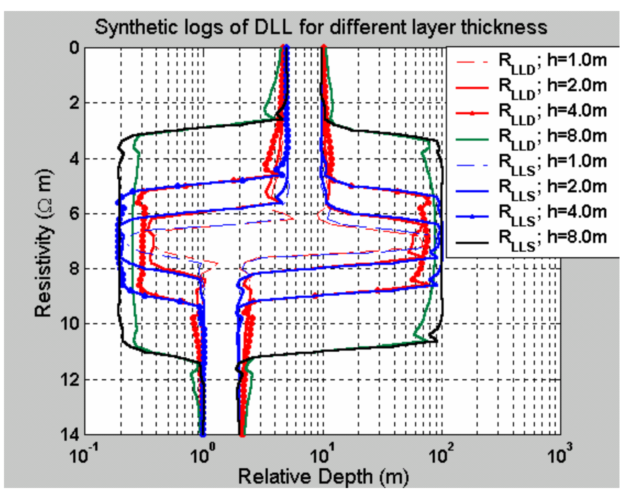

Figure 5: influence of shoulder resistivity and internal layer thickness on $R_{\_LLD}$ and R_{LLS} synthetic logs.